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MATHEMATICAL MODEL OF MOVEMENT OF A PULSING LAYER OF VISCOUS LIQUID IN THE CHANNEL WITH AN ELASTIC WALL RESTING ON WINKLER FOUNDATION

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ABSTRACT

The problem of hydroelasticity of the plate resting on Winkler foundation and forming at wall of the slot-hole channel with a pulsing layer of viscous incompressible liquid at the set harmonious law of a pulsation of pressure at its end face in flat statement is put and analytically solved. The set regional task represents nonlinear related system of the equations of Navier-Stokes for a layer of viscous incompressible liquid and the equation of dynamics of a plate (beam strip). Conditions of sticking of liquid act as regional conditions to impenetrable walls of the channel, a condition of the free expiration of liquid at end faces of the channel and a condition of a hinged supporting of a plate wall of the channel. The complex of dimensionless variables of a considered task is created and small parameters of a task are allocated. As small parameters the relative thickness of a layer of liquid and relative amplitude of a deflection of a plate are chosen. Considering asymptotic decomposition in the allocated small parameters of a task its linearization by a method of indignations is carried out. The solution of the linearized task is passed by a method of the set forms for a mode of the established harmonic oscillations. Thus, proceeding from boundary conditions for a channel plate wall, the form of its deflection is set in the form of ranks on trigonometrical functions from longitudinal coordinate. The law of a deflection of an elastic wall of the channel and distribution of hydrodynamic parameters are found in liquid. Dependent on frequency functions of distribution of amplitudes of a deflection and dynamic pressure along the channel and dependent on frequency functions of distribution of phase shift of a deflection of a wall and pressure in the channel of rather initial indignation at an end face are received. On the basis of calculations it is shown that resonant fluctuations of an elastic wall of the channel, pressure excited by insignificant pulsations at its end face, can cause essential changes of dynamic pressure and be the main reason of vibration cavitation in liquid.

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When studying a wide range of problems associated with the normal functioning of cooling and fuel supply systems in aerospace system engines, as well as in power generation systems and hydraulic drives of complex mechanical systems used in aviation and space industries [1-3] we have to face the problem of the motion of a liquid layer in a flat channel. In most cases, the researchers only study stationary liquid motion in the channel generated by absolutely solid walls. The authors of the works [4, 5] analyze motions of a layer of viscous incompressible liquid in the channel with absolutely solid walls within the framework of the theory of hydrodynamic lubrication. At the same time, studies of causes and conditions of cavitation in liquid in the channel are relevant. The works [6, 7], for instance, report on the study of cavitation in the coolant fluid of internal-combustion engines. The work [6] experimentally proves that the main reason of vibration cavitation is the elastic flexibility of the channel walls, and considers free fluctuations of a channel wall as free fluctuations of the cylindrical shell without taking into account its interaction with the liquid. In the work [7], study of reasons of cavitation is reviewed based on solving problems of interaction of an elastic beam-strip with the perfect liquid. However, consideration of viscosity of the liquid appears to be important, because it determines damping properties in this vibration system and limitation of vibration amplitudes of the channel wall at the resonance frequency. The work [8] reports on the study of fluid-elastic vibrations of the beam in a viscous liquid flow with respect to piezoelectric converters, and the authors of the work [9] studied a plane two-phase flow of a viscous fluid in a channel of variable section with solid walls by taking into account cavitation effects.

On the other hand, the work [10] is devoted to interaction of vibrating disks with a layer of viscous incompressible fluid between them, including the case when one of the disks has a form of a round elastic plate. It is shown that at resonance frequencies of channel wall fluctuations the liquid pressure amplitude can vary by several orders of magnitude and become significantly less than the saturated vapor pressure. A similar study for plates in two-dimensional formulation was carried out in the work [11], and the authors of the work [12] carried out a similar study for round three-layer plates. The work [13] deals with vibration of a round plate on the free surface of a perfect incompressible liquid. In this case, the authors examine an area in the liquid restricted by a rigid bottom and a cylindrical surface. The authors of the work [14] studied fluctuations of a round plate immersed in the water with the free surface. In the work [15], an integrodifferential equation for small transverse fluctuations of a straight elastic pipeline with the perfect incompressible fluid is obtained and examined. The pipeline fluctuations are described in a linear formulation in the beam approximation.

In order to study reasons of vibration cavitation in liquid, the authors of this work have solved the problem of nonstationary motion of a pulsating layer of viscous liquid in a flat channel with an elastic wall taking into account specifics of the expiration at end faces.

Let us take a look on the channel schematically shown in Fig. 1.

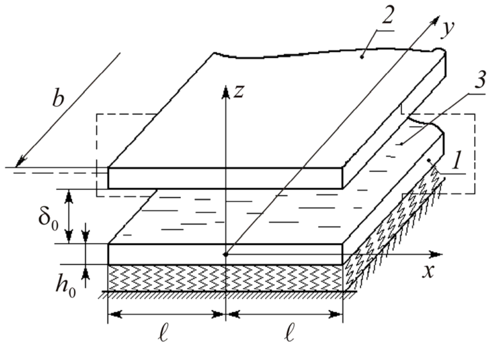

Fig. 1. Schematic view of a flat channel with elastic walls

The channel consists of two parallel impermeable walls 1 and 2 of the same geometric dimensions, between these walls a pulsating layer of viscous incompressible liquid moves according to a specified law of pressure change at the end faces. The wall 2 is considered to be absolutely solid, and the wall 1 is an elastic plate on the Winkler base with the thickness of h_0 with hinge support at the end faces. The geometric size of the channel 2ℓ is considerably smaller than its size b, and the thickness of the liquid layer (the distance between the walls) in the nonperturbed state δ_0 is much less than 2ℓ. Pressure pulsation causes bending vibrations of the wall 1, while the amplitude of its elastic displacements is much smaller than δ_0.

At end faces of the channel the outflow in the cavities filled with the same liquid can be considered as jet-like. For definiteness sake let us assume that pressure in the left cavity is constant p_0, and pressure in the right cavity has a constant component and a component that changes harmonically over time $p_0 + p_1^+(\omega t)$. The law of pressure change at the right end face is formulated as follows:

$$p_1^+ = p_{1m}^+ f_p(\omega t), \quad f_p(\omega t) = \exp(i\omega t), \qquad (1)$$

where p_{1m}^+ is the amplitude of pressure pulsations at the end faces of the channel; ω is the frequency of pulsations; $f_p(\omega t)$ is the law of pressure change.

Let us introduce into consideration the Cartesian coordinate system x, y, z connected with an absolutely solid wall 1 and shifted downwards by δ_0. Given that $2\ell << b$, let us consider the channel to be unbounded in the direction of y axis and proceed to solving the two-dimensional problem. In this case, the equations of dynamics of viscous incompressible liquid in the channel are formulated as:

$$\frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_z \frac{\partial V_x}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial z^2}\right),$$

$$\frac{\partial V_z}{\partial t}+V_x\frac{\partial V_z}{\partial x}+V_z\frac{\partial V_z}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 V_z}{\partial x^2}+\frac{\partial^2 V_z}{\partial z^2}\right),$$

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_z}{\partial z}=0, \qquad (2)$$

where p is the pressure; ρ, v are density and kinematic coefficient of the liquid viscosity; V_x, V_z are projections of the liquid velocity on coordinate axes.

The equations of liquid dynamics are complemented with boundary conditions: with conditions of sticking of liquid to the walls of the channel [10, 11, 16]

$$V_x=0,\ V_z=0 \text{ at } z=\delta_0,\quad V_x=\frac{\partial u}{\partial t},\ V_z=\frac{\partial w}{\partial t} \text{ at } z=w, \quad (3)$$

and with conditions of its free expiration at end faces, that make liquid pressure at the end face of the channel correspond to the pressure in the cavity

$$p=p_0+p_1^+(\omega t) \text{ at } x=\ell,\quad p=p_0 \text{ at } x=-\ell. \qquad (4)$$

Where w is deflection of the wall 1; u is longitudinal displacement of the wall 1.

Given that the condition $2\ell<<b$ is valid for an elastic plate-wall of the channel, equation of its motion is equation of dynamics of the beam-strip on the Winkler base

$$D\frac{\partial^4 w}{\partial x^4}+\chi w+\rho_0 h_0\frac{\partial^2 w}{\partial t^2}=-q_{zz}, \qquad (5)$$

where $q_{zz}=-p+2\rho\nu\left(\partial V_z/\partial z\right)$ is the normal stress applied to the plate from the liquid side; $D=Eh_0^3/\left(12\left(1-\mu_0^2\right)\right)$ is the cylindrical rigidity of the plate, χ is the coefficient of rigidity of the base, h_0 is the thickness of the plate, ρ_0 is the density of the plate material, μ_0 is the Poisson's ratio of the plate material, E is the Young's modulus of the plate material.

Boundary conditions of equation (5) are conditions of hinge support at the end faces

$$w=\partial^2 w/\partial x^2=0 \text{ at } x=\pm\ell. \qquad (6)$$

Let us introduce into consideration dimensionless variables

$$\psi=\delta_0/\ell<<1,\ \lambda=w_m/\delta_0<<1,\ \mathrm{Re}=\delta_0^2\omega/\nu,\ \tau=\omega t,$$

$$\xi=x/\ell,\ \zeta=z/\delta_0,$$

$$V_z=w_m\omega U_\zeta,\ V_x=w_m\omega U_\xi/\psi,$$

$$p=p_0+w_m\rho\nu\omega\left(\delta_0\psi^2\right)^{-1}P,\ w=w_mW, \qquad (7)$$

$$u=u_mU,\ p^+=w_m\rho\nu\omega\left(\delta_0\psi^2\right)^{-1}P^+.$$

Where w_m is the amplitude of deflection of the plate; W is a dimensionless deflection of the plate; u_m is the amplitude of longitudinal displacement of the plate; U is dimensionless longitudinal displacement of the plate; ψ, λ, Re are parameters that characterize the problem.

By substituting dimensionless variables (7) in the problem (1)-(6) we can formulate the problem of fluid elasticity of a flat channel in a dimensionless form, which includes equations of dynamics of a viscous incompressible liquid

$$\mathrm{Re}\left[\frac{\partial U_\xi}{\partial\tau}+\lambda\left(U_\xi\frac{\partial U_\xi}{\partial\xi}+U_\zeta\frac{\partial U_\xi}{\partial\zeta}\right)\right]=$$

$$=-\frac{\partial P}{\partial\xi}+\psi^2\frac{\partial^2 U_\xi}{\partial\xi^2}+\frac{\partial^2 U_\xi}{\partial\zeta^2}, \qquad (8)$$

$$\psi^2\,\mathrm{Re}\left[\frac{\partial U_\zeta}{\partial\tau}+\lambda\left(U_\xi\frac{\partial U_\zeta}{\partial\xi}+U_\zeta\frac{\partial U_\zeta}{\partial\zeta}\right)\right]=$$

$$=-\frac{\partial P}{\partial\zeta}+\psi^2\left[\psi^2\frac{\partial^2 U_\zeta}{\partial\xi^2}+\frac{\partial^2 U_\zeta}{\partial\zeta^2}\right],$$

$$\frac{\partial U_\xi}{\partial\xi}+\frac{\partial U_\zeta}{\partial\zeta}=0,$$

and an equation of dynamics of an elastic wall of the channel

$$\frac{Dw_m}{\ell^4}\frac{\partial^4 W}{\partial\xi^4}+\chi w_mW+\rho_0h_0\omega^2w_m\frac{\partial^2 W}{\partial\tau^2}=$$

$$=p_0+\frac{w_m\rho\nu\omega}{\delta_0\psi^2}\left(P-2\psi^2\frac{\partial U_\zeta}{\partial\zeta}\right). \qquad (9)$$

Boundary conditions (3), (4), (6) are formulated as:

$$U_\xi=\psi\frac{u_m}{w_m}\frac{\partial U}{\partial\tau},\ U_\zeta=\frac{\partial W}{\partial\tau} \text{ at } \zeta=\lambda W,\quad U_\xi=0,\ U_\zeta=0$$

$$\text{at } \zeta=1, \qquad (10)$$

$$P=P^+(\tau) \text{ at } \xi=1,\quad P=0 \text{ at } \xi=-1, \qquad (11)$$

$$W=\partial^2W/\partial\xi^2=0 \text{ at } \xi=\pm1. \qquad (12)$$

In the suggested formulation ψ is a dimensionless small parameter, the ratio of u_m/w_m is of the order of unity and, consequently, in the zero approximation of ψ the equations (8), (9) and the boundary conditions (10) are simplified, i. e. we may use the terms of order ψ and ψ^2 that are equal to zero. Considering asymptotic decompositions on a small parameter $\lambda<<1$ [17] $P=P_0+\lambda P_1+...,\ U_\xi=$ $=U_{\xi0}+\lambda U_{\xi1}+..,U_\zeta=U_{\zeta0}+\lambda U_{\zeta1}+...$ and focusing only on the first term of decomposition, we formulate a linearized problem of fluid elasticity of a flat channel, which includes:

equations of dynamics of a liquid layer

$$\mathrm{Re}\frac{\partial U_{\xi0}}{\partial\tau}=-\frac{\partial P_0}{\partial\xi}+\frac{\partial^2 U_{\xi0}}{\partial\zeta^2},\ \frac{\partial P_0}{\partial\zeta}=0,\ \frac{\partial U_{\xi0}}{\partial\xi}+\frac{\partial U_{\zeta0}}{\partial\zeta}=0, \quad (13)$$

with boundary conditions

$$U_{\xi 0}=0,\ U_{\zeta 0}=\partial W/\partial\tau \ \text{at}\ \zeta=0,$$

$$U_{\xi 0}=0,\ U_{\zeta 0}=0 \ \text{при}\ \zeta=1, \tag{14}$$

$$P_0=P^{+}(\tau)\ \text{at}\ \xi=1,\quad P_0=0\ \text{at}\ \xi=-1, \tag{15}$$

equation of motion of a plate-wall of the channel

$$\frac{Dw_m}{\ell^4}\frac{\partial^4 W}{\partial\xi^4}+\chi w_m W+\rho_0 h_0\omega^2 w_m\frac{\partial^2 W}{\partial\tau^2}= = p_0+\frac{w_m\rho\nu\omega}{\delta_0\psi^2}P_0, \tag{16}$$

with boundary conditions

$$W=\partial^2 W/\partial\xi^2=0\ \text{at}\ \xi=\pm 1. \tag{17}$$

The solution of the system of equations (13) with boundary conditions (14), (15) by assuming that hydrodynamic parameters and deflection of the plate change harmonically over time, can be written as

$$U_{\xi 0}=\frac{1}{2\varepsilon^2}\left[\frac{\partial^2 P_0}{\partial\xi\partial\tau}+\frac{\partial^2 P_0}{\partial\xi\partial\tau}\bar{\Psi}(\zeta)+\frac{\partial P_0}{\partial\xi}\bar{\Phi}(\zeta)\right], \tag{18}$$

$$U_{\zeta 0}=\frac{1}{2\varepsilon^2}\left[\frac{\partial^3 P_0}{\partial\xi^2\partial\tau}(\bar{\Psi}_1(\zeta)-\zeta)+\frac{\partial^2 P_0}{\partial\xi^2}\bar{\Phi}_1(\zeta)\right]+\frac{\partial W}{\partial\tau},$$

$$P_0=\int_\xi^1\!\!\int\left(2\varepsilon^2\alpha\frac{\partial^2 W}{\partial\tau^2}+12\gamma\frac{\partial W}{\partial\tau}\right)d\xi d\xi+$$

$$+\frac{1}{2}(\xi-1)\int_{-1}^{1}\!\!\int\left(2\varepsilon^2\alpha\frac{\partial^2 W}{\partial\tau^2}+12\gamma\frac{\partial W}{\partial\tau}\right)d\xi d\xi+P_1^{+}/2+\xi P_1^{+}/2.$$

The following symbols are used

$$\varepsilon(\omega)=\sqrt{\mathrm{Re}/2},\ F_1(\varepsilon\zeta)=\mathrm{ch}\varepsilon\zeta\cos\varepsilon\zeta,$$

$$F_3(\varepsilon\zeta)=\frac{1}{2}\mathrm{sh}\varepsilon\zeta\sin\varepsilon\zeta,$$

$$F_2(\varepsilon\zeta)=\frac{1}{2}\left[\mathrm{ch}\varepsilon\zeta\sin\varepsilon\zeta+\mathrm{sh}\varepsilon\zeta\cos\varepsilon\zeta\right],$$

$$F_4(\varepsilon\zeta)=\frac{1}{4}\left[\mathrm{ch}\varepsilon\zeta\sin\varepsilon\zeta-\mathrm{sh}\varepsilon\zeta\cos\varepsilon\zeta\right],\quad D_1=\frac{\mathrm{sh}\varepsilon-\sin\varepsilon}{\cos\varepsilon+\mathrm{ch}\varepsilon},$$

$$D_2=\frac{\sin\varepsilon+\mathrm{sh}\varepsilon}{\cos\varepsilon+\mathrm{ch}\varepsilon},$$

$$\gamma(\omega)=\frac{1}{6}\frac{\varepsilon^3(\mathrm{sh}\varepsilon-\sin\varepsilon)}{\varepsilon^2(\mathrm{ch}\varepsilon+\cos\varepsilon)-2\varepsilon(\mathrm{sh}\varepsilon+\sin\varepsilon)+2(\mathrm{ch}\varepsilon-\cos\varepsilon)},$$

$$\alpha(\omega)=\frac{\varepsilon(\varepsilon(\mathrm{ch}\varepsilon+\cos\varepsilon)-(\mathrm{sh}\varepsilon+\sin\varepsilon))}{\varepsilon^2(\mathrm{ch}\varepsilon+\cos\varepsilon)-2\varepsilon(\mathrm{sh}\varepsilon+\sin\varepsilon)+2(\mathrm{ch}\varepsilon-\cos\varepsilon)},$$

$$\bar{\Psi}(\zeta)=F_2(\varepsilon\zeta)D_1-F_1(\varepsilon\zeta)-2F_4(\varepsilon\zeta)D_2,$$

$$\bar{\Phi}(\zeta)=2F_3(\varepsilon\zeta)-F_2(\varepsilon\zeta)D_2-2F_4(\varepsilon\zeta)D_1,$$

$$\bar{\Psi}_1(\zeta)=\int_\zeta^0\bar{\Psi}(\zeta)d\zeta,\quad \bar{\Phi}_1(\zeta)=\int_\zeta^0\bar{\Phi}(\zeta)d\zeta.$$

The solution of equation (16) by taking into account the boundary conditions (17) is formulated as follows

$$W=\sum_{k=1}^{\infty}\left(R_k(\tau)+R_k^0\right)\cos((2k-1)\pi\xi)/2+Q_k(\tau)\sin k\pi\xi. \tag{19}$$

The upper index 0 in the equation (19) means the solution corresponding to a constant pressure p_0 that does not depend on τ.

By taking into consideration linearity of the equation (16) and substituting (19) and the found expression for the pressure (18) in it, as well as by decomposing the remaining terms included in its right part according to trigonometric functions, we formulate an equation for a time independent component R_k^0

$$\sum_{k=1}^{\infty}\left(\left(\frac{2k-1}{2\ell}\pi\right)^4 D+\chi\right)w_m R_k^0\cos\frac{2k-1}{2}\pi\xi= =\sum_{k=1}^{\infty}\frac{4(-1)^{k+1}}{(2k-1)\pi}p_0\cos\frac{2k-1}{2}\pi\xi, \tag{20}$$

and an equation for unknown functions of time R_k and Q_k

$$\sum_{k=1}^{\infty}w_m\left[\left(D\left(\frac{2k-1}{2\ell}\pi\right)^4 R_k+\chi\right)\cos\frac{2k-1}{2}\pi\xi+\right.$$

$$+\left(D\left(\frac{k\pi}{\ell}\right)^4+\chi\right)Q_k\sin k\pi\xi+$$

$$\left.+\rho_0 h_0\omega^2\left(\frac{d^2R_k}{d\tau^2}\cos\frac{2k-1}{2}\pi\xi+\frac{d^2Q_k}{d\tau^2}\sin k\pi\xi\right)\right]=$$

$$=\sum_{k=1}^{\infty}\left[\frac{4(-1)^{k+1}}{(2k-1)\pi}\frac{p_1^{+}}{2}\cos\frac{2k-1}{2}\pi\xi+\frac{2(-1)^{k+1}}{k\pi}\frac{p_1^{+}}{2}\sin k\pi\xi\right]-$$

$$-w_m\sum_{k=1}^{\infty}\left[\left(M_{ck}\omega^2\frac{d^2R_k}{d\tau^2}+2K_{ck}\omega\frac{dR_k}{d\tau}\right)\cos\frac{2k-1}{2}\pi\xi-\right.$$

$$\left.-\left(M_{sk}\omega^2\frac{d^2Q_k}{d\tau^2}+2K_{sk}\omega\frac{dQ_k}{d\tau}\right)\sin k\pi\xi\right]. \tag{21}$$

Given that: $d^2R_k/d\tau^2=-R_k$, $d^2Q_k/d\tau^2=-Q_k$, by putting the following expressions in:

$$M_{ck}=\frac{\rho\nu}{\delta_0\psi^2}\left[\frac{2}{(2k-1)\pi}\right]^2\frac{2\varepsilon^2\alpha}{\omega},\quad 2K_{ck}=\frac{12\gamma\omega}{2\varepsilon^2\alpha}M_{ck},$$

$$M_{sk}=\frac{\rho\nu}{\delta_0\psi^2}\left[\frac{1}{k\pi}\right]^2\frac{2\varepsilon^2\alpha}{\omega},\quad 2K_{sk}=\frac{12\gamma\omega}{2\varepsilon^2\alpha}M_{sk},$$

$$a_{1ck}=\left((2k-1)\pi/2\ell\right)^4+\chi/D-\left[\rho_0 h_0+M_{ck}\right]\omega^2/D,$$

$$a_{2ck}=2K_{ck}\omega/D,$$

$$a_{1sk} = (k\pi/\ell)^4 + \chi/D - [\rho_0 h_0 + M_{sk}]\omega^2/D,$$

$$a_{2sk} = 2K_{sk}\omega/D$$

and by equating the coefficients of cosines and sines, we formulate the following expressions using equations (20) R_k^0

$$R_k^0 = p_0\left(4(-1)^{k+1}/((2k-1)\pi)\right)\times$$

$$\times\left(((2k-1)\pi)^4 D/(2\ell)^4 + \chi\right)^{-1} w_m^{-1}, \qquad (22)$$

and (21) in order to calculate $R_k(\tau)$ and $Q_k(\tau)$

$$a_{1ck} w_m R_k + a_{2ck} w_m \, dR_k/d\tau =$$

$$= 2(-1)^{k+1}/((2k-1)\pi D)\, p_m^+ f_p(\tau), \qquad (23)$$

$$a_{1sk} w_m Q_k + a_{2sk} w_m \, dQ_k/d\tau = (-1)^{k+1}/(k\pi D)\, p_m^+ f_p(\tau). \qquad (24)$$

Particular solutions of the equations (23), (24) corresponding to the harmonic law of pressure pulsations at the end face of the channel (1) are formulated as follows:

$$R_k = \frac{2(-1)^{k+1}}{(2k-1)\pi}\frac{p_m^+}{w_m D}\left[A_{ck}\frac{df_p}{d\tau} + B_{ck} f_p\right],$$

$$Q_k = \frac{(-1)^{k+1}}{k\pi}\frac{p_m^+}{w_m D}\left[A_{sk}\frac{df_p}{d\tau} + B_{sk} f_p\right], \qquad (25)$$

where $A_{ck} = -\dfrac{a_{2ck}}{a_{1ck}^2 + a_{2ck}^2}$, $B_{ck} = \dfrac{a_{1ck}}{a_{1ck}^2 + a_{2ck}^2}$,

$A_{sk} = -\dfrac{a_{2sk}}{a_{1sk}^2 + a_{2sk}^2}$, $B_{sk} = \dfrac{a_{1sk}}{a_{1sk}^2 + a_{2sk}^2}$.

Taking into consideration (25) deflection of the wall 1 can be expressed as

$$w = w_m \sum_{k=1}^{\infty}\left\langle \frac{2(-1)^{k+1}}{(2k-1)\pi}\left\{\frac{2p_0}{((\pi(2k-1))^4 D/(2\ell)^4 + \kappa) w_m} + \right.\right.$$

$$\left. + \frac{p_m^+}{D w_m}\left[A_{ck}\frac{df_p}{d\tau} + B_{ck} f_p\right]\right\}\cos\frac{2k-1}{2}\pi\xi +$$

$$\left. + \frac{(-1)^{k+1}}{k\pi}\frac{p_m^+}{w_m D}\left[A_{sk}\frac{df_p}{d\tau} + B_{sk} f_p\right]\sin k\pi\xi \right\rangle =$$

$$= \frac{p_0}{D}\left[\sum_{k=1}^{\infty}\frac{4(-1)^{k+1}}{(2k-1)\pi}\left(\frac{2\ell}{(2k-1)\pi}\right)^4 \cos\frac{2k-1}{2}\pi\xi\right] +$$

$$+ p_m^+ A(\xi,\omega)\exp[i(\tau + \varphi(\xi,\omega))]. \qquad (26)$$

Here

$$A(\xi,\omega) = \sqrt{C(\xi,\omega)^2 + B(\xi,\omega)^2},$$

$$\varphi(\xi,\omega) = \mathrm{arctg}\left(C(\xi,\omega)/B(\xi,\omega)\right),$$

$$C(\xi,\omega) =$$

$$= \left[\sum_{k=1}^{\infty}\frac{2(-1)^{k+1}}{(2k-1)\pi}\frac{A_{ck}}{D}\cos\frac{2k-1}{2}\pi\xi + \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k\pi}\frac{A_{sk}}{D}\sin k\pi\xi\right],$$

$$B(\xi,\omega) =$$

$$= \left[\sum_{k=1}^{\infty}\frac{2(-1)^{k+1}}{(2k-1)\pi}\frac{B_{ck}}{D}\cos\frac{2k-1}{2}\pi\xi + \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k\pi}\frac{B_{sk}}{D}\sin k\pi\xi\right].$$

Using the found expression for deflection (26), we can formulate the law of change of dynamic pressure in dimensionless form:

$$P_0 = P_1^+(\tau)/2 + \xi P_1^+(\tau)/2 +$$

$$+ \frac{p_m^+}{D w_m}\left\langle \sum_{k=1}^{\infty}\left(\frac{2}{(2k-1)\pi}\right)^3 (-1)^{k+1}\left[12\gamma\left(B_{ck}\frac{df_p}{d\tau} - A_{ck} f_p\right) - \right.\right.$$

$$\left. - 2\varepsilon^2\alpha\left(A_{ck}\frac{df_p}{d\tau} + B_{ck} f_p\right)\right]\cos\left(\frac{2k-1}{2}\pi\xi\right) +$$

$$+ \sum_{k=1}^{\infty}\left(\frac{1}{\pi k}\right)^3 (-1)^{k+1}\times\left[12\gamma\left(B_{sk}\frac{df_p}{d\tau} - A_{sk} f_p\right) - \right.$$

$$\left.\left. - 2\varepsilon^2\alpha\left(A_{sk}\frac{df_p}{d\tau} + B_{sk} f_p\right)\right]\sin k\pi\xi\right\rangle, \qquad (27)$$

and in dimensional form:

$$p = p_1^+(\tau)(1+\xi)/2 + p_m^+\Pi(\xi,\omega)\sin(\tau + \varphi_p(\xi,\omega)), \qquad (28)$$

where $\Pi(\xi,\omega) = \dfrac{\rho\nu\omega}{D\delta_0\psi^2}\left(S(\xi,\omega)^2 + Q(\xi,\omega)^2\right)^{1/2}$,

$$\varphi(\xi,\omega) = \mathrm{arctg}\frac{S(\xi,\omega)}{Q(\xi,\omega)},$$

$$S(\xi,\omega) = \sum_{k=1}^{\infty}\left(\frac{2}{(2k-1)\pi}\right)^3 (-1)^{k+1}\left(12\gamma B_{ck} - 2\varepsilon^2\alpha A_{ck}\right)\times$$

$$\times\cos\left(\frac{2k-1}{2}\pi\xi\right) + \sum_{k=1}^{\infty}\left(\frac{1}{\pi k}\right)^3 (-1)^{k+1}\times$$

$$\times\left(12\gamma B_{sk} - 2\varepsilon^2\alpha A_{sk}\right)\sin k\pi\xi,$$

$$Q(\xi,\omega) = \sum_{k=1}^{\infty}\left(\frac{2}{(2k-1)\pi}\right)^3 (-1)^{k}\left(12\gamma A_{ck} + 2\varepsilon^2\alpha B_{ck}\right)\times$$

$$\times\cos\left(\frac{2k-1}{2}\pi\xi\right) + \sum_{k=1}^{\infty}\left(\frac{1}{\pi k}\right)^3 (-1)^{k}\times$$

$$\times\left(12\gamma A_{sk} + 2\varepsilon^2\alpha B_{sk}\right)\sin k\pi\xi.$$

The first component of the expression of deflection (26) is deflection under static pressure, and the second component is deflection under dynamic pressure in the channel. In the expression for dynamic pressure (28) the component $p_1^+(\tau)(1+\xi)/2$ represents the linear pressure drop along the channel, and the second component represents the pressure in liquid caused by its compression by an elastic wall of the channel. One can clearly see that the first component of the dynamic pressure along the channel does not exceed the specified pressure at an end face $p_1^+(\tau)$.

Note that using the expressions (26) and (28) we determined functions $A(\xi,\omega)$, $\Pi(\xi,\omega)$ which can be considered as frequency-dependent distribution functions of relative deflection amplitudes and dynamic pressure along the channel. Likewise, the functions $\varphi(\xi,\omega)$, $\varphi_p(\xi,\omega)$ can be considered as frequency-dependent functions of distribution of the phase change of wall deflection and pressure in the channel relative to the initial perturbation at an end face. In case of a fixed value of the longitudinal coordinate ξ, the said functions represent amplitude frequency characteristics and phase frequency characteristics of wall deflection and pressure in the specified cross-section of the channel. Thereby, using calculations of functions $A(\xi,\omega)$, $\varphi(\xi,\omega)$ we can examine elastic vibrations of a plate-wall of the channel induced by pressure pulsations at the end face (1) with amplitude p_m^+ and frequency ω, and by means of functions $\Pi(\xi,\omega)$, $\varphi_p(\xi,\omega)$ we can analyze pressure change along the channel caused by compression of the liquid by an elastic plate-wall of the channel during its bending vibrations.

As an example, let us analyze the study of elastic deflections of a plate-wall of the channel and dynamic pressure caused by compression of the liquid by an elastic wall in the center ($\xi = 0$) of the channel with the parameters: $\ell = 0.1$ m; $\delta_0/\ell = 1/15$; $b/\ell = 5$; $\rho = 1.84\cdot10^3$ kg/m^3; $\nu = 2.5\cdot10^{-4}$ m^2/c; $p_m^+ = 1$ Pa, $\rho_0 = 7.87\cdot10^3$ kg/m^3; $h_0 = 3.2\cdot10^{-3}$ m; $E = 1.96\cdot10^{11}$ Pa; $\chi = 10^6$ Pa/m; $\mu_0 = 0.3$.

The results of calculations of relative deflection amplitudes of the channel wall $A(0,\omega)$ and phase change $\varphi(0,\omega)$ depending on the disturbing frequency at an end face by retention of *1*, *2*, and *3* terms of the order are shown in Fig. 2 (curve *1*: one term of the order is retained in the calculations; curve *2*: two terms of the order are retained in the calculations; curve *3*: three terms of the order are retained in the calculations). Similar calculation results for relative pressure $\Pi(0,\omega)$ and phase change $\varphi_p(0,\omega)$ are shown in Fig. 3.

The conducted study shows that there are spikes of plate deflection amplitudes at resonant frequencies of vibration, spikes of pressure in liquid correspond to these frequencies. Meanwhile, the pressure caused by compression of the liquid by an elastic wall at resonant frequencies exceeds the specified pressure at an end face by 3-4 orders of magnitude. At these resonant frequencies phase shift changes from $-\pi/2$ to $\pi/2$, at other frequencies it ends to zero. Taking into account the second and the rest of

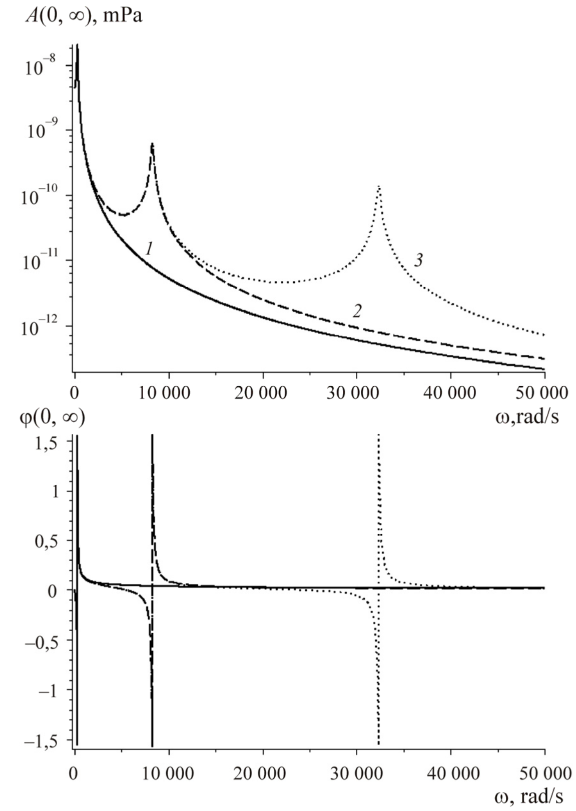

Fig. 2. Results of calculations of relative deflection amplitudes $A(0, \omega)$ of the plate and phase change $\varphi(0, \omega)$ depending on the disturbing frequency

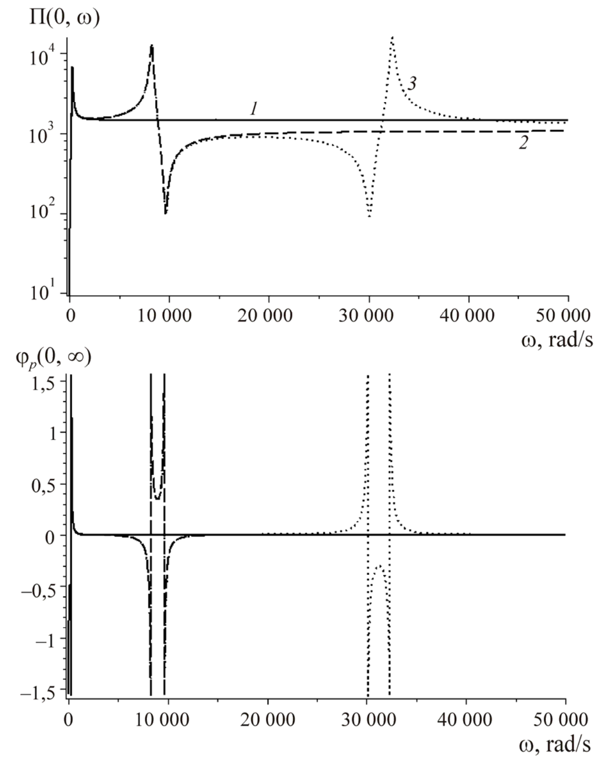

Fig. 3. Results of calculations of relative deflection amplitudes $\Pi(0, \omega)$ of the liquid and phase change $\varphi_p(0, \omega)$ depending on the disturbing frequency

the terms of the order in the expression (26) leads to additional deflection peaks and, consequently, causes occurrence of pressure which correspond to the new resonant frequencies of a higher frequency range than the previous resonant frequencies. For practical purposes, it is enough to retain the first 3-4 members of the order, since the frequency of vibrations of the channel walls and pressure pulsations do not exceed the audio range in reality. Taking into account the base elasticity leads to the increase of the resonant frequency. When we take into account the harmonic nature of changes in pressure over time, calculations show that the pressure may become lower than the saturated steam pressure at resonant frequencies. Thus, even a slight pressure pulsation at an end face leads to significant pressure changes in the channel due to compression of the liquid by an elastic wall of the channel at resonant frequencies of vibration, which is the reason of vibration cavitation in liquid. Consequently, this work has shown that fluid-elastic vibrations of a wall of the channel are the main reason of a sudden increase of the dynamic pressure in liquid at resonant frequencies of vibrations. The expressions formulated in this work for deflection of an elastic wall of the channel and hydrodynamic parameters of a viscous liquid layer can be practically used to determine resonant frequencies that correspond to conditions of cavitation generation, as well as to evaluate possibility of occurrence of resonant vibrations by using a specified frequency range of possible pressure pulsations at an end face of the channel.

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