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ON INFLUENCE OF THE RESISTANCE CRISIS ON THE FOOTBALL FLIGHT DYNAMICS

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Abstract. It is described the mathematical model of football movement when employing standard positions (kick from the center of the football ground and corner kick). An important feature of the model is an account of dependence of aerodynamic coefficients of drag and lift forces on the ball velocity. It is found the optimal conditions which provide a football hit a rival goal for the minimal time. Also the problem of the impact theory is considered which allow to describe the football boot kick on the ball. It is proposed the method of determination of aerodynamic coefficients in three-dimensional movement of a ball.

Key words: biomechanics in sports, model of football movement, aerodynamic coefficients, optimal solution

1. Introduction

The major role in football belongs to the trajectory of a ball for a correct pass or to score a goal. Often realization of so called standard positions leads to success. The trajectory of a ball represents a three-dimensional curve which depends on the initial conditions of ball's movement. The aim of this work is the analysis of optimal initial conditions at which a ball hits the goal for the minimal time. In literature on sports biomechanics there are papers on mathematical modeling of movement of volley-ball and tennis-ball [1] – [4]. In this work a theoretical research of a football's flight phase is made and several problems are solved which are connected with execution of standard positions: kick from the center of the football ground and corner cock. An important feature of the model is an account of dependence of aerodynamic coefficients of drag and lift forces on the ball velocity. This account allows to receive results which correlate with experiments. Also the problem of the impact theory is considered which allows to describe the football boot kick on the ball. It is proposed the method of determination of aerodynamic coefficients in three-dimensional movement of a ball. These results have not only scientific but also an applied importance. They can be useful for coaches and football-players in training of standard positions. For clear understanding of obtained results the scheme of the football ground is presented (Fig. 1).

2. System of forces acting on a football. Aerodynamic coefficients of drag and lift forces

In this work we consider flat and spatial trajectories of the ball movement. In Fig. 2 a flat trajectory of a ball movement in vertical plane Oxy at the initial velocity V_0, lying in plane Oxy is represented. The rotation of the ball takes place around the axis parallel to axis z.

The important kinematical parameters of a movement are: the velocity vector of translation $\mathbf{V}$ directed at a tangent of the center of mass trajectory and the angular velocity vector $\boldsymbol{\omega}$ directed along the axis of rotation to that side from where the rotation is seen counter-clockwise. The rotation can be arbitrary depending on the kick on the ball.

During the ball flight there are forces acting on the ball: $\mathbf{P} = m\mathbf{g}$ – the force of gravity, where m – the mass of the ball, $\mathbf{g}$ – the acceleration of the free fall; $\mathbf{R}$ – the drag and $\mathbf{Q}$ – the lift force. The forces $\mathbf{R}$ and $\mathbf{Q}$ depend on velocity squared and are defined as follows:

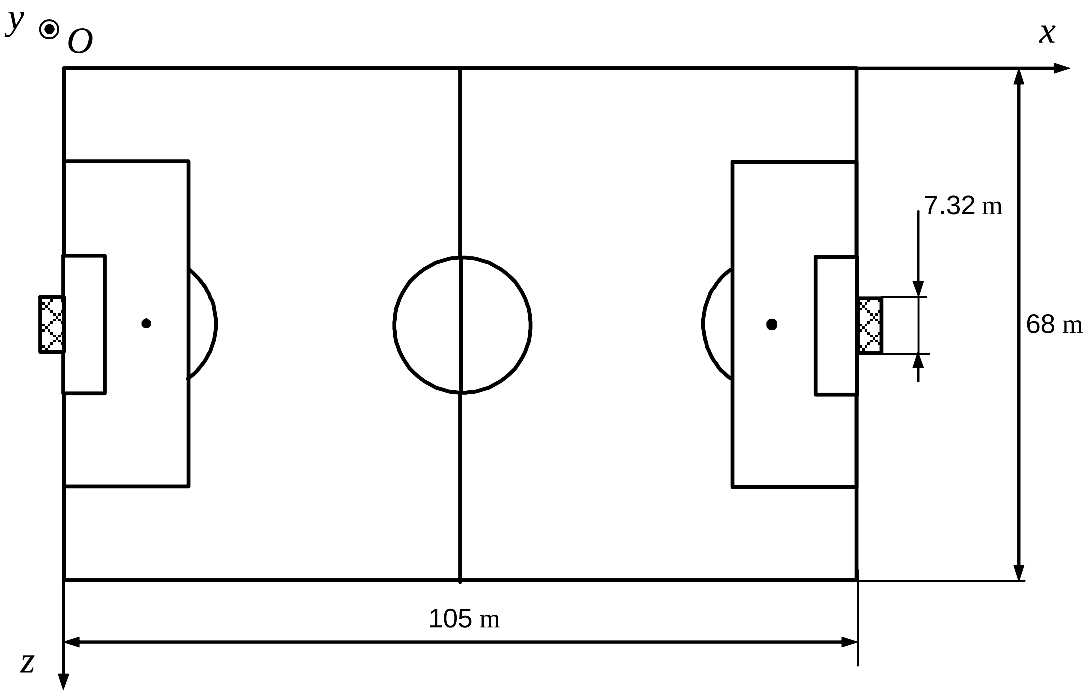

Fig. 1. The football ground.

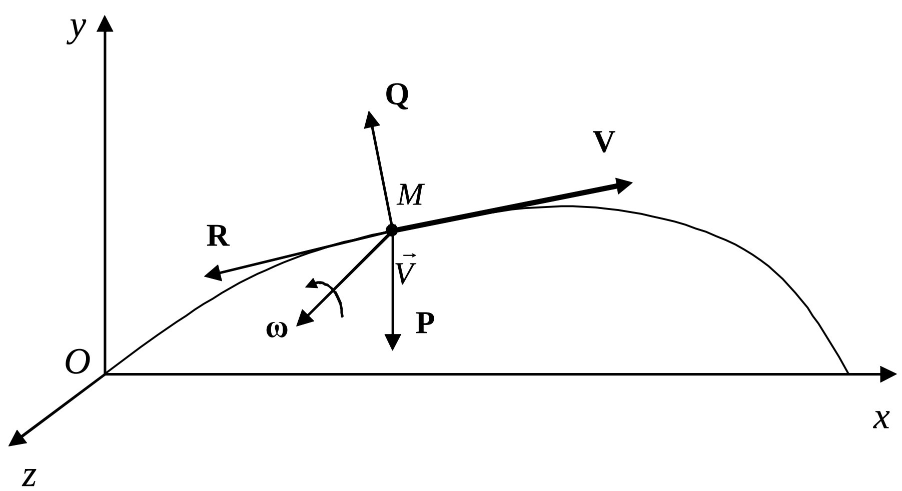

Fig. 2. The system of forces acting on a football at the reverse rotation.

$$R = \frac{1}{2}\rho\, C_D\, S V^2\,, \tag{2.1}$$

$$Q = \frac{1}{2}\rho\, C_L\, S V^2\,, \tag{2.2}$$

where $\boldsymbol{\rho}$ – the air density, S –the area of the largest section perpendicular to the velocity vector $\mathbf{V}$ (middle section), CD and CL – the aerodynamical coefficients: drag coefficient and lift coefficient. On account of the small magnitude the friction force moment with respect to the center of mass was ignored. In this case the angular velocity vector is constant in time during the flight. The force of gravity $\mathbf{P}$ is directed vertically down. The drag force $\mathbf{R}$ is opposed to the velocity vector $\mathbf{V}$:

$$\mathbf{R} = -R\frac{\mathbf{V}}{|\mathbf{V}|}\,. \tag{2.3}$$

It is more difficult to define the direction of the lift force $\mathbf{Q}$ due to Magnus effect. There is no theoretical substantiation of this effect but the main mechanism of the lift force occurrence lies in the fact that in the flight of the ball the contrary air flow interacts with the air flow due to rotating ball. In mixture zone the domain of elevated pressure forms. For example, in the reverse rotation of a ball the velocity of the lower point of the ball coincides with the direction of $\mathbf{V}$. Therefore the mixture zone is located in the vicinity of the lower part of the ball where the domain of elevated pressure arises. The term “lift force” is used also under other directions of the lateral aerodynamic force. In experiment the lift force is measured as a force directed perpendicular to vectors $\boldsymbol{\omega}$ and $\mathbf{V}$. Also from experiment it is known that force is directed to that side from where the shortest turn of vector $\boldsymbol{\omega}$ to vector $\mathbf{V}$ is seen counter-clockwise. This allows to use the rule of cross product of two vectors [4]:

$$\mathbf{Q} = Q\frac{\boldsymbol{\omega}\times\mathbf{V}}{|\boldsymbol{\omega}\times\mathbf{V}|}\,. \tag{2.4}$$

The central part of problem of investigation of solid movement in continuum is determination of aerodynamic coefficients. The drag coefficient for the sphere without rotation was found experimentally in large range of velocities. In paper [5] the drag coefficient C_D dependence of Reynolds number Re ($Re=Vd/\nu$, d – the diameter of a ball, ν – the kinematic air viscosity) was investigated. This dependence is shown in Fig. 3. In this diagram it can be seen that at Re values from 1000 to 140000 (it corresponds to football velocity $0.1 \div 14$ m/s) drag coefficient C_D is almost constant and it is equal to 0.45. In football the velocity changes in the large range, thereby the account must be taken of coefficient C_D dependence of the great values of Reynolds number. As can be seen from Fig. 3 the sharp decrease of coefficient C_D begins at value $\log\ Re > 5.4$ – “resistance crisis”. This effect allows to increase significantly the ball flight distance at increase of the initial velocity.

In Fig. 4 C_D and C_L dependence of the angular velocity of ball rotation for different Reynolds numbers (up to $Re \cong 10^5$) is represented [6]. It is clear that at $n = \frac{\omega\, d}{2V} > 1$ (ratio of the rotational velocity to the velocity of translation) coefficients C_D and C_L depend weakly on angular velocity and Reynolds number. According to Fig. 4 it is possible to take $C_D = 0.6$ and $C_L = 0.35$ at the fast ball rotation.

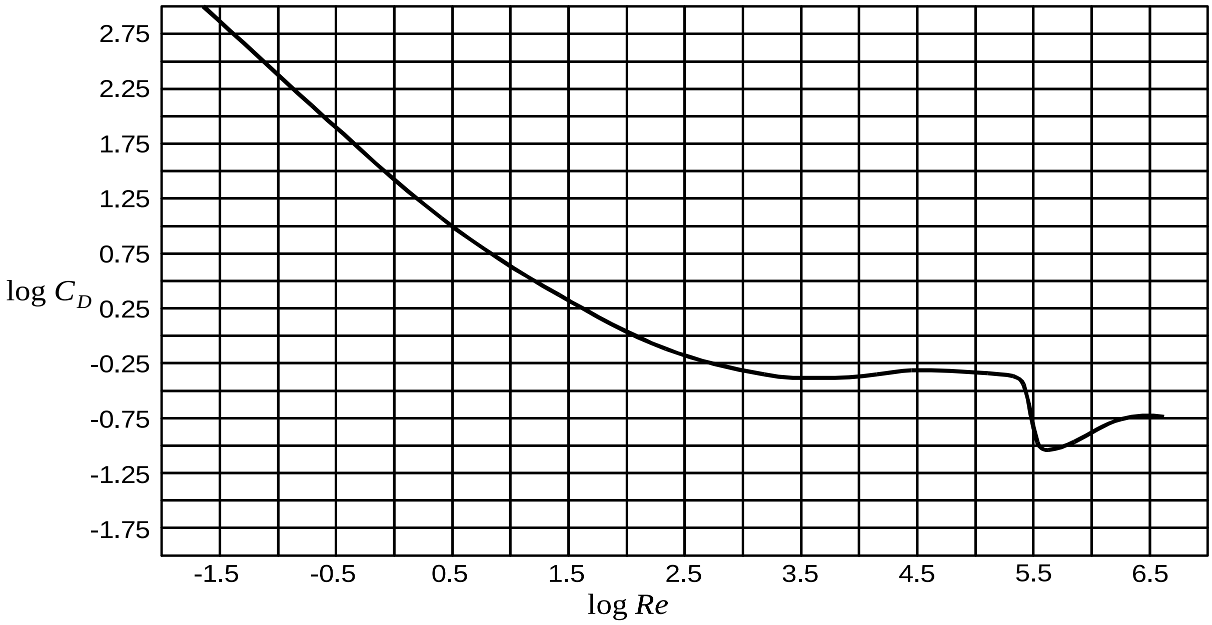

Fig. 3. Drag coefficient dependence of Reynolds number.

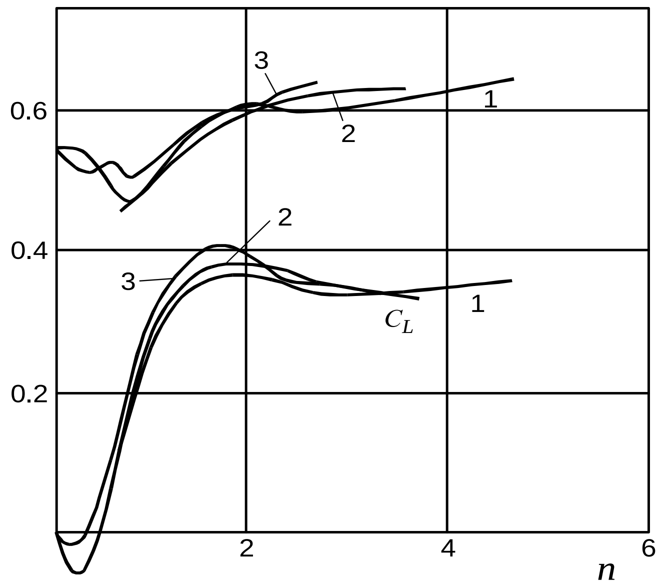

Fig. 4. Drag coefficient CD and lift coefficient CL dependence of the ratio of the rotational velocity to the velocity of translation: 1 – Re = 6.15·104, 2 – Re = 7.74·104, 3 – Re = 10.70·104.

3. Differential equations of the ball center of mass movement. Cauchy problem

By the theorem of the center of mass movement

$$m\,\mathbf{a}_c = \mathbf{P} + \mathbf{R} + \mathbf{Q}\,, \tag{3.1}$$

where $\mathbf{a}_c$ – the ball center of mass acceleration ($\mathbf{a}_c = \frac{d\mathbf{V}}{dt}$), $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ – forces described in section 2. By projecting the left and right hand sides (3.1) on system axes we can receive with the help of (2.1) – (2.4) the system of equations of center of mass movement (for convenience the projections of center of mass velocity $\mathbf{V}$ on axis x, y, z (Fig. 2) and denoted as u, v, w):

$$m\frac{du}{dt} = -R\frac{u}{V} + Q\frac{(\boldsymbol{\omega}\times\mathbf{V})_x}{|\boldsymbol{\omega}\times\mathbf{V}|},$$
$$m\frac{dv}{dt} = -R\frac{v}{V} + Q\frac{(\boldsymbol{\omega}\times\mathbf{V})_y}{|\boldsymbol{\omega}\times\mathbf{V}|} - mg,$$
$$m\frac{dw}{dt} = -R\frac{w}{V} + Q\frac{(\boldsymbol{\omega}\times\mathbf{V})_z}{|\boldsymbol{\omega}\times\mathbf{V}|},$$
$$u = \frac{dx}{dt},\ v = \frac{dy}{dt},\ w = \frac{dz}{dt}, \tag{3.2}$$
$$V = \sqrt{u^2 + v^2 + w^2},$$

where R, Q are defined by (2.1) and (2.2), x, y, z – the ball center of mass coordinates. The given parameters of the problem are: $g = 9.8$ m /s^2, $\rho = 1.23$ kg/m^3, $m = 0.4$ kg, $S = \pi\frac{d^2}{4}$ ($d = 0.22$ m). In the case of the central impact $C_D = C_D(Re)$, $C_L = 0$, at the eccentric impact $C_D = 0.6$, $C_L = 0.35$. For the initial–value problem it is necessary to preset coordinates and velocity of the center of mass at the initial time moment:

$$t = 0:\quad \begin{array}{ll} x = x_0, & u = u_0, \\ y = y_0, & v = v_0, \\ z = z_0, & w = w_0, \end{array} \quad . \tag{3.3}$$

Cauchy problem (3.2) – (3.3) was solved numerically using the method of step-by-step integration (Euler's method). The integration step was chosen so that calculation result was independent of the step parameter ($\Delta t = 0.001$ s).

4. Ball trajectories calculation at standard positions

For a finding optimal ball trajectories it is necessary to solve a problem of optimization. At kick on a ball we may vary three parameters: the module of the ball initial velocity V_0, the angle between the plane of a ground (*Oxz*) and the vector of the initial velocity α and the angle between the plane of a goal (*Oyz*) and the vector of the initial velocity β (Fig. 1). The aim of this optimization is reception of ball trajectories, moving on which the ball will hit the goal under the goal crossbeam for the minimal time. Then the problem is written as:

$$t(V_0, \alpha, \beta) \rightarrow \min, \tag{4.1}$$

on condition that

$$H' \leq y(z = L) < H, \tag{4.2}$$

where H – is the goal height minus the ball radius, H' – is the minimal allowable height of a ball in the goal. Also it is necessary to impose restriction on the maximal speed of the kick:

$$V < V_{\max}, \tag{4.3}$$

were $V_{\max}$ – is the maximal allowable initial ball velocity.

4.1. Center kick

It happened several times that a football-player kicked a ball from the center of a ground and scored a goal. Basically it was goalkeeper's fault in most occasions, but forward's skill is the main thing. All football funs know that kick of Pellet, when he scored a goal to Bulgaria right from the center of the football ground.

We calculated the possible trajectories of a ball and founded an amazing fact. If drag coefficient C_D is constant and equal to 0.45 (as usually accepted), the ball initial velocity should be more than 50 m/s to reach the goal. Even the strongest players can receive hardly such velocity.

Test of C_D coefficient dependence of Reynolds's number (Fig. 3) shows that so-called drop of resistance starts at $Re_* = 3.5 \cdot 10^5$ i.e. value of aerodynamic drag coefficient slumps. The physical reason of this event is that air flows around a ball moving at a high speed separates from the ball surface closer to the rear part. At the same time the eddy diameter decreases and improves the ball streamlining. For football it starts at the velocity:

$$V_* = \frac{Re_* \nu}{d}. \qquad (4.4)$$

Since air the kinematic viscosity $\nu = 0.15 \cdot 10^{-4}$, the ball diameter $d = 0.22\,\text{m}$, so critical speed $V_* \cong 14\,\text{m/s}$. We consider C_D dependence of the ball velocity. This dependence is shown in Table 1.

Table 1.

V_0 m/c	14	15	16	17	18	19	20	25	30	35	40	45
C_D	0.45	0.41	0.38	0.35	0.28	0.2	0.14	0.12	0.09	0.09	0.1	0.095

If the velocity is under $14\,\text{m/s}$, then $C_D = 0.45$. This is a result of data processing in Fig. 3.

The optimal ball trajectory is shown in Fig. 5. We used Table 1 data to calculate this trajectory taking into account C_D dependence of the ball velocity. The minimal initial velocity of our ball should be 30 m/s at kick angle $\alpha = 29^O$ to score a goal. The ball flight time is 2.7 s. By the way it took 4 seconds to score the "quickest" goal from the center of the ground. According to our calculations the ball flight time from the center to the goal is $t < 3$ s. Real and estimated time comparison gives a certain assurance in the exactness of our calculations.

Curve 1 in Fig. 5 represents the ball trajectory, line 2 is drawn at the goal' height. The origin of the coordinates is in the ground corner (Fig. 1). It is clear that the trajectory is a ballistic type of curve: it is gentle at the beginning and deep at the end. A goalkeeper can catch a ball staying basically in the goal range. Our calculations let us estimate a safe distance for a goalkeeper to come back to his goal. We tried to determine the height of optimal trajectory and the time of ball flight dependence of its initial velocity (Figs. 6 and 7).

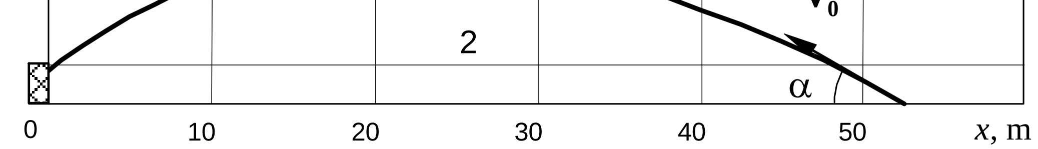

Fig. 5. Optimal ball trajectory at center kick $V_0 = 30\,\text{m/s}$, $\alpha = 29^\circ$.

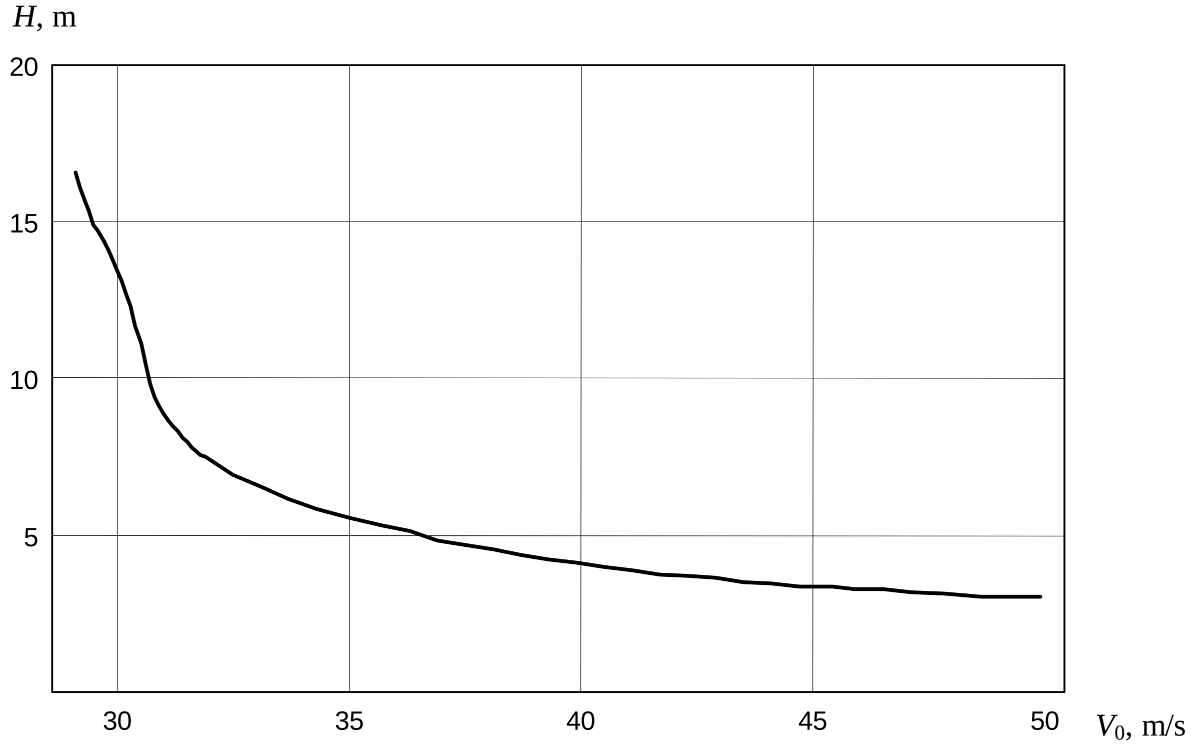

Fig. 6. Dependence of maximum ball trajectory height of its initial velocity.

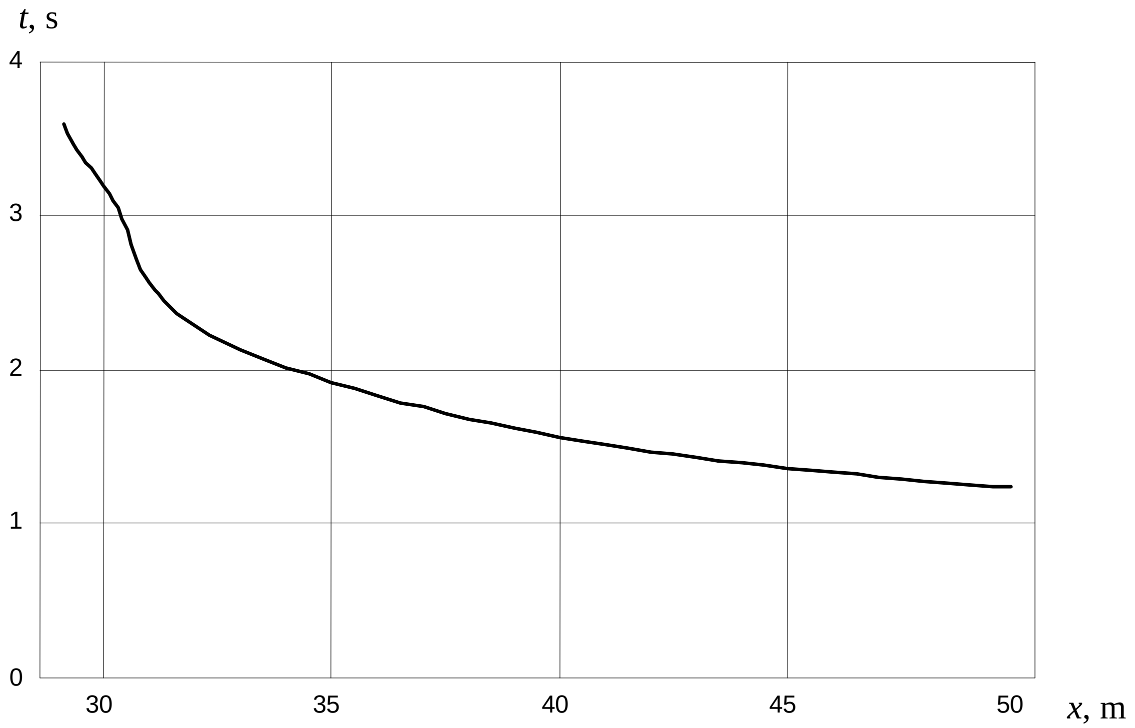

Fig. 7. Ball flight time in dependence of its initial velocity.

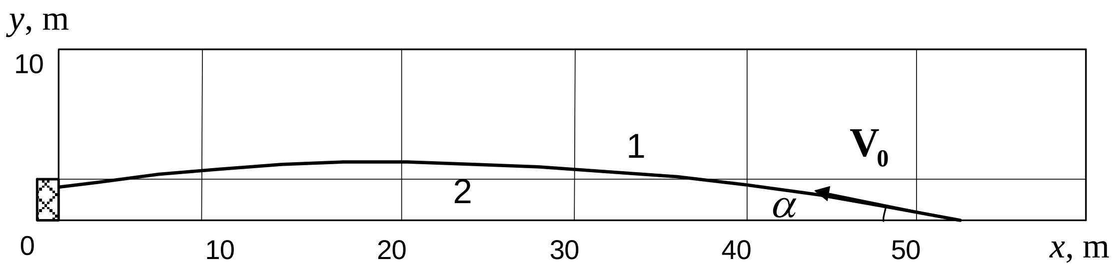

Fig. 8. Ball trajectory at center kick $V_0 = 45\,\mathrm{m/s}$, $\alpha = 11^\circ$.

A dependence of maximum ball trajectory height of its initial velocity is shown in Fig. 6. Figure 7 represents a ball flight time in dependence of its initial velocity. Our measurements have shown that at height $H > 10$ meters the ball flight time is approximately 3 seconds and these results correspond to our calculations. If we increase the initial kick velocity up to $V_0 = 45\,\mathrm{m/s}$ (as follows from Fig. 7), the ball flight time decreases more than twice (see the trajectory in Fig. 8). In this case the trajectory is more gentle, its height is 3.5 m and the flight time is 1.4 sec.

It is obviously that the initial velocity increase from 28 m/s to 35 m/s reduces the flight time in 1.8 times, so a possibility to score a goal increases also as compared to computational results. We used records of TV football match to measure the flight time and estimate the trajectory height of a ball kicked from the goal. According to our analysis the maximum velocity can reach 35 m/s. Although it is possible that the strongest football-players could impart to a ball the higher velocity.

4.2. Corner kick

In this problem we calculated the possible ball trajectories for corner kick. In our calculations (as follows from Fig. 4) we assumed that the drag coefficient $C_D = 0.6$ and the lift coefficient $C_L = 0.45$.

The ball trajectories in vertical and horizontal projections are shown in Fig. 9 and Fig. 10. The minimal initial velocity should be 30 m/s to score a goal. So big value of velocity is explained by the rotation of a ball, thereby drag coefficient C_D is increased. Also the trajectory is extended due to a deviation of a ball from the vertical plane. According to Fig.9 the maximum flight height is about 6 m and according to Fig.10 the maximum ball deviation from the goal line is about 2 m.

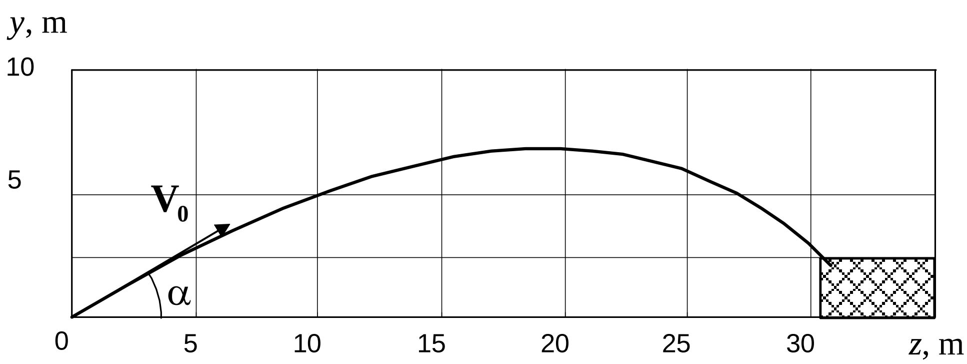

Fig. 9. Ball trajectory at corner kick in goal plane projection (*Oyz*), $V_0 = 30\,\mathrm{m/s}$, $\alpha = 30^\circ$, $\beta = 19^\circ$.

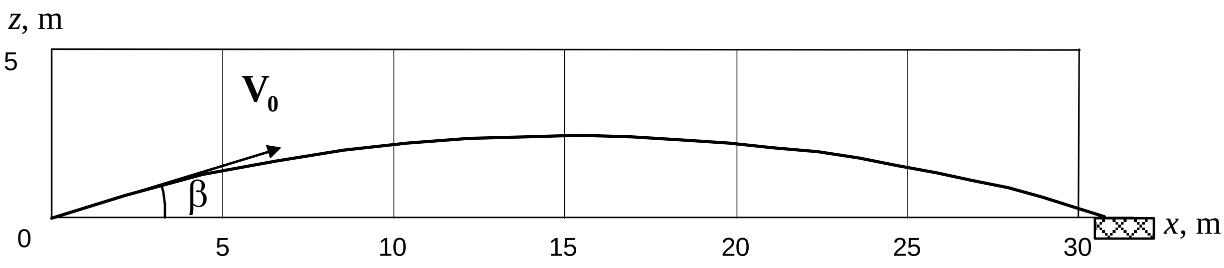

Fig. 10. Ball trajectory at corner kick in horizontal plane projection (*Oxz*), $V_0 = 30\,\mathrm{m/s}$, $\alpha = 30^{\circ}$, $\beta = 19^{\circ}$ (top view).

5. Eccentric kick on a ball

Using an eccentric kick a football-player can generate rotary and translational motion. It is necessary to examine an impact interaction of player's foot and a ball to determine the initial velocity. Let us assume that the velocity of football boot does not change at impact (player's foot mass exceeds ball's mass many times over) and there is no slipping of football boot on a ball.

A ball is at rest before impact. The ball's mass center after impact acquires velocity **U** and the ball also starts the rotation around the mass center with the angular velocity ω. The angle φ that determines the impact point and the boot velocity V are known.

We have to find impact angle α, the ball's mass center velocity after impact U and the angular velocity ω (Fig. 11).

Let us use the impact theory to solve the task. We write down the theorems of momentum and of angular momentum for the impact:

$$mU_\tau = S_\tau,$$

$$I_C\omega = S_\tau r, \qquad (5.1)$$

where S_τ is the tangent component of impulse of the impact force, *m* is the ball mass, r is the ball radius, $I_C = \frac{2}{5}mr^2$ is the moment of ball's inertia with respect to the central axis.

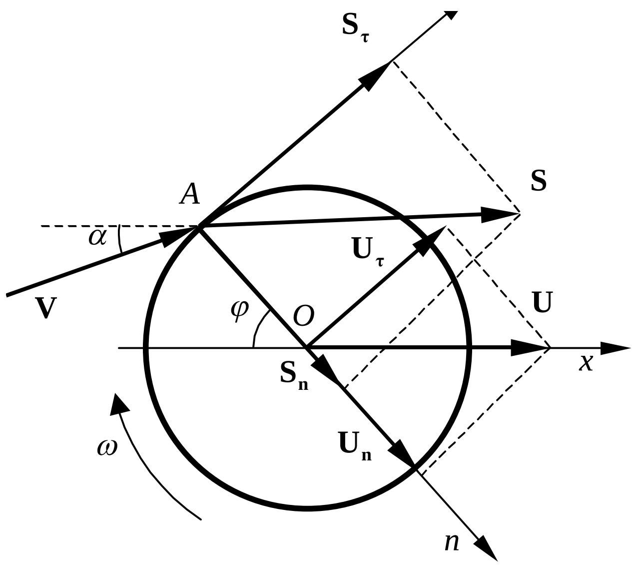

Fig. 11. Eccentric kick on a ball.

The coefficient of restitution is equal to the ratio of relative velocities of a ball after and before the impact:

$$k = \frac{U_n - V_n}{V_n}, \tag{5.2}$$

where V_n is the boot velocity $\mathbf{V}$ projection on axis *n*:

$$V_n = V\cos(\alpha + \varphi). \tag{5.3}$$

The tangent component of velocity $\mathbf{V}$:

$$V_\tau = V\sin(\alpha + \varphi). \tag{5.4}$$

Considering no slipping of football boot on a ball the theorem of addition of velocities yields:

$$V_\tau = U_\tau + \omega\ r. \tag{5.5}$$

We can find all sought quantities from equations (5.1) - (5.4):

$$tg(\alpha + \varphi) = 2.5(1 + k)\,tg\varphi, \tag{5.6}$$

$$U = \frac{(1 + k)V\cos(\alpha + \varphi)}{\cos\varphi}, \tag{5.7}$$

$$\omega = \frac{5}{2} r(1 + k)V\cos(\alpha + \varphi)\,tg\varphi. \tag{5.8}$$

Equation (5.5) defines angle α, (5.7) and (5.8) − translational and rotary velocities of a ball. For the central impact we have $\alpha = 0,\ \varphi = 0$. Using formulas (5.6) and (5.8) we can receive the following values for the ball's mass center velocity U and angular velocity ω:

$$U = (1 + k)V,\ \omega = 0. \tag{5.9}$$

We have got the experimental data of coefficient of restitution k for volley-ball rebound from solid surface $k = 0.65$ [2].

We use this value to estimate the football take-off velocity. With (possible for football player) the kick velocity $V = 20$ m/s we can calculate the ball's take-off velocity after impact $U = 33$ m/s, it is sufficiently to score a goal at center and corner kick.

6. Method of aerodynamic coefficients determination

There are no experimental data for coefficients value C_D and C_L at high velocity with ball rotation. According to Fig. 4 they are known up to $V = 6.8\,\text{m/s}$ i.e. up to usual ball velocity. It is actually to conduct an experiment to receive those coefficients.

The velocities higher than $V = 6.8\,\text{m/s}$ were used for examination of ball rotation. Reynolds's number basically does not influence on drag and lifting force coefficients at high rotation rate (see Fig. 4). Nevertheless there is a need for new experiments to determine coefficients C_D and C_L. We offer using methods suggested in paper [1] to determine C_D for volley-ball flight and developed in paper [2] to determine C_D and C_L for ski-jumper flight. This model is based on object movement record analysis that determines its mass center velocities and accelerations. Differential equations of the mass center movement are used to determine drag and lift force coefficients. Only plain trajectories are considered in papers [1] and [4].

We consider the three-dimensional movement of the football. The differential equations of the center of mass movement are:

$$\begin{cases} m\ddot{x}_c = R_x + Q_x, \\ m\ddot{y}_c = R_y + Q_y, \\ m\ddot{z}_c = -mg + R_z + Q_z. \end{cases} \tag{6.1}$$

Substituting results of numerical differentiation of ball coordinates in (6.1) yields:

$$\begin{aligned} m\frac{d^2x_c}{dt^2} &= -\frac{\rho}{2}C_D SV^2\frac{u}{V} + \frac{\rho}{2}C_L SV^2\frac{(\boldsymbol{\omega}\times\mathbf{V})_x}{|\boldsymbol{\omega}\times\mathbf{V}|}, \\ m\frac{d^2y_c}{dt^2} &= -\frac{\rho}{2}C_D SV^2\frac{v}{V} + \frac{\rho}{2}C_L SV^2\frac{(\boldsymbol{\omega}\times\mathbf{V})_y}{|\boldsymbol{\omega}\times\mathbf{V}|}, \\ m\frac{d^2z_c}{dt^2} &= -mg - \frac{\rho}{2}C_D SV^2\frac{w}{V} + \frac{\rho}{2}C_L SV^2\frac{(\boldsymbol{\omega}\times\mathbf{V})_z}{|\boldsymbol{\omega}\times\mathbf{V}|}, \\ u = \frac{dx_c}{dt},\ v &= \frac{dy_c}{dt},\ w = \frac{dz_c}{dt}, \\ V &= \sqrt{\ddot{x}_c^2 + \ddot{y}_c^2 + \ddot{z}_c^2}\ . \end{aligned} \tag{6.2}$$

We suggest to video-record the ball flight from two points at the right angles. This record will determine the coordinates of the center of mass as functions of time:

$$\begin{aligned} x_c &= x_c(t), \\ y_c &= y_c(t), \\ z_c &= z_c(t). \end{aligned} \tag{6.3}$$

Thus only two unknown values C_D and C_L are contained in three equations (6.3). Those values could be determined from first two equations (6.3) and the third equation could be used to check the method accuracy. It should be fully identical while substituting the coefficients determined.

This paper shows only the determination method of C_D and C_L.

7. Conclusions

The mathematical model of the football mass center motion under the action of gravity, drag and lift forces is developed. Several standard ball positions are examined. The optimal trajectory is calculated for ball kicked from the center of the ground to score a goal. The drop of resistance is the main point to consider. Calculations without considering drop of resistance cause the results that do not correspond to practice. According to the calculations the initial velocity increase causes the essential reduction of the ball flight time. In this case ball moves basically along whole trajectory with the drop of resistance and this cause the lesser resistance. Optimal corner kick trajectory with ball rotation is calculated to aim a ball into the goal. The football kick is examined. The ball initial velocity after kick is estimated. A method of drag and lift coefficients determination is suggested. This method is based on ball flight video-records processing. The results could be used by trainers and football-players to improve performance in the standard positions.

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ВЛИЯНИЕ КРИЗИСА СОПРОТИВЛЕНИЯ НА ДИНАМИКУ ПОЛЕТА ФУТБОЛЬНОГО МЯЧА

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При игре в футбол важнейшую роль играет траектория мяча, обеспечивающая правильный пас или поражение ворот соперника. Часто к успеху приводит реализация так называемых стандартных положений. Траектория мяча представляет собой пространственную кривую, вид которой зависит от начальных условий движения мяча. Цель работы – исследование оптимальных начальных условий, при которых мяч попадет в ворота соперника за минимальное время. В литературе по биомеханике спорта известны работы по математическому моделированию движения волейбольного мяча и теннисного шарика [1] – [4]. В настоящей работе проведено теоретическое исследование фазы полета футбольного мяча и решены задачи исполнения стандартных положений – удар с центра поля по воротам и углового удара. Особенностью полученных решений является то, что учтена зависимость аэродинамического коэффициента сопротивления от скорости мяча. Это позволило получить результаты, согласующиеся с экспериментальными данными. Рассмотрен также удар бутсой по мячу и представлена методика определения аэродинамических коэффициентов лобового сопротивления и подъемной силы для общего случая пространственной траектории мяча. Полученные результаты имеют не только научное, но и прикладное значение. Они могут быть использованы тренерами и футболистами при отработке техники исполнения стандартных положений. Библ. 6.

Ключевые слова: биомеханика спорта, модель движения футбольного мяча, аэродинамические коэффициенты, оптимальное решение

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