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THE NUMERICAL MODEL OF DYNAMIC MECHANICAL BEHAVIOR OF BRITTLE MATERIALS BASED ON THE CONCEPT OF THE KINETIC THEORY OF STRENGTH

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ABSTRACT

A model of the dynamic mechanical behavior of brittle materials based on the ideas of the kinetic theory of strength is developed. The proposed model is a generalization of the classical "quasi-static" Nikolaevsky plasticity model (non-associated flow law with the plasticity criterion in the form of Mises-Schleicher) to the strain rate interval corresponding to the dynamic loading. In contrast to the traditional approach to constructing dynamic models, in which the dependence of the model parameters on the strain rate is specified, the proposed model suggests to use the relaxation time and time of destruction as the key parameters.

The presented model allows taking into account the change in the strength and rheological properties of brittle materials with an increase in the loading rate. This ensures a correct transition from the quasi-static regime of loading to the dynamic one in the range of strain rates within $10^{-3} < \dot{\varepsilon} < 10^{3}$ s^{-1}.

Within the framework of the proposed model it is assumed that there exist the experimental data about the dependences of the strength and rheological characteristics of the material on the times of different scales discontinuities nucleation. However, in view of the complexity of obtaining this information, we propose a way of obtaining the estimates of these dependencies by transforming the dependences of the mechanical properties on the strain rate that can be gained with standard tests.

The developed dynamic model can be implemented within various Lagrangian numerical methods using an explicit integration scheme (including the finite element and discrete element methods) and is relevant for solving a new class of applied problems related to natural and technogenic dynamic impacts to structures of artificial building materials, including concretes, ceramic elements of structures and natural rock materials.

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1. Introduction

It is known that macroscopic elastic, rheological and strength properties of brittle materials are sensitive to the strain rate and may differ significantly from the corresponding characteristics of mechanical behavior which are determined under "quasi-static" loading conditions [1-9]. Brittle materials show a strong dependence of rheological and strength properties on the strain rate in the range from 10^2 s^{-1} to 10^7 s^{-1}. At the same time, effective elastic moduli of brittle materials change noticeably at strain rates above 10^3 s^{-1}.

A widely-used tool for a thorough study of dynamic mechanical behavior of materials, including fracture dynamics, is Lagrangian numerical methods with the use of explicit integration schemes. The most known "Lagrangian" numerical method used for solving such problems is the finite element method (FEM) [9-13]. A well-developed mathematical apparatus of this method allows to implement complex rheological models of the mechanical response of materials and to solve difficult problems of deformable solid mechanics. We would like to note that when using this method for solving dynamic problems there are well-known difficulties associated with modeling of multiple fractures, followed by intense mass-transfer and interfusion. Currently, in addition to FEM, numerical methods of discrete approach to the medium description, in particular, discrete element methods (DEM) are widely used to simulate dynamic processes of multiple fractures and fragment transfer [14-18]. Despite the known advantages of failure simulation, DEMs have a number of limitations including a lower order of accuracy in space in comparison with the traditionally used finite element implementations and insufficient development of mathematical formalism, limited by the use of quasi-static models of mechanics of materials. It is possible to overcome some important limitations of these methods and to combine their advantages, for example, by creating combined (FEM/DEM) methods [19, 20]. It is possible to create such "hybrid" methods because of similarity of basic principles of implementation of elasticity, plasticity and fracture models in FEM and DEM that use explicit integration schemes. Therefore, the correction of existing rheological models and creation of new ones considering properties of dynamic response of brittle materials is relevant for the development of a wide class of "Lagrangian" numerical methods.

Inelastic behavior of brittle materials is mainly caused by accumulation of micro-fractures (formation of pores and cracks of different sizes, their growth or healing, merging, collapse, etc.) [21-25]. The involvement of crystal-lattice defects in deformation process of such materials happens only at high pressures and temperatures [24, 26-28]. Therefore, in order to describe dynamic inelastic behavior of brittle materials we use modified plasticity models, which take into account high sensitivity of inelastic response parameters to pressure, as well as complexity of the relationship between shear plastic strain and volumetric plastic strain (non-associated flow rules). In this regard, characteristics of the used plastic potential and limiting surface define the field of application of the model. For instance, the work [29] describes a dynamic rheological model for simulation of asphalt concretes based on the non-associated plastic flow rule with the Vermeer type limiting surface.

Such models of the dynamic behavior of brittle isotropic materials, as the Taylor-Chen-Kuszmaul model [30, 31], the Holmquist-Johnson-Cook model [32], the "Continuous Surface Cap" (CSC) model [33], the Karagozian and Case concrete model (KCC) [34], the Riedel-Tom-Hermayer (RHT) model [35] and several others [36] became widely used. These models include a significant number of model parameters and, if correctly determined, adequately describe characteristics of dynamic deformation and fracture of brittle materials, including sensitivity of properties to pressure and rate of load application. Among the dynamic models, we would like to distinguish models that are based on the use of the modified Drucker-Prager criteria for description of inelastic strain and fracture. One of these models in particular (Drucker-Prager Cap), modified to describe dependence of the behavior of brittle materials on the strain rate, is described in the work [37]. This model considers the influence of excessive pressure that causes pores to collapse [38], dimensional effect [39, 40] and the influence of strain rate on the value of cohesion [41]. Different modifications of the Drucker-Prager plasticity model are widely used for simulation of the dynamic inelastic response of brittle materials [42-44]. We would also like to note that along with "isotropic" models, the researchers actively develop dynamic models of orthotropic brittle materials that consider the anisotropy of elastic and inelastic mechanical properties (including strength properties) [9, 45]. In this work we focus on locally isotropic materials in which mechanical properties of elementary volumes are assumed to be orientationally independent.

The described models are based on the traditional approach that considers sensitivity of characteristics of the inelastic response of materials to dynamics of changes of the local stress-strain behavior by introducing dependencies of parameters of plasticity and fracture models on the strain rate. At the same time, the strain rate (unlike the physical mass rate) is a technical parameter that characterizes the volume-averaged rate of dimensional change of the sample or its fragment. Usually, for experimental determination of dependencies of parameters of the used plasticity or fracture model on the strain rate, integral values $\dot{\varepsilon}$ for the entire sample are used. Herewith, local values $\dot{\varepsilon}$ (for example, in the area of fracture formation) may differ significantly from the integral value. An alternative way to describe behavior of materials under dynamic loading is the approach developed by such researchers as Zhurkov, Regel, Petrov, Morozov [46-49]. In the framework of this approach, the typical degradation time of the material T is used as the main parameter of the dynamic deformation response of the material. The T parameter is a physical one and usually characterizes process of formation and development of the main crack (fracture) in a sample or a fragment of material or formation of system of micro-fractures that cause macroscopically inelastic behavior of this fragment. It is obvious that the time parameter T depends on the scale, and hierarchy of the corresponding scales is associated with hierarchy of elements of the internal material structure [50-52]. According to Zhurkov, this parameter is associated with thermal fluctuation effects in the crystal lattice and defined by the temperature and applied load. In later works of Petrov and Morozov, an integral physical parameter with dimension of time which has a more general meaning and is called incubation time is used as the typical time of degradation or fracture. For example, in the work [53], the T parameter is described as the typical time of stress relaxation. The authors of this work point out that stress relaxation mechanisms can differ qualitatively for different relaxation processes (plastic strain, fracture) and different materials, but this is not an obstacle for description of these processes on the basis of a general approach.

In this work, the kinetic approach to the construction of a model of dynamic mechanical behavior of materials is implemented based on the example of the Nikolaevsky plasticity model (non-associated plastic flow rule with plasticity criterion in the form of Mises-Schleicher) and the Drucker-Prager criterion of fracture [54, 55]. The initial implementation of this model is applicable for solving quasi-static problems. This is the first work that proposes dynamic modification of this model that allows to take into account characteristic features of deformation of brittle materials at strain rates up to ~10^3 s^{-1}.

2. Description of the Model of Quasi-Static Mechanical Behavior of Brittle Materials

Macroscopic description of inelastic strain and fracture of brittle heterogeneous materials is usually based on models of mechanical behavior that use the first, second, and third invariants of the stress tensor. This is associated with special aspects of the description of inelastic strain caused by formation and development of structural fractures (cracks of different sizes, pores). The increase of concentration and sizes of fractures causes the effect of dilatancy (non-elastic volumetric strain).

The authors of this work consider one of the widely used "isotropic" models of inelastic strain response of brittle materials, which is the Nikolaevsky model (non-associated plastic flow rule). Implementations of this model within different Lagrangian numerical methods are successfully used in studying characteristic features of deformation of brittle heterogeneous materials and media [24, 54-57]. This model is based on the non-associated plastic flow rule with the plasticity criterion in the form of Mises-Schleicher $\Phi = \omega J_1 + \sqrt{J_2} = 3\omega\sigma_{mean} + \sigma_{eq}/\sqrt{3}$, where J_1 and J_2 are the first invariant of the stress tensor and the second invariant of the deviatoric stress tensor, ω is the dimensionless parameter proportional to the internal friction coefficient α ($\alpha = 3\omega$), σ_{mean} is the mean stress, σ_{eq} is the equivalent stress.

Condition of the extreme limit state is the fulfillment of inequality:

$$\Phi \geq Y, \quad (1)$$

where Y is the cohesion.

In the Nikolaevsky model, the linear connection between volumetric and shear rate of plastic strain is postulated:

$$\dot{I}_1^p = 2\Lambda\sqrt{\dot{I}_2^p}, \quad (2)$$

where $\dot{I}_1^p$ is the first invariant of the tensor of plastic strain rate, $\dot{I}_2^p$ is the second invariant of the deviator of the tensor of plastic strain rate, Λ is the coefficient of dilatancy.

The Wilkins algorithm [58] is widely used for implementation of the Nikolaevsky model within numerical methods that use explicit integration schemes. According to the Wilkins algorithm, the solution of an elastic-plastic problem includes two stages:

1) An incremental solution of the elastic problem. Examples of numerical implementation of the elastic problem as part of DEM can be found in earlier works of the authors [54, 59].

2) Reduction of stress tensor deviator components to the yield surface in case of the fulfillment of inequation (1) at the current step of integration of the motion equations (see Fig. 1).

Mathematically, the reduction can be expressed in terms of stress deviators:

$$D'_{\alpha\beta} = D_{\alpha\beta} \cdot M, \quad (3)$$

where $D_{\alpha\beta} = \sigma_{\alpha\beta} - \sigma_{mean}\delta_{\alpha\beta}$ denotes stress tensor deviator components (initial values obtained after solving the elastic problem), $D'_{\alpha\beta}$ means reduced (corrected) stress tensor deviator components (hereinafter the symbol "′" means corrected values of corresponding parameters), $M \leq 1$ is the reduction factor.

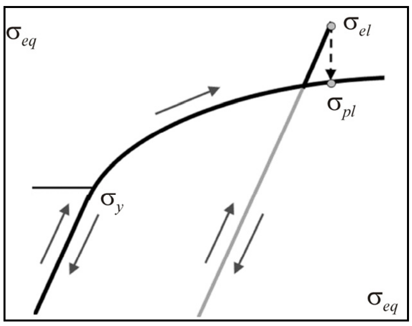


Fig. 1. Scheme of the Wilkins algorithm

Thus, the linear Hooke's law is used as the defining relation in the area of elastic response of the simulated elementary volume of the brittle material. The Wilkins algorithm is used for description of stress relaxation beyond the elastic limit [58]. Within the Nikolaevsky model, the stress tensor is reduced to the corresponding point of the limiting surface using the following expression:

$$\sigma'_{\alpha\beta} = \left(\sigma_{\alpha\beta} - \delta_{\alpha\beta}\sigma_{mean}\right)M + \delta_{\alpha\beta}\sigma'_{mean}, \tag{4}$$

where $M = \sqrt{J'_2/J_2}$, $\delta_{\alpha\beta}$ – is the Kronecker symbol. The reduced ("corrected") values of stress tensor invariants are expressed as follows [55]:

$$\begin{cases} \sqrt{J'_2} = \dfrac{\sigma'_{eq}}{\sqrt{3}} = \sqrt{J_2} - \dfrac{G\left(\Phi - Y\right)}{K\Lambda\omega + G}, \\ J'_1 = 3\sigma'_{mean} = J_1 - \dfrac{K\Lambda\left(\Phi - Y\right)}{3K\Lambda\omega + G}, \end{cases} \tag{5}$$

where G is the shear modulus, K is the bulk modulus. We would like to point out that in this model the value of elastic moduli of the elementary volume of the material is assumed to be constant and independent of the value of the accumulated inelastic strain (which reflects the level of damage of the elementary volume).

In this work, the two-parameter Drucker-Prager criterion is used as a local fracture criterion. The Drucker-Prager criterion takes into account sensitivity of shear strength to hydrostatic pressure which is typical for brittle materials and has the same structural form as the Mises-Schleicher criterion:

$$\sigma_{eq}0.5\left(a+1\right) + \sigma_{mean}1.5\left(a-1\right) \geq \sigma_c, \tag{6}$$

where σ_c is the compressive strength, $a = \sigma_c/\sigma_t$, σ_t is the tensile strength, σ_{mean} and σ_{eq} are mean stress and equivalent stress, respectively.

3. Description of the Model of Dynamic Mechanical Behavior of Brittle Materials

The main point of the kinetic model of dynamic inelastic behavior of brittle materials developed in this work is to take into account the finite time of relaxation of local stresses caused by formation of fractures of different sizes. The static model described above adequately describes the inelastic response of materials subjected to loading at low strain rates when inertial effects can be ignored (quasi-static state). The quasi-static approximation assumes that relaxation of local stresses and the associated redistribution of stresses in the sample are carried out in a negligibly small time (in one time step of integration scheme in numerical modeling). Under dynamic loading, however, the contribution of inertial terms in redistribution of kinetic and potential energy in the sample volume becomes significant or even fundamental. Therefore, characteristics of the integral response of samples depend on the strain rate under dynamic loading. This proves the necessity of taking into account the finite time of relaxation of local stresses, since its value will determine the dynamics of stress redistribution.

Modern kinetic models of inelasticity and strength of brittle materials usually use integral fracture criteria which are based on the calculation of the loading pulse increment for a specific amount of time τ, called fracture incubation time [49, 60]. The parameter τ is interpreted as the time period during which a crack is formed under constant load equal to the static tensile strength of the material σ_c^{st}. This integral fracture criterion can be expressed as:

$$\int_{t-\tau}^{t} \sigma\left(t'\right)dt' = \sigma_c^{st}\tau, \tag{7}$$

where $\sigma\left(t'\right)$ is the scalar force parameter of the stress state, t is the time. In different implementations of the criterion, the maximum primary stress or a combination of invariants of the stress tensor are used as force parameter.

It is obvious that fulfillment of the criterion (7) is possible when the applied load reaches or exceeds the static strength of the material σ_c^{st} and does not become lower that σ_c^{st} for some finite amount of time. If the applied load is constant during the period of time τ and equal to the static strength of the material, the fracture will occur during the period of time τ (typical fracture incubation time).

It is difficult to determine experimentally the parameter τ. Moreover, its quantitative value depends on the spatial scale of the examined sample [49]. Therefore, the authors of this work suggest a simplified implementation of the dynamic fracture criterion (7):

$$\sigma\left(t\right) \geq \sigma_c^{dyn}\left(t - t_0\right)H\left(t_0 + \tau - t\right), \tag{8}$$

where σ_c^{dyn} is the value of the dynamic strength of the material, $H(t_0 + \tau - t)$ is the Heaviside function. Such formulation of criterion (7) means that when the scalar force parameter σ reaches the static strength (σ_c^{st}) at t_0, a fracture starts to form. The duration of this process is determined by change dynamics of the parameter σ at $t > t_0$. By the time of crack formation (at $t = t = t_{fr}$), the value of the parameter σ will be equal to a certain value $\sigma_c^{dyn} \geq \sigma_c^{st}$ (the specific value σ_c^{dyn} depends on change dynamics of the stress state during the time period ($t_{fr} - t_0$)). Let us assume that the strain rate does not change significantly during crack formation. In this case, the rate of increase of the parameter σ during the time period ($t_{fr} - t_0$) will not change significantly as well. Under this assumption, the fracture time ($t_{fr} - t_0$), the local strain rate, and the dynamic strength are uniquely interconnected. And the value of any of these parameters (σ_c^{dyn}, ($t_{fr} - t_0$), $\dot{\varepsilon}$) can be uniquely determined if we know the values of the other two.

In case of uniaxial compression with a constant rate, the fracture time can be calculated by using the formula:

$$T_f = t_{fr} - t_0 = \frac{\sigma_c^{dyn} - \sigma_c^{st}}{E\dot{\varepsilon}}, \qquad (9)$$

where σ_c^{st} and σ_c^{dyn} are static and dynamic values of the material strength under uniaxial compression, E is the Young's modulus, $\dot{\varepsilon}$ is the axial strain rate, T_f is the fracture time (normally different from the fracture incubation time).

A similar relation can be formulated for uniaxial dynamic tension with a constant rate. It should be noted that there is a sufficiently large amount of data on the dependence of the dynamic strength of samples of brittle materials on the strain rate under uniaxial compression and tension [1-8]. These data allow to determine dependencies of $T_f(\sigma_c^{dyn})$ under specified loading conditions. It is worth mentioning that these dependencies are non-linear and monotonically decreasing (thus, the function $T_f(\sigma_c^{dyn})$ is single-valued).

Let us suppose that we know the dependence $T_f(\sigma_c^{dyn})$ found at the constant strain rate for a specified type of the stress state (normally, different from uniaxial loading). Let us assume that this dependence is monotonically decreasing (as in the case of uniaxial loading). In this case, the above assumption of the constant rate of increase of local stresses during the time period T_f allows us to suggest the following numerical implementation of the dynamic fracture criterion (8).

Within this implementation, we suppose that fracture of the material does not occur at t_0 when the value of the static strength $\sigma = \sigma_c^{st}$ is reached, but after some period of time, during which internal transitions take place in the examined local area of the material (incubation of fracture). The result of these processes is formation of new free surfaces (material fracture) which causes decrease of the resistance to external loading. Meanwhile, the examined local area of the material stays undamaged during the fracture timeT_f. As noted above, approximation of constancy of the strain rate during T_f gives us the unique value of the dynamic strength that corresponds to the current rate of change of the scalar force parameter σ and, consequently, the unique value of T_f (see Fig. 2).

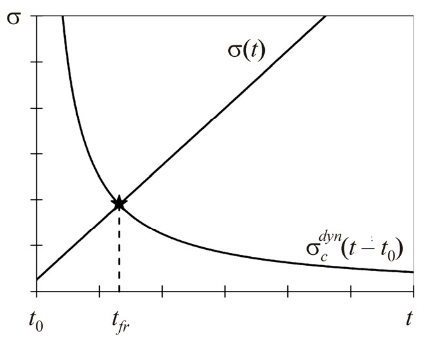


Fig. 2. Scheme of dynamic fracture

Then, to determine the moment of ending of the fracture preparation at each moment of time $t > t_0$ after reaching the value of the static strength the following inequation is checked:

$$\sigma(t) \geq \sigma_c^{dyn}(T_f), \qquad (10)$$

where σ(*t*) is the current local value of the force parameter, $T_f = t - t_0$, and the value of σ_c^{dyn} is determined by using the "calibration" dependence $T_f(\sigma_c^{dyn})$. When the condition (10) is fulfilled (at the moment of time t_{fr}), the dynamic strength of the material is considered to be reached, the incubation process is considered to be complete and the material is locally destroyed. The corresponding values $T_f = t_{fr} - t_0$ and σ_c^{dyn} are considered as local values of the fracture timeand dynamic strength. We would like to point out that the suggested scheme of numerical implementation of the dynamic criterion (8) is common for different structural types of force parameters.

In this work, the combination of the first two invariants of the stress tensor suggested by Drucker and Prager is used as the force parameter σ. The condition (10) in this case is formulated as:

$$\sigma_{DP}(t) = \sigma_{eq}(t)0.5(a+1) + \sigma_{mean}(t)1.5(a-1) \geq$$
$$\geq \sigma_c^{dyn}(t - t_0), \qquad (11)$$

where $a = \sigma_c^{dyn} / \sigma_t^{dyn}$, $\sigma_c^{dyn} = \sigma_c^{dyn}(t - t_0)$ is the dynamic compression strength, $\sigma_t^{dyn} = \sigma_t^{dyn}(t - t_0)$ is the dynamic tensile strength. It is possible to use the criterion (11) if the dependencies $\sigma_c^{dyn}(T_f)$ and $\sigma_t^{dyn}(T_f)$ are known.

Experimental data show that not only the strength properties but also inelastic properties of brittle materials, such as cohesion *Y*, internal friction coefficient α and dilatancy coefficient Λ [5] are sensitive to the load rate. This is caused by the fact that the inelastic response of brittle materials is associated with formation of a system of internal discontinuities (fractures and micro cracks) that lasts during a finite time. In order to adequately describe the inelastic strain of brittle materials subjected to dynamic loading, it is necessary to consider dependence of the model parameters on duration of formation of the system of internal discontinuities. Since the inelastic response of the material is associated with relaxation of local stresses, duration of formation of micro-fractures will hereinafter be referred to as the stress relaxation time or relaxation time T_r. As in the case of dynamic fracture, the specific value of relaxation time depends on the change dynamics of the stress state. In the special case of constancy of the stress state at the point that corresponds to the yield strength, relaxation time is the kinetic constant of the material, similar to the fracture incubation time τ.

Since the mechanisms of inelastic strain in this case are similar to the mechanisms of local fracture, the model of dynamic inelastic response of brittle materials can also be created by using the formalism of the kinetic theory of strength. This model assumes that the strain rate and the rate of change of the stress state parameter σ remain constant during relaxation time (a specific type of parameter σ may differ from the same one in the dynamic fracture criterion). In this case, the dynamic yield strength is a unique monotonically decreasing function of the relaxation time. For a special case of uniaxial loading, the type of this dependence can be obtained by an equation similar to (9) using yield strength instead of tensile strength and the modulus $E^* = E/\sqrt{3}$ instead of E.

In the general case, the form of the dynamic plasticity criterion is identical to (8):

$$\sigma(t) \geq \sigma_y^{dyn}(T_r) H(t_0 + \tau - t), \qquad (12)$$

where σ_y^{dyn} is the dynamic yield strength.

The specific type of this criterion is determined by the applied plasticity model. As with the fracture model, we assume that the stress relaxation does not happen at the time t_0 when the value of the static yield strength $\sigma = \sigma_y^{st}$ is reached, but after a certain period of time (relaxation time).

The Wilkins algorithm involves an incremental solution of the elastic problem at each step of integration of motion equations and reducing the resulted values of stress to the corresponding point of the limit surface according to the relations (4)-(5) when the inequation $\Phi \geq Y^{st}$ is fulfilled. Within numerical implementation of the static model, the reduction is carried out in one step of time integration. In the dynamic model, the values of stress are gradually reduced to the yield surface during relaxation time. We suppose that the dependence of the cohesion value on the relaxation time $Y^{dyn}(T_r)$ (which is a monotonically decreasing function) is known for the examined material. When the inequation $\Phi \geq Y^{st}$ is fulfilled at this step of integration, the current value of the parameter Φ is considered as the current (dynamic) value of the shear elastic limit: $Y^{dyn} = \Phi$. In this case, the "calibration dependence" $Y^{dyn}(T_r)$ allows to determine the current value of the relaxation time $T_r = T_r(\Phi)$ (that corresponds to the current dynamics of stress change). Assuming that the stress relaxation (reduction of stresses to the point Y^{st} of the limit surface) during the time period T_r occurs in linear rule; reduction of the value of the parameter Φ in one time step Δt can be expressed on the basis of linear interpolation:

$$\Delta\Phi = \left(\Phi - Y^{st}\right)\frac{\Delta t}{T_r}. \qquad (13)$$

The increment ΔΦ is used in relations (4)-(5) of the dynamic implementation of the Nikolaevsky plasticity model instead of "static" difference (Φ-Y).

The current values of other parameters of the plasticity model (ω^{dyn} and Λ^{dyn}) in the relations (4)-(5) are determined by means of "calibration" dependencies $\omega^{dyn}(T_r)$ and $\Lambda^{dyn}(T_r)$ using the current value of the relaxation time T_r. It is assumed that the curves $\omega^{dyn}(T_r)$ and $\Lambda^{dyn}(T_r)$ are known and monotone. It is important to point out that the calibration dependencies $Y^{dyn}(T_r)$, $\omega^{dyn}(T_r)$, and $\Lambda^{dyn}(T_r)$ must correspond to each other, i. e. the specific values of Y^{dyn}, ω^{dyn}, and Λ^{dyn} at a specific value of T_r must correspond to the same material state (i. e. must be obtained under the same loading conditions).

In the general case, the cohesion value is not only a function of the relaxation time (or that of the strain rate), but also a strain function ($Y^{dyn} = Y^{dyn}(\varepsilon^{pl}, T_r)$). For instance, in the Nikolaevsky model, the function $Y(\varepsilon)$ determines material hardening beyond the elastic limit. It is known that the hardening function changes significantly under dynamic loading of the material. As limit stresses (elastic limits, strength limits) increase, the proportion of plastic strain of the material decreases and, consequently, the slope of the hardening function increases. Therefore, in order to adequately simulate the strain response of the material under dynamic loading, it is necessary to take into account reduction of the proportion of plastic strain at high loading rates. The authors of this work suggest to take into account reduction of the proportion of plastic strain by introducing the function $\varepsilon^{pl}(T_r)$ using the same time parameter T_r as for the other model parameters. It is obvious that the function $\varepsilon^{pl}(T_r)$ must be monotonically increasing, i.e. it must tend to the upper limit while tending towards quasi-static loading conditions.

As stated above, the elastic properties of the material are also sensitive to the strain rate, but this effect is noticeable at quite high strain rates (>10^3 s^{-1}) and is not discussed in this work.

It should be noted that the suggested model of the dynamic behavior of brittle materials is a generalization of static models of inelastic strain and fracture of the material that includes the finite time of relaxation processes (even though it is small). It makes it possible to simulate both quasi-static and dynamic processes within a single formalism.

4. Determination of Dependencies of Inelastic/Strength Properties of Brittle Materials on Relaxation/Fracturing Time

The suggested modification of the Nikolaevsky model allows to describe the mechanical behavior of brittle heterogeneous locally isotropic materials subjected to dynamic loading. However, it is necessary to know the type of dependence of each model parameter on the relaxation time in order to solve specific problems. Ideally, this information should be gained by means of generalization of experiments on dynamic strain of the examined samples of the required dimensions using different types of stress states.

As stated above, the experiments on determination of the fracturing and relaxation time for different types of stress states and different materials are quite complicated. This causes insufficient data on the dependence of strength and rheological properties of materials on the fracturing and relaxation time.

On the other hand, in recent decades a considerable amount of experimental data on the dependence of the properties of brittle materials on the strain rate under uniaxial strain and some other types of loading has been accumulated. Analysis of these data shows that, despite different absolute values of the dynamic characteristics, dependencies of the specified dynamic values of these parameters (obtained by normalization to corresponding static values) on the strain rate can be quite accurately approximated by uniform dependencies for a wide range of brittle materials [1-5]. By using uniform dependencies of the material properties on the strain rate, it is possible to determine (for example, by using the formula (9)) the required dependencies of the model parameters on the relaxation and fracture timefor a specific brittle material. The authors of this work use experimental data on the dynamic loading of brittle materials subjected to uniaxial compression and uniaxial tension (these data are summarized in the work [5]) in order to determine such dependencies.

The dependencies of compression and tensile strength on the strain rate are well known from the experimental data on a wide range of brittle materials. In this work we refer to the dependencies of compression and tensile strength on the strain rate summarized in the work [5] (see Fig. 3, *a*, curves *1* and *2*, respectively):

$$\frac{\sigma_c^{dyn}(\dot{\varepsilon})}{\sigma_c^{st}} = 1 + 0.238\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}\right)^{0.348}, \quad (14)$$

$$\frac{\sigma_t^{dyn}(\dot{\varepsilon})}{\sigma_t^{st}} = 1 + 0.093\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}\right)^{0.617}, \quad (15)$$

where $\dot{\varepsilon}$ is the strain rate in s^{-1}, $\dot{\varepsilon}_r = 1\ s^{-1}$ is the normalization factor.

These data on the dynamic strength of brittle materials can be expressed in terms of the stress relaxation time (in this case, in terms of the fracturing time). For instance, the fracture timeof the specific material (e.g., of sandstone: E = 16 GPa, σ_c^{st} = 70 MPa, σ_t^{st} = 31,5 MPa) under uniaxial compression was calculated by using the formula (9):

$$T_f(\dot{\varepsilon}) = \frac{\left(\sigma_c^{dyn}(\dot{\varepsilon})/\sigma_c^{st} - 1\right)}{\dot{\varepsilon}}\frac{\sigma_c^{st}}{E}. \quad (16)$$

The obtained dependence $\frac{\sigma_c^{dyn}(T_f)}{\sigma_c^{st}}$ is a decreasing dependence of a nonlinear kind. At low values of fracture time(that correspond to high strain rates) it decreases rapidly, and at high values of T_f (that correspond to quasi-static strain rates) it asymptotically tends to 1.

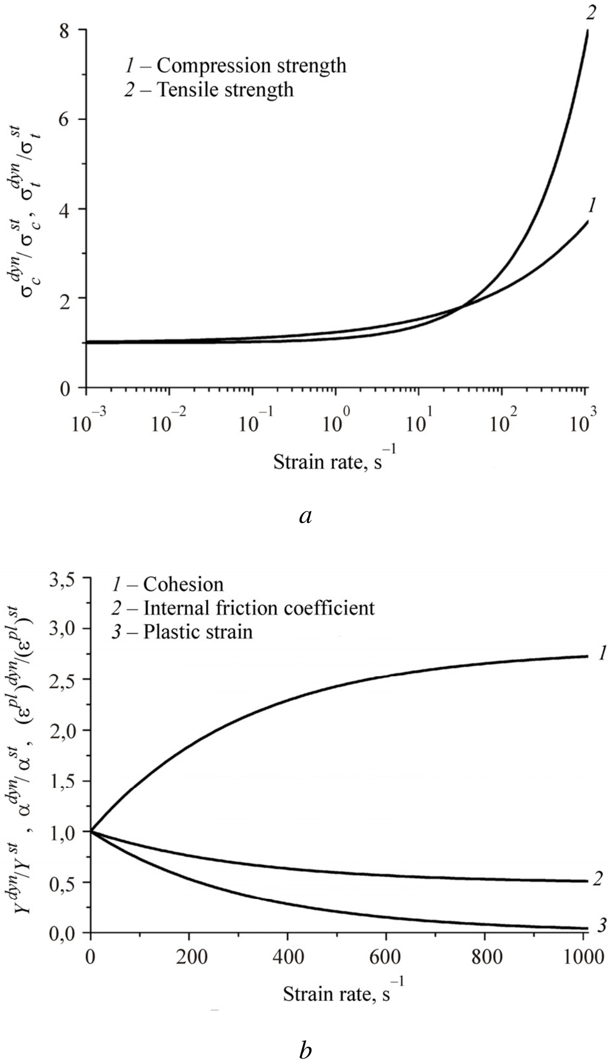


Fig. 3. Dependencies of the compression and tensile strengths (*a*) and cohesion, internal friction coefficient and plastic strain (*b*) on strain rate for concretes and rocks

The constructed set of points $\frac{\sigma_c^{dyn}(T_f)}{\sigma_c^{st}}$ that corresponds to the strain rate range from $10^{-3}\ s^{-1}$ to $10^3\ s^{-1}$ cannot be satisfactorily approximated by a simple analytic function. The best approximation is possible when the dependence $\frac{\sigma_c^{dyn}(T_f)}{\sigma_c^{st}}$ is divided into two parts. The part with low values of T_f was approximated by a logarithmic dependence, and the part with high values of T_f (a flat area) was approximated by a linear function:

$$\frac{\sigma_c^{dyn}(T_f)}{\sigma_c^{st}} = \begin{cases} 0.48 - 0.48\ln\left(\frac{T_f}{T_f^0} - 0.005\right) \text{ by } T_f < 2.157\cdot10^{-4}\text{s}; \\ 1.575 - 0.57\frac{T_f}{T_f^0} \text{ by } T_f > 2.157\cdot10^{-4}\text{s}; \end{cases} \quad (17)$$

where the fracture time T_f is expressed in seconds, $T_f^0 = 0.00175$ s.

In the same way the dependence $\frac{\sigma_t^{dyn}\left(T_f\right)}{\sigma_t^{st}}$ for sandstone was determined, which was also divided into two parts with low and high values of T_f and approximated by two functions:

$$\frac{\sigma_t^{dyn}\left(T_f\right)}{\sigma_t^{st}} = \begin{cases} -2.1-1.1\ln\left(\frac{T_f}{T_f^0}-0.0075\right) \text{ by } T_f < 1.127\cdot10^{-4}\text{s}; \\ 1.055-0.05\frac{T_f}{T_f^0} \text{ by } T_f > 1.127\cdot10^{-4}\text{s}; \end{cases} \quad (18)$$

The expressions (17)-(18) are obtained for sandstone by using uniform curves of the dependence of strength on the strain rate. For generalization of (17)-(18) to a class of brittle materials, the "scaled" parameters $\frac{T_f}{F_c} = T_f^{ref}$ in (17) and $\frac{T_f}{F_t} = T_f^{ref}$ in (18) should be used as the function argument, where $F_c = \frac{\sigma_c^{st}}{E} \Big/ \frac{\left(\sigma_c^{st}\right)^{ref}}{E^{ref}}$, $F_c = \frac{\sigma_t^{st}}{E} \Big/ \frac{\left(\sigma_t^{st}\right)^{ref}}{E^{ref}}$, $\left(\sigma_c^{st}\right)^{ref}$ is the compression strength of the "reference" material, $\left(\sigma_t^{st}\right)^{ref}$ is the tensile strength of the "reference" material, E^{ref} is the Young's modulus of the "reference" material, T_f^{ref} is the fracture timeof the "reference" material under appropriate type of loading. In this case, the reference material is sandstone.

In order to simulate the dependence of the cohesion value on the strain rate we also used experimental data (summarized in the work [5]) on various brittle materials accurately approximated by the following dependence (see Fig. 3, *b*, curve *1*):

$$\frac{Y^{dyn}(\dot{\varepsilon})}{Y^{st}} = 2.8 - 1.8e^{\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}}, \quad (19)$$

where $\dot{\varepsilon}_r = 317\ \text{s}^{-1}$ is the normalization factor.

Transition from the strain rate to the relaxation time for a random brittle material was carried out using a formula similar to (9):

$$T_r\left(Y^{dyn}/Y^{st}\right) = f\left(Y^{dyn}/Y^{st}\right)\frac{Y^{st}}{E/\sqrt{3}}, \quad (20)$$

$$f\left(Y^{dyn}/Y^{st}\right) = \frac{\left(Y^{dyn}(\dot{\varepsilon})/Y^{st}-1\right)}{\dot{\varepsilon}}. \quad (21)$$

Since we assume that the original dependence $\frac{Y^{dyn}(\dot{\varepsilon})}{Y^{st}}$ is applied to a wide range of brittle materials, the function $f\left(Y^{dyn}/Y^{st}\right)$ is an invariant part of the expression for determination of the relaxation time T_r. The multiplier $\frac{Y^{st}}{E/\sqrt{3}}$ is a scale factor determined by the "static" properties of the material. The dependence $f\left(Y^{dyn}/Y^{st}\right)$ can be approximated by the following linear function in a first approximation:

$$f\left(Y^{dyn}/Y^{st}\right) = 0.00787 - 0.00216\left(\frac{Y^{dyn}}{Y^{st}}\right), \quad (22)$$

numerical coefficients of which have the dimension of time.

The dependence $T_r\left(Y^{dyn}/Y^{st}\right)$ determined by using the equation (20) is used in the formula (13) for determination of the increment of the Φ parameter within numerical implementation of the dynamic plasticity model.

The rest of the parameters of the model (α, Λ, ε^{pl}) are also functions of the strain rate and relaxation time T_r. For instance, it is known that dependence of the internal friction coefficient on the strain rate is the same for a wide range of brittle materials [5] and can be approximated by the following function (see Fig. 3, *b*, curve *2*):

$$\frac{\alpha^{dyn}(\dot{\varepsilon})}{\alpha^{st}} = 0.49 + 0.51e^{\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}}, \quad (23)$$

where $\dot{\varepsilon}$ is the strain rate expressed in s^{-1}, and the normalization factor in exponent is the same as in (19).

By using the relation between T_r and $\dot{\varepsilon}$ determined by (20), it is possible to determine the dependence of the internal friction coefficient on the relaxation time for the "reference" material. This dependence can be applied to the class of brittle materials by introducing a dimensionless material index:

$$R = \frac{Y^{st}}{E/\sqrt{3}}\frac{E^{ref}/\sqrt{3}}{\left(Y^{st}\right)^{ref}}. \quad (24)$$

The expression for determination of the relaxation time of a random brittle material can then be written as:

$$T_r\left(Y^{dyn}/Y^{st}\right) = T_r^{ref}\left(Y^{dyn}/Y^{st}\right)\cdot R, \quad (25)$$

where

$$T_r^{ref}\left(Y^{dyn}/Y^{st}\right) = f\left(Y^{dyn}/Y^{st}\right)\cdot\frac{\left(Y^{st}\right)^{ref}}{E^{ref}/\sqrt{3}}. \quad (26)$$

By knowing the dependence of the internal friction coefficient on the relaxation time for the reference material and by using the R coefficient, it is possible to construct a "uniform" dependence of the internal friction coefficient on the relaxation time (in this case, the "reference" material is sandstone $(Y^{st})^{ref}$ = 15.4 MPa, E^{ref} = 16 GPa):

$$\frac{\alpha^{dyn}(T_r)}{\alpha^{st}} = 0.212 + 0.193 \cdot e^{\frac{T_r}{R \cdot T_r^0}}, \qquad (27)$$

where the fracture timeis expressed in seconds, and $T_r^0 = 6.7 \cdot 10^{-6}$ s.

The dependence of the dilatancy coefficient on the relaxation time can be determined in the same way. However, in the absence of sufficient information on the dependence of the dilatancy coefficient on the strain rate, the following calculations use the dependence of the dilatancy coefficient on the relaxation time of the same kind as for the internal friction coefficient:

$$\frac{\Lambda^{dyn}(T_r)}{\Lambda^{st}} = \frac{\alpha^{dyn}(T_r)}{\alpha^{st}} = 0.212 + 0.193 \cdot e^{\frac{T_r}{R \cdot T_r^0}}. \qquad (28)$$

The dependence of contribution amount of the plastic component to the total strain on $\dot{\varepsilon}$ was included in calculations given below by introduction of the exponential law of reducing this contribution as $\dot{\varepsilon}$ increases (see Fig. 3, *b*, curve *3*):

$$\frac{\left(\varepsilon^{pl}\right)^{dyn}(\dot{\varepsilon})}{\left(\varepsilon^{pl}\right)^{st}} = e^{\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}}, \qquad (29)$$

where $\dot{\varepsilon}$ is the strain rate expressed in s^{-1}, and the normalization factor in exponent is the same as in (19). As in the case of the internal friction coefficient, the dependence $\frac{\left(\varepsilon^{pl}\right)^{dyn}(T_r)}{\left(\varepsilon^{pl}\right)^{st}}$ for the "reference" material was determined and summarized by using the material index *R*:

$$\frac{\left(\varepsilon^{pl}\right)^{dyn}(T_r)}{\left(\varepsilon^{pl}\right)^{st}} = 0.378 \cdot e^{\frac{T_r}{R \cdot T_r^0}} - 0.546, \qquad (30)$$

where the fracture timeis expressed in seconds, and the normalization factor is the same as in (27)-(28).

5. Verification of the Dynamic Model

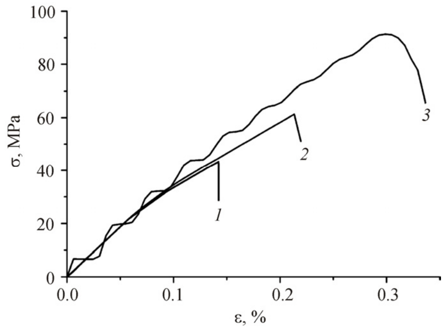
To demonstrate the correctness and adequacy of the suggested model of dynamic strain of brittle materials, a number of numerical tests were carried out by using one of the Lagrangian numerical methods, the movable cellular automaton method (MCA) [54, 61, 62] in particular. The movable cellular automaton method is an actively-developing method from the DEM group, the main characteristics of which are approximation of the homogeneous distribution of strains and stresses in the element volume and many-particle formulation of element–element interaction forces obtained on the base of Wiener-Rosenblueth model of cellular automaton interaction [63, 64].

Tests of two-dimensional rectangular samples 6×9 mm for uniaxial compression and tension at a constant rate were simulated. The strain rate varied from 10^{-3} s^{-1} (which was supposed to correspond to quasi-static loading) to 10^3 s^{-1}. The maximum value of the observed load resistance of the sample (the area of the side surface was examined) was considered as the strength limit of the sample. We examined the samples of two model materials (see Table) the mechanical properties of which are close to mechanical properties of sandstone and concrete of normal strength grades.

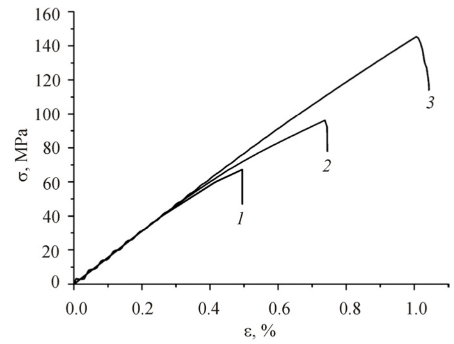
Mechanical properties of concrete
and sandstone samples for tests

	Poisson ratio	Density, kg/m^3	*E*, GPa	σ_y^{st}, MPa	σ_c^{st}, MPa	σ_t^{st}, MPa	α	Λ
Concrete	0.194	4.660	38.6	21.8	45.8	21	0.63	0
Sandstone	0.28	2.200	16	40	70	31.5	0.57	0.36

Fig. 4, *a* and *b* show calculation diagrams of uniaxial compression of model samples of concrete and sandstone at different strain rates. The shown diagrams were obtained by approximation of the plane stress state. The figures show the correct change of integral properties of the material with increase of the strain rate or, to be more exact, an increase of elasticity and strength limits, as well as a decrease of the plastic component of strain (increase of the strain hardening coefficient).



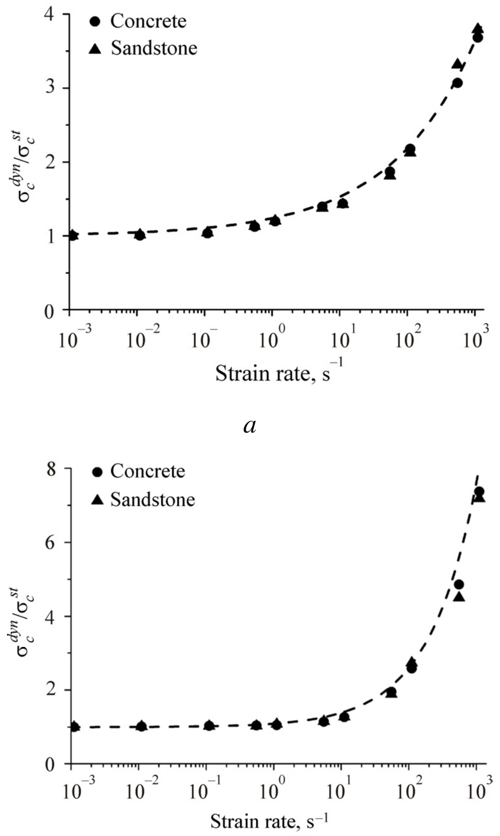
a



b

Fig. 4. Uniaxial compression loading diagrams of concrete (*a*) and sandstone (*b*) samples at various strain rates $\dot{\varepsilon}$: 10^{-3} s^{-1} – *1*; 10 s^{-1} – *2*; 100 s^{-1} – *3*

The analysis of diagrams of uniaxial compression and tensile of the examined materials allowed us to determine dependencies of compression and tensile strength on the strain rate (see Fig. 5). One can see that the results of the numerical modeling correspond to the experimental data [5], used for determination of the dependence between the relaxation time and model parameters ($T(Y)$, $T_{\dot{\varepsilon}}\left(\sigma_c^{dyn}\right)$, $T_{\dot{\varepsilon}}\left(\sigma_t^{dyn}\right)$). It proves the adequacy of the developed dynamic numerical model of inelastic strain and fracture of brittle materials.



a

b

Fig. 5. Dependencies of specified compressive strength (*a*) and specified tensile strength (*b*) of the concrete and sandstone samples on the strain rate: Dash line is the experimental data approximation

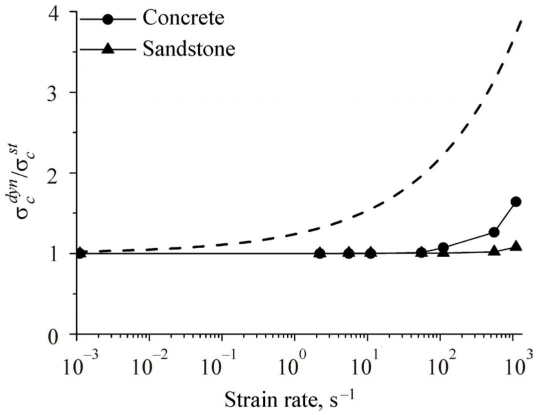


Fig. 6. Dependence of the compressive strength on the strain rate without taking into account the dependence of material properties on the strain rate for concrete and sandstone samples. Dash line is the experimental data approximation

To demonstrate the necessity of considering changes of the strength and inelastic properties of materials with increase of the strain rate, we simulated the tests of concrete samples for uniaxial compression that did not take into account the dependence of model parameters on the strain rate. The strain rate varied from 10^{-3} s^{-1} to 10^3 s^{-1}. The obtained results are shown in Fig. 6. You can see that when we use "quasi-static" values of model parameters, the strength properties of the samples change insignificantly when the strain rate increases. Only the part with high strain rates shows a slight increase of the sample strength. This effect is caused by the response rate of the samples, which begins to play a significant role during high-rate deformation of the material. Thus, the modeling results have shown that it is important and necessary to take into account the finite time of formation of fractures and cracks of different sizes in brittle materials when solving dynamic loading problems.

The dynamics and final fracture pattern are important results of numerical simulation of strain and fracture processes of brittle materials. When modeling uniaxial compression tests of model samples of concrete and sandstone, the classical fracture pattern was observed: the samples were destroyed at "quasi-static" compression rates by formation of main cracks from which secondary cracks develop. With increase of the strain rate, the degree of fragmentation of the samples increases, and small fragments split off from the side surfaces. Similar fracture patterns are observed during testing real samples of brittle materials [36, 65]. The obtained results show that the dynamic model developed in this work can be used to study dependencies of the integral fracture energy of "quasi-brittle" materials, as well as dependencies of size distribution of fragments on the amplitude and loading rate.

6. Conclusion

The main idea of the presented approach to the construction of models of dynamic mechanical behavior of brittle materials is taking account of the finite time of formation of different discontinuities (of relaxation time T_r and fracture timeT_f) in the material based on the kinetic theory of strength. In comparison with traditional dynamic models, here, the main control parameter that determines the dynamics of the material response is not the local strain rate, but the physical parameter which is the time of formation of discontinuities. We would like to point out that this does not require a strict dividing of material strain problems (and corresponding mathematical models) into static and dynamic ones, which allows to provide a correct transition from quasi-static loading conditions to high-rate dynamic loading, and vice versa. Moreover, despite the fact that the developed approach is implemented by means of examples of specific models of plasticity and fracture of brittle materials, it can also be applied to different models of inelastic response and fracture, implemented within various Lagrangian numerical methods (not only finite and discrete element methods, but also methods of finite differences). The further development of this approach may be associated with taking account of dependencies of elastic moduli of the material on the level of damage (which will allow to more accurately describe the elastic response of the material during relatively large inelastic strains) and anisotropy of local mechanical properties.

Since control parameters of the model (duration of discontinuities formation processesat the various scales) are determined by the scale-dependent value of the fracture incubation time [53], these parameters are also considered to be scale-dependent. Therefore, it is necessary to consider the scale factor during experimental determination of the key dependences of parameters of the numerical model on the duration of formation of discontinuities. To be more exact, characteristic size of the samples used to determine dependence of strength and rheological properties on the strain rate (or on the discontinuities formation process duration) should be comparable to the size of the finite/discrete element, behavior of which will be determined by these dependencies.

The approach to the construction of dynamic models developed in this work is relevant for solving a new class of applied problems related to natural and technogenic dynamic impacts to structures of manufactured building materials, including concretes, ceramic elements of structures and natural rock materials. Specifically, implementation of the presented approach within the particle method allows to solve problems of predicting the moment of fracture of materials and structures depending on the amplitude and rate of loading by taking into account structural properties of the materials. In addition to this, based on solving the reverse problem, the presented approach allows to estimate the characteristic size of internal elements of the material that determine the fracture pattern at the considered scale.

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