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OPTIMAL DESIGN OF REMOVABLE LAMINAR MAXILLARY DENTURES

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Abstract: The removable laminar denture cannot completely restore ability to masticate due to low threshold value of pain sensitivity of the mucosa under the denture. The optimal design problem consists in finding such a denture basis thickness that provides the maximal masticatory force without pain in the mucosa and the absence of fatigue cracks in the denture. The finite element model of the denture together with the mucosa is used to determine stresses in the denture and pressure on the mucosa. The dependence of the optimal basis thickness on elastic properties of a basis material is investigated. The influence of the Poisson's ratio was found negligibly small and the optimal thickness with respect to the Young's modulus curve in logarithmical coordinates appeared to be very close to the straight line. It was ascertained that for the basis with the optimal thickness the maximum relative masticatory pressure (the masticatory pressure with respect to the pain threshold pressure) loads both the alveolar process and the palatine torus areas.

Key words: removable laminar denture, mucosa, threshold of pain sensitivity, optimal design

Introduction

During mastication, the basis of the removable laminar denture distributes the load applied to the artificial teeth on the mucosa of the prosthetic bed. As the abutment function is not natural for the mucosa, low threshold of pain sensitivity protects it from negative action of the masticatory pressure. Nevertheless functional loading of the laminar denture during a long time leads to an inhomogeneous resorption of the bone tissue under the denture basis [1-4]. At the same time, low pain threshold of the mucosa limits the magnitude of the masticatory load. Therefore, the laminar denture cannot completely restore ability to masticate. The aim of this work is to augment masticatory ability and to slow bone resorption by means of optimal design of the laminar denture structure.

Formulation of the optimal design problem

Let $\mathbf{F}$ be the masticatory resultant force (further simply "the masticatory force" for briefness), i.e. the resultant force acting on the artificial teeth of the laminar maxillary denture from tooth-antagonists during mastication. Let F^{th} be the threshold magnitude of the $\mathbf{F}$, i.e. $F = F^{th}$, when a patient feels pain in any point of the mucosa. Then we can evaluate the degree of restoration of masticatory ability and consequently masticatory efficiency of the denture by the magnitude of the relative threshold masticatory force

$$F_o^{th} = \frac{F^{th}}{F_m} \cdot 100\% \,, \tag{1}$$

where F_m is the mean value of F for healthy person. F_m approximately equals $150 - 200\ N$ [4]. We take the lower margin of this range, i.e. $F_m = 150\ N$.

Let p_o be the relative masticatory pressure, that is

$$p_o(\mathbf{x}) = \frac{p(\mathbf{x})}{p_{th}(\mathbf{x})} \cdot 100\%, \tag{2}$$

where $p(\mathbf{x})$ is the pressure of the denture basis in a point $\mathbf{x}$ of the mucosa and $p_{th}(\mathbf{x})$ is the pain threshold pressure in this point. The magnitude of F_o^{th} will increase and simultaneously a process of bone resorption will slow if distribution of p_o over the prosthetic bed becomes uniform. Therefore, we can consider the magnitude of F_o^{th} not only as a measure of masticatory efficiency of the denture but also as a measure of slowing bone resorption.

Thus, the optimal design problem is to maximize F_o^{th}

$$F_o^{th} \to \max \tag{3}$$

varying some denture structure parameters.
Since by definition

$$F_o^{th} = F_o\Big|_{\max\limits_{\mathbf{x}\in S} p_o(\mathbf{x}) = 100\%}, \tag{4}$$

where S is the prosthetic bed domain, then the equivalent formulation of the optimal design problem is

$$F_o \to \max \tag{5}$$

with constraint

$$\max_{\mathbf{x}\in S} p_o(\mathbf{x}) = 100\%. \tag{6}$$

If we increase the threshold magnitude of the masticatory force, stresses in the denture will grow. Therefore, the following two constraints are fatigue strength conditions. The artificial teeth are made of plastic and the denture basis is made of the same plastic or metal. For these plastic materials, we use the theory of maximum strain energy due to distortion [5]. In accordance with this theory, the maximum equivalent stress in the basis or in the artificial teeth must be less than the respective fatigue limit

$$\max_{\mathbf{x}\in V_b} \sigma_o^b(\mathbf{x}) < 100\%, \tag{7}$$

$$\max_{\mathbf{x}\in V_t} \sigma_o^t(\mathbf{x}) < 100\%, \tag{8}$$

where

$$\sigma_o^b(\mathbf{x}) = \frac{\sigma_e^b(\mathbf{x})}{\sigma_{-1}^t} \cdot 100\% , \tag{9}$$

$$\sigma_o^t(\mathbf{x}) = \frac{\sigma_e^t(\mathbf{x})}{\sigma_{-1}^t} \cdot 100\% . \tag{10}$$

Here V_b and V_t are the domains occupied by the basis and the teeth, respectively, $\sigma_e^b(\mathbf{x})$ and $\sigma_e^t(\mathbf{x})$ are the equivalent stresses in a point $\mathbf{x}$ of the domains V_b and V_t, respectively, σ_{-1}^b and σ_{-1}^t are the fatigue limits for materials of the basis and teeth, respectively.

In most cases, optimizing some structure, we vary its geometric parameters and structure materials properties. In our case, the form and dimensions of the artificial teeth are fixed and the palatine form defines the form of the basis contact surface. Thus, the single geometric parameter to be controlled is the basis thickness t. This thickness is assumed to be constant over all domain S since the technology of making the denture basis with the constant thickness is simpler.

For thickness t we have the following natural constraint

$$0 < t \le 2 \text{ mm} . \tag{11}$$

When the basis thickness exceeds approximately 2 mm the disturbance of patient's speech due to difficult moving of the tongue in the reduced volume of the oral cavity becomes appreciable [1].

A material failed due to fatigue is known to remain elastic up to the fracture moment. Therefore, the Young's modulus E and the Poisson's ratio ν are the only material properties to be varied. Let us accept the following constraints for these quantities:

$$1000 \le E \le 1000\,000 \quad \text{MPa} , \tag{12}$$

$$0.3 \le \nu \le 0.45 , \tag{13}$$

since the ranges defined by inequalities (12) and (13) involve values of E and ν for all known basis materials.

Method of finding stresses in the denture and pressure on the mucosa

To solve the optimization problem stresses in the denture and pressure on the mucosa under the masticatory load must be determined. For this, we used the model of the denture together with the mucosa analogous to the model used in [6,7]. In correspondence with this model, the artificial teeth were considered as one elastic curved beam [8] and the denture basis together with the mucosa were considered as the elastic shell on elastic layer [9,10], covering rigid foundation (bone). The modulus of an elastic layer, i.e. the coefficient k in the Winckler's relation

$$p(\mathbf{x}) = kw(\mathbf{x}) , \tag{14}$$

was calculated by formula [7]

$$k = \frac{E_m(1-\nu_m)}{(1+\nu_m)(1-2\nu_m)t_m} , \tag{15}$$

where w is the shell deflection and E_m, ν_m, t_m denote the Young's modulus, the Poisson's ratio and the thickness of the mucosa, respectively.

Stresses in the denture and pressure on the mucosa were calculated numerically by the finite element method (FEM) [11,12].

Method of search of the optimal basis thickness

We suggested the following algorithm of search of the optimal basis thickness for some given basis material.

1. At first we seek the maximum relative pressure on the mucosa and the maximum relative equivalent stresses in the basis and teeth as a function of the basis thickness under the unit value of the masticatory force, i.e. relations: $\max\limits_{S} p_o\big|_{F=1}(t)$, $\max\limits_{v_b} \sigma_o^b\big|_{F=1}(t)$ and $\max\limits_{v_t} \sigma_o^t\big|_{F=1}(t)$.
2. As the elastic problem of determination of stresses in the denture and pressure on the mucosa has a linear solution with respect to the masticatory force F, one can find the threshold force F^{th} as a function of the basis thickness by formula

$$F^{th}(t) = \frac{100}{\max\limits_{S} p_o\big|_{F=1}(t)}\,. \tag{16}$$

3. Then the maximum relative equivalent stresses in the basis and the teeth as functions of the basis thickness under the threshold value of the masticatory force are

$$\max_{v_b} \sigma_o^b\Big|_{F=F^{th}(t)}(t) = \max_{v_b} \sigma_o^b\Big|_{F=1}(t)\cdot F^{th}(t)\,, \tag{17}$$

$$\max_{v_t} \sigma_o^t\Big|_{F=F^{th}(t)}(t) = \max_{v_t} \sigma_o^t\Big|_{F=1}(t)\cdot F^{th}(t)\,. \tag{18}$$

4. Knowing a dependence $F^{th}(t)$ (or $\max\limits_{S} p_o\big|_{F=1}(t)$) we determine the optimal basis thickness t_{opt} for given material with the help of some unidimensional search method by condition

$$F^{th}(t_{opt}) = \max_{0<t\le 2} F^{th}(t) \tag{19}$$

or (that is the same) by condition

$$\max_{S} p_o\big|_{F=1}(t_{opt}) = \min_{0<t\le 2}\,\max_{S} p_o\big|_{F=1}(t)\,. \tag{20}$$

In this study, we used the sufficiently thrifty method of golden section search.

5. For t_{opt} we check if the constraint (11) is satisfied.
6. Substituting t_{opt} in expressions (17), (18) we check if the constraints (7), (8) are satisfied.

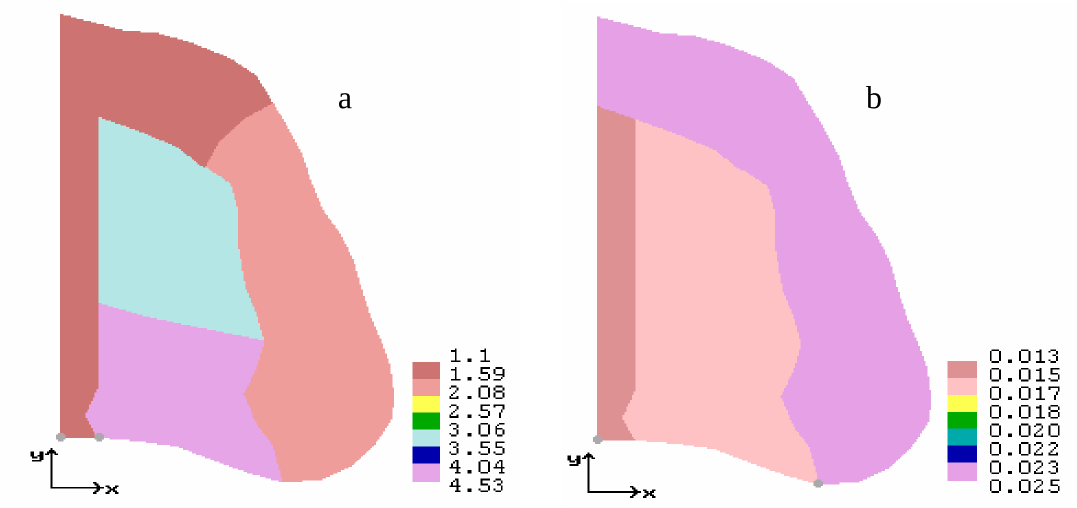

Fig. 1. Mean thickness of the mucosa and mean pain threshold pressure on the mucosa: a – thickness (mm), b – pain threshold pressure (MPa) [20].

Table 1. Elastic and Fatigue Properties of Materials.

Material	Young's modulus E, MPa	Poisson's ratio ν	Reference	Fatigue limit σ_{-1}, MPa	Reference
Cobalt-chromic alloy KHS	220000	0.3	[14]	100	[15]
Titanic alloy VT1-00	112000	0.32	[16]	120	[15]
PMMA plastic AKR-15	2900	0.43	[17]	17.5	[18]
Oral mucosa	3.1	0.45	[19]	–	–

Table 2. Threshold Masticatory Resultant Force for Various Values of the Basis Thickness.

Basis material	Optimal thickness t_{opt}, mm	F^{th}_{max}, %	Recommended thickness, mm [15, 21]	F^{th}_{o}, %
Cobalt-chromic alloy KHS	0.29	87.96	0.4	75.41
Titanic alloy VT1-00	0.41	88.12	0.3	87.31
PMMA plastic AKR-15	1.87	91.06	1.5	90.15

Results and discussion

The optimal basis thickness was found for the following basis materials: cobalt-chromic alloy KHS, titanic alloy VT1-00, and PMMA plastic AKR-15. Elastic and fatigue properties of these materials and the elastic properties of the mucosa are presented in Table 1 [14-19]. The thickness of the mucosa and the pain threshold pressure on it are shown in Fig.1 [20]. The finite element mesh used in calculations is shown in Fig.2.

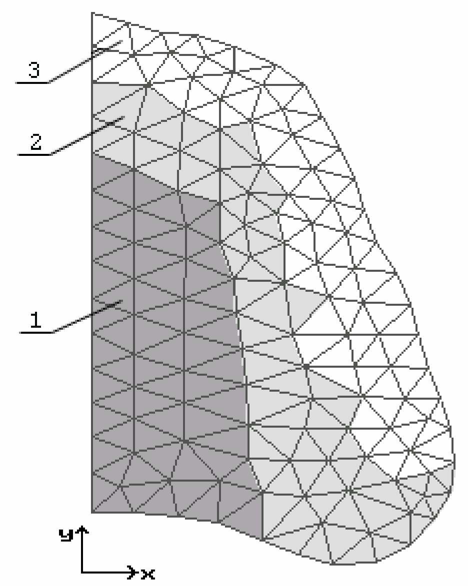

Fig. 2. Finite element mesh of the domain investigated: 1 – plastic or metal layer of the denture basis, 2 – plastic layer of the denture basis, 3 – artificial teeth.

Fig.3, 4a illustrate the algorithm of search of the optimal basis thickness for the case of the cobalt-chromic basis. Fig.3 shows curves $\max\limits_{S} p_o\big|_{F=1}(t)$, $\max\limits_{v_b} \sigma_o^b\big|_{F=1}(t)$ and $\max\limits_{v_t} \sigma_o^t\big|_{F=1}(t)$ obtained with the help of the FEM model, and Fig.4a shows curves $F_o^{th}(t)$, $\max\limits_{v_b} \sigma_o^b\big|_{F=F^{th}(t)}(t)$ and $\max\limits_{v_t} \sigma_o^t\big|_{F=F^{th}(t)}(t)$ obtained from three preceding ones by formulas (1), (16-18), consequently. We can see that the optimal thickness ($t_{opt} = 0.29$ mm) exists and the constraints (7), (8), (11) are satisfied not only in the optimal point but also in its significant neighborhood. As one can see in Fig. 4 analogous results occur for the titanic basis ($t_{opt} = 0.41$ mm) and for the plastic one ($t_{opt} = 1.87$ mm). As follows from Fig.4, a plot $F_o^{th}(t)$ has the following feature in a neighborhood of the optimal point independently on a basis material. If the thickness t decreases from t_{opt} then the threshold force $F_o^{th}(t)$ drops very slowly, and if t increases from t_{opt} then $F_o^{th}(t)$ drops rapidly. It means that it is possible to use the basis with the thickness significantly less (in accordance with Fig. 4, more than twice) than the optimal one without any serious negative effects. At the same time, it is not recommended to do the basis thickness more than the optimal one.

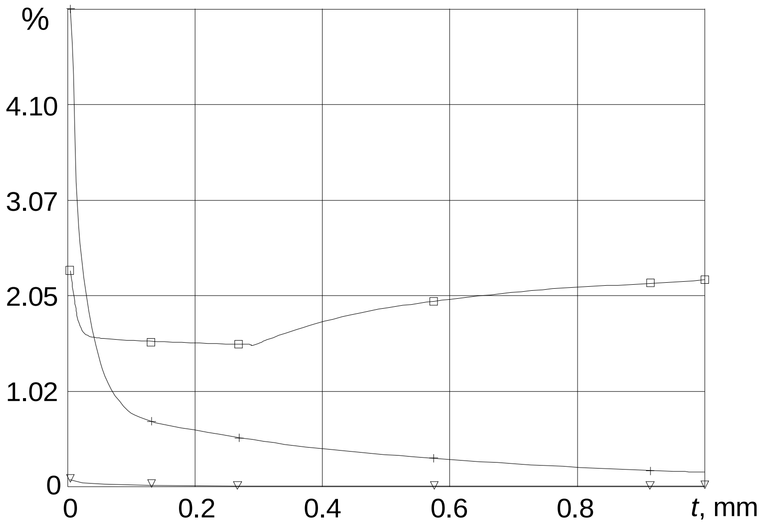

Fig. 3. Maximum relative masticatory pressure on the mucosa and maximum relative equivalent stresses in the denture basis and in the artificial teeth as functions of the basis thickness under the unit magnitude of masticatory resultant force for cobalt-chromic basis:

□ – $\max\limits_{S} p_o\big|_{F=1}(t)$, + – $\max\limits_{v_b} \sigma_o^b\big|_{F=1}(t)$, ▽ – $\max\limits_{v_t} \sigma_o^t\big|_{F=1}(t)$.

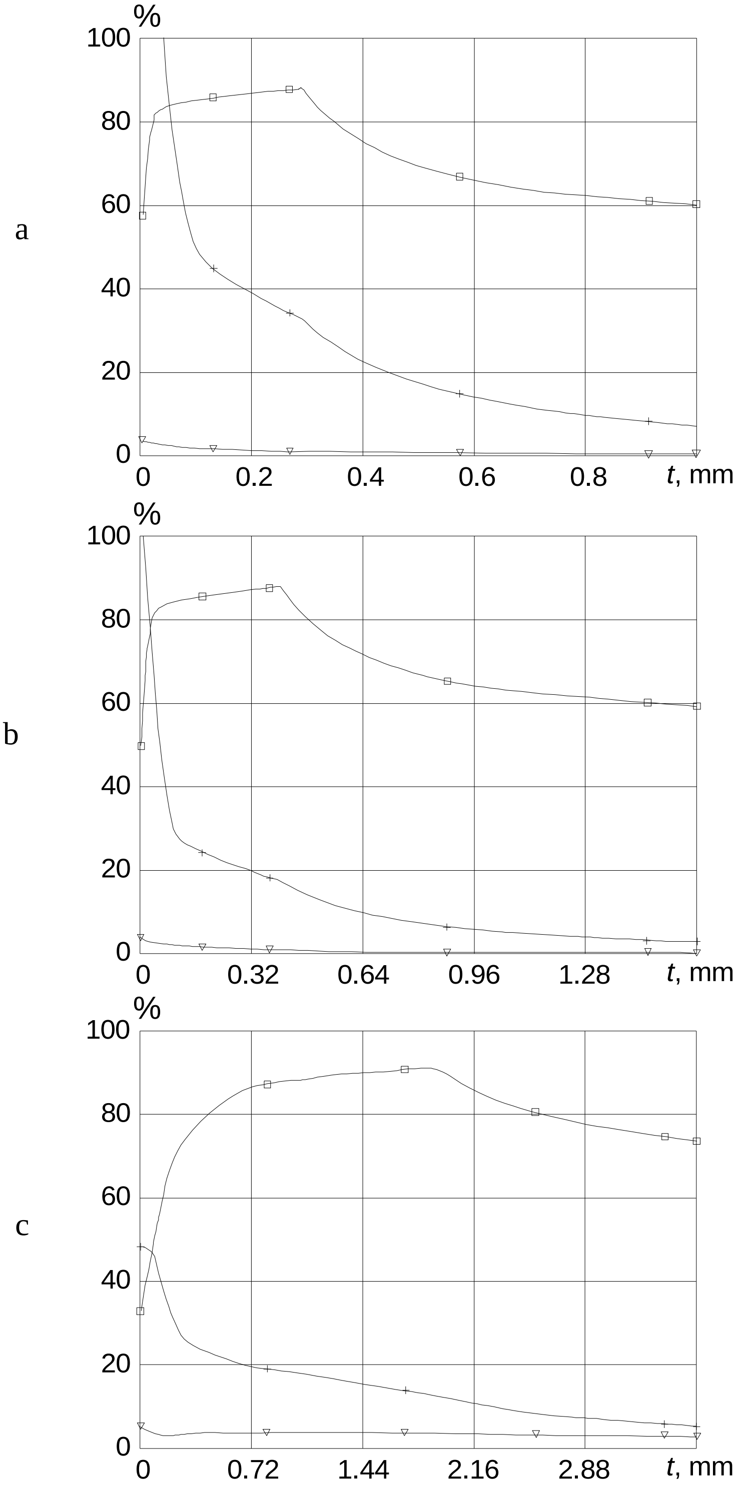

Fig. 4. Relative masticatory threshold force and maximum relative equivalent stresses in the denture basis and in the artificial teeth under threshold magnitude of masticatory force as functions of the basis thickness: a –cobalt-chromic basis; b – titanic basis; c – plastic basis,

□ – $F_o^{th}(t)$, + – $\max\limits_{v_b} \sigma_o^b \Big|_{F=F^{th}(t)}(t)$, ▽ – $\max\limits_{v_t} \sigma_o^t \Big|_{F=F^{th}(t)}(t)$.

The magnitudes of the relative threshold force F_o^{th} for the optimal values of the thickness and for values that are recommended in works [15,21] are presented in Table 2. The recommended value of the thickness for the cobalt-chromic basis appeared high and it resulted in significant decreasing F_o^{th} (i.e. masticatory efficiency of the basis). On the contrary, though recommended values of the thickness for titanic and plastic bases were obviously less than the optimal ones, F_o^{th} decreased insignificantly.

To evaluate the influence of elastic material properties on both the optimal basis thickness t_{opt} and the maximum relative threshold force F_{max}^{th} these quantities as functions of the Young's modulus E were obtained for the range (12) of E under two margin values of the Poisson's ratio ν in the closed interval (13) (Fig.5, 6). The dependency $t_{opt}(E)$ is presented in Fig.5 in logarithmical and usual coordinates. The conclusions following from these results are:

- curves $\log t_{opt}(\log E)$ are very close (Fig.5) to the straight line

$$\log t_{opt} = -0.46 \log E + 1.89 , \tag{19}$$

- the influence of the Young's modulus E on t_{opt} is significant (Fig.5),
- the influence of the Poisson's ratio ν on t_{opt} is negligibly small (Fig.5).

Thus in orthopedic practice one can easily calculate the optimal thickness for given basis material by sufficiently accurate (Fig.5b) formula

$$t_{opt} = 76.88 \cdot E^{-0.46}, \tag{20}$$

that follows from (19).

The dependence $F_{max}^{th}(E)$ under two margin values of the Poisson's ratio is presented in Fig.6 in semilogarithmical coordinates. These results demonstrate the negligible influence of ν and the small influence of E on the magnitude of the maximum relative threshold force F_{max}^{th}. Namely, when the E decreases from $1000\,000$ MPa (very "rigid" material) to 1000 MPa (very "soft" material) the maximum force F_{max}^{th} increases from 88% to 92%.

It is possible to ascertain the cause of existence of the optimal basis thickness by analyzing distribution of the relative masticatory pressure p_o over the prosthetic bed under the threshold magnitude of the relative masticatory force F_o^{th}. This distribution is shown in Fig.7 for three thickness values of the cobalt-chromic basis. When the basis thickness is small ($t = 0.005$ mm), the mucosa covering the alveolar bone appears to be loaded by the maximum relative pressure (i.e. by the pressure $p_o = 100\%$) (Fig.7a). When the basis thickness is great ($t = 1$ mm), such maximum relative pressure is observed in the area of the medial palatine torus (Fig.7b). When the thickness is optimal ($t_{opt} = 0.29$ mm) the maximum relative masticatory pressure loads both these areas (Fig.7c). Analogous results occur for two other materials. At the same time, the mucosa between the alveolar process area and the palatine torus area is proved to be weakly loaded even in the case of the optimal thickness of the basis (Fig.7c). Consequently, the abutment capability of the mucosa is not completely exhausted.

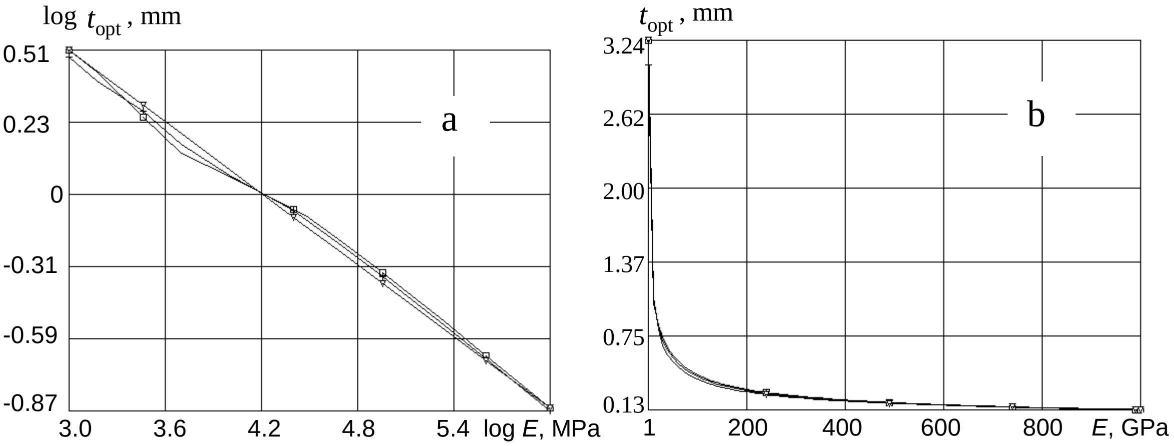

Fig. 5. Young's modulus dependence of the basis optimal thickness in logarithmical (a) and usual (b) coordinates under two values of the Poisson's ratio:
□ – ν = 0.3, + – ν = 0.45, ▽ – the straight line $\log t_{opt} = -0.46 \log E + 1.89$.

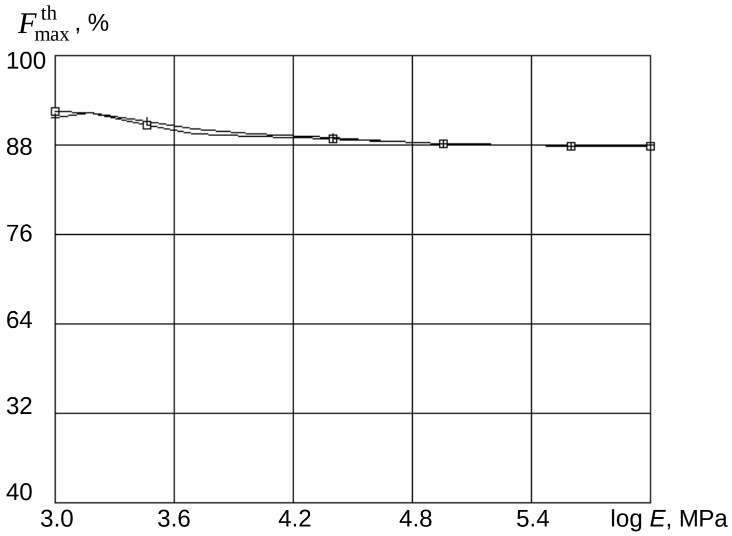

Fig. 6. Young's modulus dependence of maximum masticatory threshold force under two values of the Poisson's ratio: □ – ν = 0.3, + – ν = 0.45.

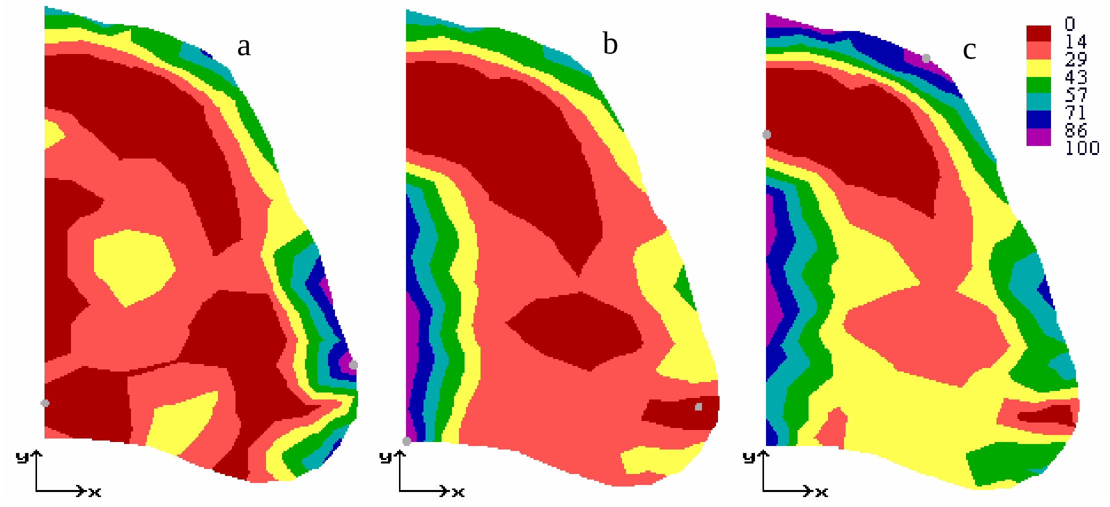

Fig. 7. Distribution of relative masticatory pressure on the mucosa (%) over the prosthetic bed for various values of thickness *t* of cobalt-chromic basis: a – *t* = 0.005 mm; b – *t* = 1 mm; c – t_{opt} = 0.29 mm.

Conclusions

- The optimal design problem of the removable laminar denture was formulated.
- A computer based finite element model of the removable laminar maxillary denture together with the mucosa of the prosthetic bed was created.
- An efficient method of solving the optimization problem using linearity of the elastic problem solution was suggested.
- The optimal values of the denture basis thickness were determined for cobalt-chromic, titanic and plastic basis materials.
- Dependence of the optimal basis thickness on the elastic properties of basis materials was investigated. It was found that only the Young's modulus essentially influences on the optimal value of the thickness. An analytical expression connecting the optimal thickness with the Young's modulus was obtained.
- It was shown that for any basis material the optimal solution meant loading by the maximum relative masticatory pressure both the alveolar process and the palatine torus areas.

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ОПТИМИЗАЦИЯ КОНСТРУКЦИИ СЪЕМНОГО ПЛАСТИНОЧНОГО ПРОТЕЗА ВЕРХНЕЙ ЧЕЛЮСТИ

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Особенностью съемных пластиночных протезов является неполное восстановление жевательной способности вследствие низкого порога болевой чувствительности слизистой оболочки протезного ложа. В данной работе рассматривается задача поиска оптимальной толщины базиса пластиночного протеза, которая обеспечивала бы максимальную величину жевательной нагрузки. При этом должны выполняться следующие естественные ограничения: давление на слизистую оболочку протезного ложа не должно превосходить порогового значения, при котором возникает ощущение боли; интенсивность напряжений в любой точке базиса не должна превосходить предела усталости; толщина базиса должна быть положительной и не превосходить некоторого предельного значения, при котором становятся заметными нарушения речи.

При моделировании поведения протеза под жевательной нагрузкой базис протеза рассматривается как упругая оболочка, искусственные зубы – как упругая криволинейная балка, а слизистая оболочка – как упругий слой, лежащий на жестком основании (кости). Напряжения в протезе и давление на слизистую оболочку определяются численно с помощью метода конечных элементов.

Оптимальные решения получены для случаев изготовления базиса из следующих материалов: кобальт-хромового сплава КХС, титанового сплава ВТ1-00, этакриловой пластмассы АКР-15. Показано, что для всех материалов оптимальное решение соответствует максимальной нагруженности слизистой оболочки в области альвеолярного отростка и в области костного шва, тогда как для толщин, отличных от оптимальных, максимально нагруженной оказывается либо та, либо другая область. Так как базис протеза может быть изготовлен и из других материалов, были получены кривые зависимости оптимальной толщины от модуля упругости материала базиса при различных значениях коэффициента Пуассона этого материала. Анализ поведения этих кривых показал: коэффициент Пуассона практически не влияет на значение оптимальной толщины базиса; зависимость оптимальной толщины от модуля упругости в логарифмических координатах близка к прямой линии. Это позволило получить аналитическое выражение последней зависимости, которое может быть рекомендовано для применения в медицинской практике. Библ. 21.

Ключевые слова: съемный пластиночный протез, слизистая оболочка, порог болевой чувствительности, оптимальное проектирование

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