A SELF-REGULATION MODEL OF BLOOD CIRCULATION AT ACTIVE DEFORMATION BEHAVIOUR OF VESSELS

S.V. Shilko

V.A. Belyi Metal-Polymer Research Institute of National Academy of Sciences of Belarus, 32a, Kirov Street, 246050, Gomel, Belarus

Abstract: In terms of moving boundaries localization a model of circulation system adaptation to varying functioning conditions through self-regulating permeability based on passive and active (muscle) reaction of vessels is proposed.

Key words: blood circulation, arterial pressure, adaptation, moving boundaries, vessel deformation, muscle tonus, anisotropy

Introduction

It is to be ascertained that most of organism pathologies are connected with different disturbances in blood supply. Investigation of self-regulation of blood flow is very actual and must take into account a number of factors, including ones connected with peculiarities of living tissue deformation [1-5].

Self-regulation contributes to constancy of blood flow under considerable variations of arterial pressure. It is observed in fact in all organs and tissues and is most prominent in brain and kidneys, where blood flow is invariable under varying arterial pressure within 80-160 torrs.

However a biomechanical aspect of this process has been insufficiently investigated due to several reasons:

- extremely complex deformation mechanism of biotissues;
- branching of the circulatory system (CS);
- sensitivity of hemodynamic characteristics to chemical and physical nature factors.

That is why formulation and solution of hemodynamic problem with account of different-scale vessels anisotropy and physical nonlinearity of biotissues are of great significance. Of interest are, in particular, modelling and estimation of the efficiency of blood flow self-regulation as a consequence of active deformation behaviour of precapillaries on the example of quite lesser arteries (arterioles) which are just discussed in the present paper.

CS self-regulation signs

The CS is known to be divided into the central part and the peripheral circulation. A number of useful reactions providing for optimum functioning of the system under different effects takes place. Analysis of clinical observations and experimental studies summarized in [1-4] makes grounds to specify an important adaptive reaction of the CS assisting in optimum blood supply under varying outer conditions, namely functional load, temperature and environmental conditions.

The reaction is displayed in self-regulation of blood flow proceeding from priority of brain and other vital organs supply at the expense of reducing blood flow at the periphery (e.g., in the area of skin integument). This adaptive reaction is the result of either reduced or increased permeability of blood channel. When we speak about perfusion of tissues, then efficient regulation is supported by the presence of a great amount of subsidiary capillary involved in blood supply when necessary. So far, periphery blood supply is accompanied by expedient change of active vessel topography. In other words, optimal solid-liquid phase interfaces are formed in biotissues.

It follows that observed redistribution of blood flow could be described in terms of localization principle (stabilization of moving boundaries). Like in other biostructures with moving boundaries [4], localization result is not given a priory, but is determined in the process of numerical studies by a certain criterion, similar with condition of attained full-strength condition of solid tissues.

To choose main characteristics of localization and a regulated optimum parameter, let us employ the method of systematic analysis of adaptive systems suggested in [7].

Vessel lumina (inner diameter) is used in this case as a movable boundary effecting volume of blood flow in circulation zone. So, the regulated parameter is inner contour diameter in the considered cross section. Its value is defined by a competing effect of blood pressure tending to expand the vessel and opposite directed forces from the side of surrounding tissues, elastic (passive) reaction of walls and active reaction (tonus) of smooth-muscle elements of vessels.

The role of active reaction conditioned by tonus is especially pronounced in arterioles having most developed muscle layer.

Experimental data prove that a threshold pressure value p_0 exists below which vessel permeability is absent due to closure of walls at uncompensated initial tonus of muscular tissue [4]. Hence, vessel tonus, i.e. stress originated at muscular contraction, is the governing parameter.

Proceeding from above, localization criterion will be a condition of minimum deviation from sufficient blood supply (provision of physiologically normal minute volume) to vital organs.

CS model in terms of moving boundaries

To investigate CS self-regulation let us consider a model of arterial blood supply including different-scale vessels (in Fig.1a the minimum set of single arterial vessels is shown). Heart fulfilling pumping function supplies blood to the main vessel - aorta. Then blood passes sequentially medium and small arteries, arterioles and a net of capillaries at the final stage of perfusion. Because of different sizes of vessels a stepwise change of blood flow takes place in the branching points. Thus, vascular channel can be presented as an inhomogeneous structure with varying permeability (Fig.1b).

Suppose that effective diameter d characterizing permeability of a structure, blood pressure p and blood flow velocity v through some vessel section are interrelated through elastohydrodynamic relations. Besides, let pressure values p^E be known in some vessel points. The model of adaptive reaction under discussion is reduced to formulation of an extreme problem with bounds as inequalities. It is necessary to find distribution of channel's effective diameter $d(x)$ along vessel length l which minimizes the error functional of both theoretical (calculated) and experimental diameter d values

$$J_1(d) = \int_0^l (d^E(x) - d^T(x))^2\, dx \qquad (1)$$

with limits as inequalities defining wall closure upon critical pressure p_0 attainment

$$p < p_0 \;\Rightarrow\; d = 0; \qquad p = p_0 \Rightarrow d > 0 \qquad (2)$$

in case of meeting the condition of relevant blood flow volume Q in a chosen cross-section $S(x)$ during time T

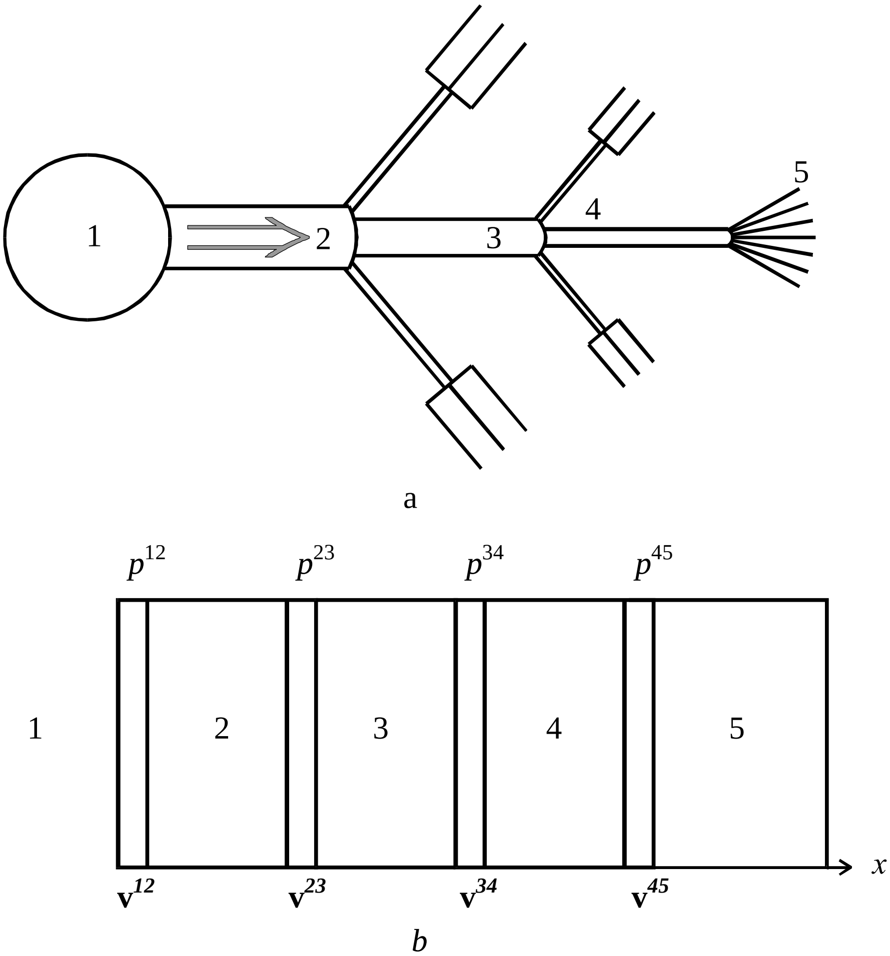


Fig. 1. Scale levels of arterial blood flow (a) and its interpretation (b) in terms of moving boundaries: 1 – heart; 2 – aorta; 3 – arteries; 4 – arterioles; 5 – capillaries.

$$\int_T S(x,t)v(x,t)dt = Q\,, \qquad (3)$$

where $S(x,t) = \dfrac{\pi d^2(x,t)}{4}$.

To fulfil condition (3) let us introduce the penalty function into functional (1)

$$J_2(d) = \frac{1}{\varepsilon}(\int_T (S(t)v(t)dt - Q)^2\,, \qquad (4)$$

where $\varepsilon > 0$ is the penalty parameter.

Even when p^E is measured correctly, distribution $d(x)$ is calculated with fluctuations due to incorrectness of analogous problem. To stabilize the solution one can demand its smoothness, i.e. elimination of $d(x)$ oscillations on the length l. Since oscillations are characterized by high values of the first derivative, the following functional can be used as a stabilizer

$$J_3(d) = \alpha \int_l d'(x)^2\, dx\,, \qquad (5)$$

where $a > 0$ is the regularization parameter. So, we finally have

$$min_d\{ J_1(d) + J_2(d) + J_3(d)\}. \qquad (6)$$

Deformation behaviour of vessels

To specify relations (1) - (6) let us discuss the possibility of determining dependence of vessel diameter on arterial pressure $d = d(p)$. Vessel walls possess complex structure being built of concentric layers of elastin laminae and collagen fibres. Upon their deformation, the relaxation properties are generated. For most of biotissues relaxation time limit τ is, nevertheless, very small - about 10^{-8} s, that is, such materials can be to be considered elastic. It simplifies much stress-strain analysis.

Experiments, e.g. in [9], also proved that artery walls behave a like nonlinear homogeneous anisotropic compressible material whose properties are described by six elastic constants at each loading level. More complex situation is with arterioles which are unique arterial vessels having a muscular layer and, consequently, showing an active deformation behaviour. Particularly, arterioles with 70-100 μm diameter have an additional outer layer formed by three rows of muscular cells (5 μm thick) [3].

Direct tests of such small-size vessels are rather problematic, so only estimates of mechanical characteristics obtained for large arteries can be employed in calculations.

As for arteriole and capillary scale, thickness of vessel walls is commensurable with the radius. Loads on the vessels involve arterial pressure *p* applied to the inner surface, axial force *N* characterizing vessel wall tension *in vivo*, pressure of neighbouring tissues and (for arterioles) additional circumferential stresses σ_θ conditioned by outer muscular layer tonus. Analytical solutions (like dependencies for thin-walled shells and Lame formula for homogeneous isotropic cylinders) which do not consider the pointed peculiarities of deformation and loading probably introduce a noticeable error. So far, we discussed a model of a blood vessel as a two-layer cylinder with coaxially located main (inner) and muscular (outer) layers from anisotropic nonlinear material (Fig. 2). Geometrical characteristics of the model were given based on the known data listed in Table 1 for shoulder artery and arteriole [1].

Table 1. Initial data for vessel deformation calculations.

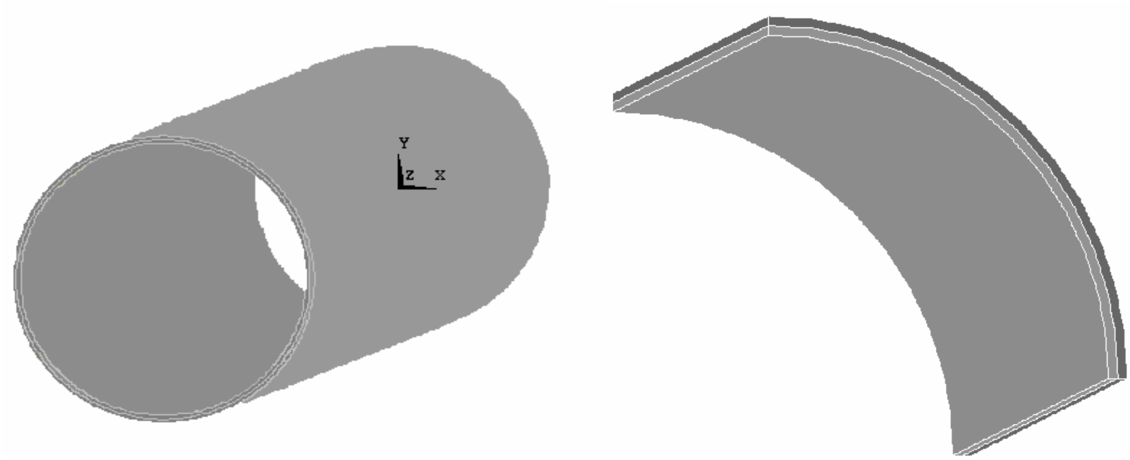
Vessel type	Geometrical characteristics				Tonus stress	Tension force, *N*
	Diameter, mm	Length, mm	Total wall thickness, mm	Muscular layer thickness, mm		
Shoulder artery	3	20	0.5	0	0	$0.32\cdot10^{-6}$
Arteriole	0.007	0.09	0.001	0.0002	σ_θ	$75\cdot10^{-6}$

To simulate valve function of the vessel the initial stress $\overset{\circ}{\sigma}_\theta$ in the outer layer was set which corresponded to threshold blood pressure value p_0 below which vessel permeability becomes equal to zero.

When deformation behaviour was studied, elastic constants cited in Table 2 were obtained with taking into account experimental data for a human shoulder artery [9].

Table 2. Elasticity constants of shoulder artery material.

	N = 0						*N* = 0.32 N					
p, torr	E_θ, MPa	$E_{z\theta}$, MPa	$E_{r\theta}$, MPa	μ_{rz}	$\mu_{z\theta}$	$\mu_{r\theta}$	E_θ, MPa	$E_{z\theta}$, MPa	$E_{r\theta}$, MPa	μ_{rz}	$\mu_{z\theta}$	$\mu_{r\theta}$
50	0.6	0.3	0.2	0.75	0.5	0	1.70	2.25	0.60	0.10	0.90	0.40
100	1.3	0.6	0.3	0.60	0.5	0	2.30	2.40	0.70	0.25	0.79	0.25
150	2.2	1.1	0.4	0.55	0.5	0	3.00	2.65	0.75	0.30	0.75	0.15
200	2.9	1.6	0.6	0.6	0.5	0	3.50	2.80	0.76	0.40	0.70	0.10
250	3.7	1.9	0.8	0.75	0.5	0	3.75	2.85	0.78	0.45	0.65	0.12



a *b*

Fig. 2. Vessel model (a) and fragment of its wall (b).

Conclusions

Biomechanical analysis of blood flow self-regulation made grounds for formulation of a problem on blood flow redistribution in terms of localization principle (control of moving boundaries). The main characteristics of localization process can be distinguished, namely: regulated parameter - inner diameter (lumen) of the vessel, regulating parameter - vessel tonus, and optimum criterion as a physiologically normal minute volume of circulation. To describe deformation behaviour of vessels, including regulating function of arterioles a model has been proposed consisting of a two-layer hollow shell from anisotropic nonlinear material with a prestressed outer layer.

Analysis of vessel deformation and calculation of blood flow characteristics using the hydrodynamic theory will follow in ensuing publication.

Acknowledgements

The author would like to express his gratitude to the Scientific Center of Surgery of RAMS (Moscow) for inducement of the present work.

References

1. САВИЦКИЙ Н.Н. **Биофизические основы кровообращения и клинические методы изучения гемодинамики**. Ленинград, Медицина, 1963 (in Russian).
2. ОСИПОВ В.П. **Основы искусственного кровообращения**. Москва, Медицина, 1976 (in Russian).
3. ДЖОНСОН П. **Периферическое кровообращение**. Москва, Медицина, 1982 (in Russian).
4. ЧЕРНУХ А.М., АЛЕКСАНДРОВ П.Н., АЛЕКСЕЕВ О.В. **Микроциркуляция**. Москва: Медицина, 1975 (in Russian).
5. NASYROV R.V., BAKUSOV L.M. A model of central venous hemodynamics. **Russian Journal of Biomechanics**, 3(3): 52-55, 1999.
6. ШИЛЬКО С.В. Адаптивность дентального аппарата и напряженное состояние зуба человека. **Механика композитных материалов и конс**трукций, 5(1): 49-59, 1999 (in Russian).
7. ПЛЕСКАЧЕВСКИЙ Ю.М., ШИЛЬКО С.В., СТЕЛЬМАХ С.В. Этапы развития и уровни организации структуры материалов: адаптивные композиты. **Доклады Национальной академии наук Беларуси**, 43(5): 119-123, 1999 (in Russian).

8. ТИХОНОВ А.Н., АРСЕНИН В.Я. **Методы решения некорректных задач**. Москва, Наука, 1979 (in Russian).
9. ТИКНЕР Е.Г., САКС А. Теория статического упругого поведения кровеносных сосудов. **Механика**, 19(6): 138-155, 1968 (in Russian).

МОДЕЛЬ АВТОРЕГУЛЯЦИИ КРОВООБРАЩЕНИЯ ПРИ АКТИВНОМ ДЕФОРМАЦИОННОМ ПОВЕДЕНИИ СОСУДОВ

С.В. Шилько (Гомель, Беларусь)

Обсуждается деформационный механизм регуляции периферийного артериального кровотока, способствующей оптимальному функционированию сердечно – сосудистой системы. Предлагаемая модель основана на формулировке задачи локализации (управления подвижными границами) просвета сосудов различного масштабного уровня. Для выбора основных характеристик процесса локализации - регулируемого и управляющего параметров, а также критерия оптимальности использована методика системного анализа адаптивных материалов и конструкций. В качестве критерия локализации постулируется минимальное отклонение от должного уровня кровоснабжения (физиологически нормального минутного объема) важнейших отделов организма.

Исследуется актуаторная функция прекапилляров как способность изменять проницаемость посредством мышечного тонуса при наличии управляющего сигнала. С этой целью учитывается начальное напряжение во внешнем слое, соответствующее пороговому значению давления крови, ниже которого просвет сосуда становится нулевым. Для расчета радиальных перемещений стенок сосудов рассматривается двухслойный цилиндр из ортотропного нелинейно - упругого материала с преднапряженным внешним слоем. Характеристики материала (модули Юнга и коэффициенты Пуассона) определяются на основании известных экспериментальных данных. Библ. 9.

Ключевые слова: кровообращение, артериальное давление, адаптация, подвижные границы, деформация сосудов, мышечный тонус, анизотропия

Received 22 November 1999