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MATHEMATICAL MODELLING OF ORTHOPEDIC RECONSTRUCTION OF CHILDREN'S CONGENITAL MAXILLARY ANOMALY

A. G. Masich, Y. I. Nyashin

Department of Theoretical Mechanics, Perm State Technical University, 29a, Komsomolsky Prospect, 614000, Perm, Russia

Abstract: The present work briefly outlines the main stages of preoperative orthopedic reconstruction procedure involving the process of bringing down the palate fragments. A uniform growing beam is used for biological simulation of the fragmented hard palate. The problem of determining the growth parameters involved in the physical equations of the model is solved. The calculations are made to estimate the optimal scheme of loading provided by the orthopedic apparatus.

Key words: hard palate, cleft, orthopedic reconstruction, biomechanical model, growth strain.

Introduction

Congenital maxillary anomaly (uranoschisis, labium leporium) is one of the most commonly encountered and serious disorders of dentofacial system. It disturbs such vitally important functions as sucking, swallowing, breathing, speech, phonematic hearing, and is certainly a distressing cosmetic defect.

In the framework of physiological approach, the uraniscus being one of the structure elements of the upper jawbone system plays the role of a partition, separating the oral and nasal cavities. In the context of biomechanics, the hard palate due to the presence of counterforces provides an optimal distribution of stresses generated and transferred to the upper jaw bones through a lump of food and teeth in the process of action of masticatory apparatus while eating and with teeth tightly squeezed. The anomalies of face skeleton and soft tissues cause violation of normal functions, since the action of loads upon muscle-bone system in deformed masticatory apparatus dramatically changes. As a result, this leads to a more severe deformation of dentofacial bones and teeth arches. With such defects, the tongue adds to a stronger protrusion of palatal fragments into nasal cavity. The mastication process proves to be unable to provide optimal distribution of forces over the bone system of hard palate, which, in turn, leads to an overload and underload in all parts of maxilla bone - muscle system.

The importance of one or another pathology is essentially determined by its frequency. According to statistical reports of different countries the amount of children born with congenital dentofacial pathology defects ranges at the average from 1 to 500 or 1 to 1000 newborns.

It is well understood that children born with maxillary congenital anomaly need specialised emergency, which involves a very important stage of eliminating a disconnection of nasal and oral cavities. The success of measures aimed at recovering anatomic forms and functions of congenital deformed organs largely depends on such factors as the urgency of the first specialised aid and efficiency of methods used to correct pathomorphological, functional and anatomic disorders.

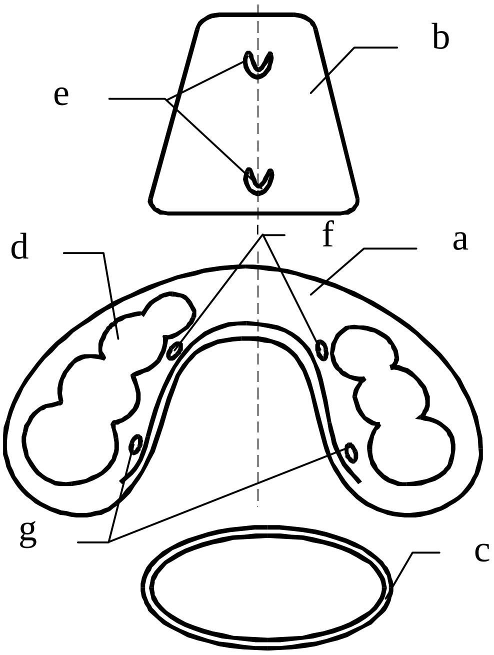

Fig.1. Schematic representation of the apparatus for correction of hard palate anomaly: a) teeth-gum plate; b) nasal plate; c) elastic ring; d) the teeth holes in the teeth-gum plate; e) supporting loops of nasal plate; f) medial supporting loops; g) distal supporting loops.

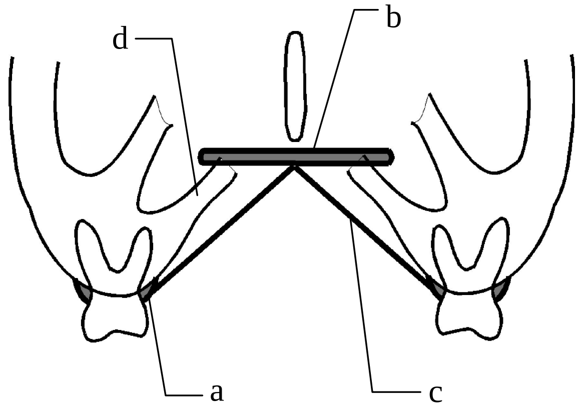

Fig.2. Scheme for application of apparatus in the case of double -sided palate cleft: a) teeth-gum plate; b) nasal plate; c) elastic ring; d) palate fragment.

In 1978 Prof. T. V. Scharova and Prof. E. Yu. Simanovskaya, the associates of Children's Stomatology Department of Perm State Medical Academy elaborated the original method of stepwise preoperative orthopedic reconstruction of defective maxilla [1]. The key to this method is to bring down the hard palate fragments from nasal to oral cavity with the help of demountable orthopedic apparatus (author's certificate № 848020). The outline of its main elements is shown in Fig.1. Fig.2 schematically shows the apparatus after applying to separated fragments in the case of double - sided cleft. The apparatus consists of nasal plate and teeth-gum plate operated by a rubber ring with predetermined diameter. The mechanical force developed by the rubber ring mounted on six supporting loops is transferred to a nasal plate, which exerts an downward force on the palate. This results in a gradual bringing down of the palate fragments from the nasal to oral cavity and finally stimulates osteogenesis along their free edge. A period of use of the apparatus depends on the child's age, the kind of a cleft, the degree of deformation and underdevelopment of the palate fragments, as well as on the general state of a child's health and may last from 3 months to 1.5 year [1]. The efficiency of the apparatus is supported by the facts that the palate fragments change their position. and the lateral dimensions of the cleft decrease. The basis for demountable apparatus is the principle of mechanical action. The main mechanical factor is the pulling force developed by elastic rubber ring as soon as it has been mounted on the supporting loops.

Since the degree of morphological tissue maturity varies with the age of a child, it is essential to employ differential approach when determining the magnitude of functional loading for each individual. The objective of our investigation is to develop a mathematical model of fragmented hard palate subjected to loading as a complicated biomechanical system.

A mathematical substantiation of the optimal load on the bone tissue, which will not interfere with the growth of underdeveloped maxilla fragments is a key aspect to orthopedic reconstruction.

Physical equations describing growth processes in living tissues

The grounds for physical equations of the model is the experimentally established fact that the bone tissue as any living structure is capable of a spontaneous growth, especially in childhood, and a growth due to mechanical stresses arising in tissues. In the recent years, the effect of the mechanical factors on the growth processes has become the subject of comprehensive experimental and theoretical investigation [2]. Thus, the models of growing material of viscoelastic nature have found increasing favour in theoretical analysis. These models are based on the hypothesis that the strain rate is related to the mechanical stresses in a media by

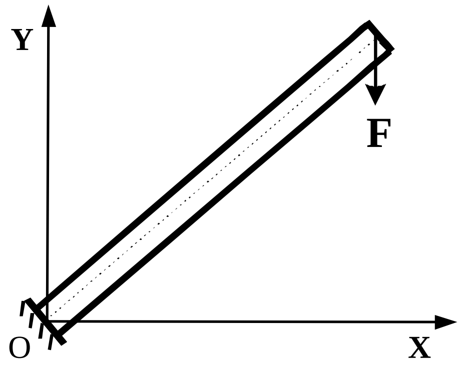

Fig.3. Elementary model of fragmented hard palate.

$$\tilde{e}_\rho = \tilde{A} + \tilde{\tilde{M}} : \tilde{\sigma}. \qquad (1)$$

Here $\tilde{e}_\rho$ is the rate of growth, $\tilde{\sigma}$ is the stress tensor, the tensor $\tilde{A}$ characterises the actual growth of a material (in the absence of stresses), the tensor $\tilde{\tilde{M}}$ is responsible for contribution of stress into deformation of growth. The mechanical deformation creates prerequisites for setting new material and is a significant growth limiting factor.

At present, there are exist few mathematical models reported in the literature dealing with the problems of biological growth, for example, the model of distortion of thin objects (curvature of the man's spine, bending of stem and plant root) growing under conditions of axial compression [3-5] and the mathematical model of bone regenerator tissue obtained as a result of distraction osteosynthesis [2].

Determination of growing cantilever beam configuration

As a biomechanical model capable of describing the clinical picture of the cleft palate response we have used a uniform growing beam with one end fixed and the other subjected to a load (Fig.3). This essentially simplifies the understanding of the processes occurring in the growing medium and reduces the number of parameters for a mathematical model, many of which cannot be defined with sufficient accuracy.

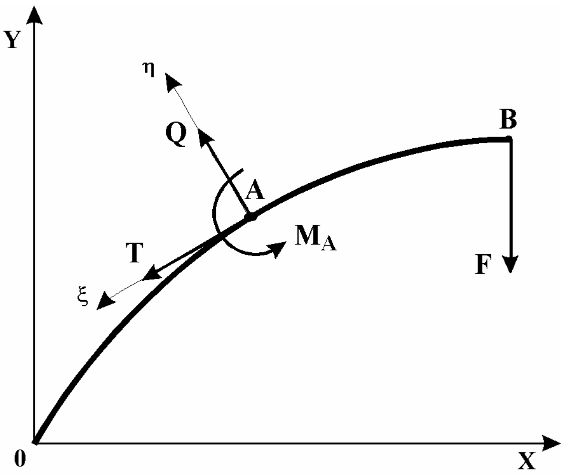

Fig. 4. Local coordinates (ξ,η). related to the axial element of the axis; schematics of forces T, Q and moment M_A acting on imaginary cut section AB of the beam.

Neglecting the instantaneous elastic strain the constitutive relation can be written as

$$e_\rho = \rho + K\sigma, \qquad (2)$$

where σ is the axial stress normal to cross section, e_ρ is the strain rate at points lying on the beam axis, ρ and K are the growth constants.

To calculate deformation of the beam shown in Fig.3 we introduce local coordinates (ξ,η) related to the axial element of the beam (Fig.4). Then constitutive equation (2) can be rewritten as

$$e_\rho(\xi,\eta) = \rho + K\sigma(\xi,\eta)\,. \tag{3}$$

For this relation we require the fulfilment of the hypothesis for the plane sections

$$e_\rho(\xi,\eta) = e_0(\xi,\eta) + \dot{\chi}(\xi,\eta)\eta \tag{4}$$

and neglect the shear strains. Here $e_0(\xi,\eta)$ defines the strain rate of the beam axial element $d\xi$ and $\dot{\chi}(\xi,\eta)$ is the bending strain rate of the element $d\xi$.

The strain parameters $e_0(\xi,\eta)$ and $\dot{\chi}(\xi,\eta)$ are determined using general equilibrium conditions (Fig.5) [5]. The bending moment $M_A(\xi,\eta)$ and axial (longitudinal) force $T(\xi,\eta)$ defined by equilibrium conditions on the separated section of the beam are determined at the same time by the internal forces at the section, namely:

$$\begin{aligned} &\int_S \sigma(\xi,\eta)dF = T(\xi,\eta), \\ &\int_S \sigma(\xi,\eta)\eta dF = M_A(\xi,\eta). \end{aligned} \tag{5}$$

Here S is the cross section area of the beam, the axial force $T(\xi,\eta)$ defines the growth strain at the section, and the bending moment $M_A(\xi,\eta)$ is the measure of bend at the section.

The equilibrium conditions for the separated (right hand) part of the cantilever beam (Fig.4) are written as

$$\begin{aligned} &\sum \mathrm{X} = F_x - T(\xi,\eta)\cos\gamma - Q(\xi,\eta)\sin\gamma = 0, \\ &\sum \mathrm{Y} = F_y - T(\xi,\eta)\sin\gamma + Q(\xi,\eta)\cos\gamma = 0, \\ &\sum M_A = M_A(\xi,\eta) - F_y(x_B - x_A) + F_x(y_B - y_A) = 0 \end{aligned} \tag{6}$$

where F_x and F_y are the projections of the force **F** into the x and y axis, respectively, γ is the slope angle of a tangent to the beam at the point A and $Q(\xi,\eta)$ is the transverse force in the cross section. In the following the effect of the force $Q(\xi,\eta)$ is neglected.

Finally we obtain:

$$\begin{aligned} e_0(\xi,\eta) &= \frac{T(\xi,\eta)\cdot K}{S} + \rho, \\ \dot{\chi}(\xi,\eta) &= \frac{M_A(\xi,\eta)\cdot K}{I_A}, \end{aligned} \tag{7}$$

where I_A is the moment of inertia of the cross section. Relation (7) defines configuration of the beam at each time instant.

Thus, at each current moment we can define the strain $\varepsilon_0(\xi,\eta)$ of axial element $d\xi$:

$$e_0(\xi,\eta) = \frac{d\varepsilon_0(\xi,\eta)}{dt} \tag{8}$$

and its deflection $w(\xi,\eta)$ along local axis:

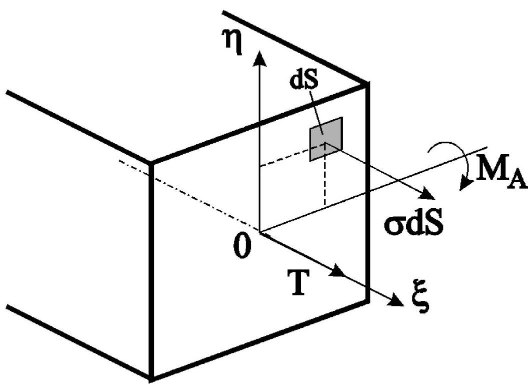

Fig.5. General equilibrium conditions under beam bending [3].

$$\chi(\xi,\eta) = \frac{\partial^2 w(\xi,\eta)}{\partial \eta^2}. \qquad (9)$$

The stated problem has been solved numerically for a beam of rectangular cross-section of width b=15mm and thickness h=3mm. The load on the free end of the beam distributed over the width is 10g.

Determination of growth parameters: mathematical formulation of the problem and analysis of experimental data

The problem of evaluating the optimal forces involved in the process of bringing down the palate fragments can be solved, provided that the physical parameters characterising the properties of the bone basis of the cleft palate are known.

The properties of the medium can be determined from stretching experiments. In major cases, the bone tissue experiments are performed on the macroscopic samples cut from different bone sections and having different orientation [6]. However, the experiments allowing us to determine the growth parameters on the basis of the separated fragments of the hard palate present a serious problem, especially with children. Therefore, there is an urgent need for estimating these parameters with the help of mathematical simulation methods.

In the above case, there are two parameters of growth: the parameter of actual growth of the medium ρ (in the absence of stresses) and the parameter of growth viscosity K due to mechanical stresses σ generated in the medium.

Unfortunately, the currently available literature provides neither exact nor approximate information on the values of these parameters. The only relevant work [7] indicates, however, that the actual growth deformation is positive $\rho>0$ which is characteristics of tissues in the period of intensive growth, for example in childhood. According to numerous observations of biological objects in a wide range of loads the tensile axial stresses exert accelerating effect (and vice versa, decelerating effect in the case of compressive stresses) on the growth in the same direction, which suggests that inequality $K>0$ must be satisfied [7].

The experimental part of our investigation consists in mapping the configuration of the cleft palate from the control diagnostic models (CDM) of the patients obtained from the maxillary impression. Then using the chronometer-type (clockwork) indicator and BMI-1C № 812107 toolmaker's microscope the coordinate of deviation from some basis plane (occlusion plane) is measured. The measurement error is ~0.01mm.

Let us consider a mathematical formulation for the problem of determining the growth parameter ρ and K. A key aspect of the problem definition is to find the unknown growth parameters from the prescribed initial $y^*(x)$ and final $y_{end}(x)$ configuration of the beam subjected to a known force on the free end. Thus the problem reduces to finding an optimal solution with two unknowns.

At the first step it is necessary to construct the objective function $\Phi(K,\rho)$. Since the purpose of optimization is searching for parameters, which can maximally approximate the initial configuration to the final, the functional must clearly involves the residual with respect to coordinate:

$$r_1 = y(x, K, \rho) - y_{end}(x). \qquad (10)$$

Here $y(x, K, \rho)$ is the beam configuration at the finite moment t_{end} (the time of treatment) obtained by calculating the variation on of the growing beam configuration under the action of the force **F**; $y_{end}(x)$ is the prescribed final configuration (Fig.6). Thus,

$$\Phi_1(\mathrm{K},\rho) = \frac{C_1}{L_{end}{}^3} \int [\, y(x,\mathrm{K},\rho) - y_{end}(x)\,]^2\, dx. \tag{11}$$

Since we consider a growing beam whose length changes during deformation, it is reasonable to introduce the addition term to the quantity Φ_1 taking into account the so-called length discrepancy:

$$r_2 = L(K,\rho) - L_{end}, \tag{12}$$

where L_{end} is the finite length of the beam obtained from the experiment, and $L(K,\rho)$ is the length of the beam obtained by solving the problem of beam configuration.

Hence:

$$\Phi_2(K,\rho) = C_2[1 - \frac{L(K,\rho)}{L_{end}}]^2, \tag{13}$$

where C_2 is a dimensionless weight coefficient.

With the assumption used, the generalised objective function $\Phi(K,\rho)$ can be written as:

$$\Phi(K,\rho) = \Phi_1(K,\rho) + \Phi_2(K,\rho). \tag{14}$$

Thus we can state the problem of functional minimization $\Phi(K,\rho)$, allowing us to determine the growth constants ρ and K such that:

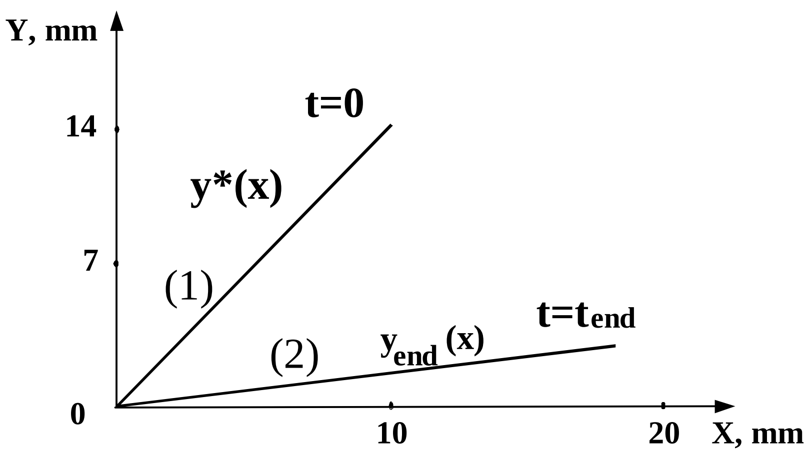

Fig.6. Experimental straight lines obtained from CDM of the patient D. having one-sided palate cleft at the beginning of treatment (1) and at the end (2).

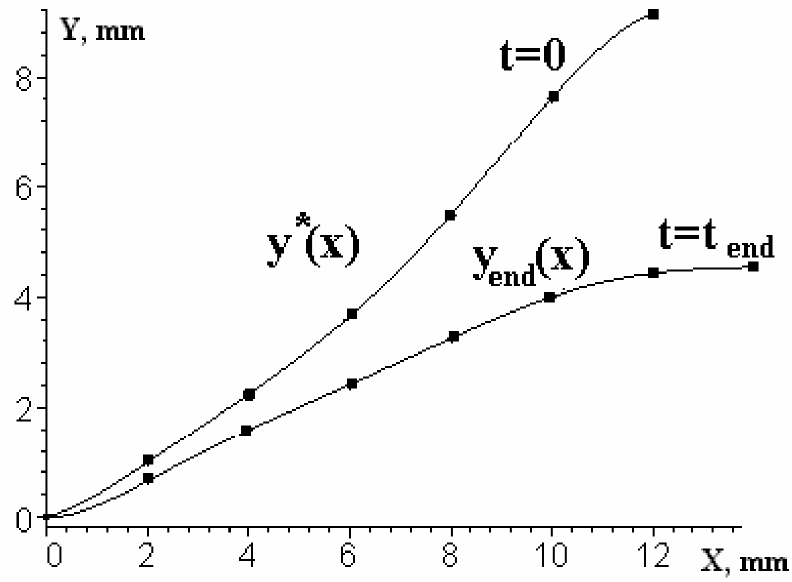

Fig.7. Experimental points for patient M. obtained from the rear section of double-sided cleft of the hard palate (symmetric one half).

$$\Phi(K,\rho)=\frac{C_1}{L_{end}^{\;3}}\int[\,y(x,K,\rho)-y_{end}(x)\,]^2\,dx+C_2[1-\frac{L(K,\rho)}{L_{end}}]^2\xrightarrow[K,\rho]{}\min,\quad(15)$$

under the constraint: determination of growing cantilever beam configuration (3)-(8).

We have calculated the growth parameters for the case shown in Fig.6. The control points were obtained from the CDM of the patient D. made for a rear section of one-sided cleft at the beginning of treatment (t=0) and at the end (t=18 months) and approximated by straight lines. The calculated values of the actual growth parameter ρ and growth viscosity K are, respectively, ρ =0.002543 ($\frac{1}{\text{months}}$) and K =0.002096 ($\frac{mm^2}{g\cdot\text{months}}$).

The developed model can be also adapted to the case of real section geometry. Fig.7 illustrates the section configuration in the rear part of double-sided cleft of the patient M., obtained with the help of the above procedure. The experimental data processing was accomplished using the licensed mathematical MAPLE V package.

Mathematical formulation of optimal apparatus design problem. Biomechanical analysis

Once the growth parameters have been determined the question arises as to what magnitude of optimal force should be applied at the free end of the beam to form the palatal arch in the best possible way. Here of particular importance is the clinical fact that the maximum stress in the living structure produced by the action of applied external load must be not higher than 20 g/mm^2 [1], that would otherwise injure the mucous membrane and bone tissues of the hard palate. The maximum load F estimated under this physiological constraint was ~40 g. At present we have formulated and solved the problem of determining the optimal magnitude of the force $\mathbf{F}_{\mathbf{opt}}$. With consideration of restrictions on the applied force F and the angle β, made by the force **F** with the horizontal axis x (Fig.8), the functional to be minimized is similar to the functional $\Phi(K,\rho)$. From the biomechanical model in Fig 8 it follows that the vertical force **F** is decomposed into two components **T** and **N**. The compressive force **T** interferes with the growth. However, the actual tissue growth prevails over the action of the force **T** so that from clinical viewpoint we observe the osteohenesis

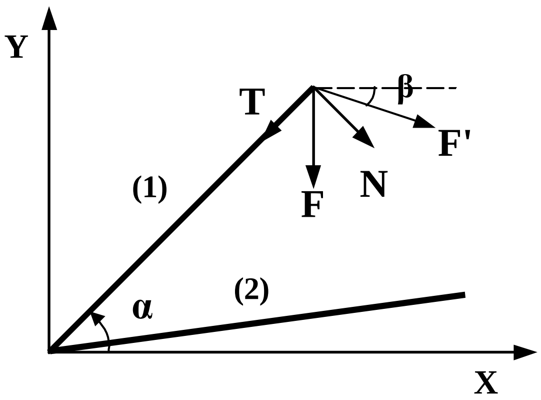

Fig.8. Graphical analysis of constraints on the force **F.**

processes. The force **N** turns the fragment and brings it to the position (2). The force **F** must be obviously applied in such a way as to help the actual physiological growth of the tissue and appropriately rotate the palate fragments, for example, to have the direction of the force **F'** as shown in Fig.8. From these considerations it follows that angle β has to be restricted to

$$0 \le \beta \le 90^\circ - \alpha . \qquad (16)$$

Hence, with the above restrictions the optimisation problem is reduced to finding F and β such that

$$\Psi(F,\beta)=\frac{D_1}{L_{end}^{\;3}}\int[\,y(x,F,\beta)-y_{end}(x)\,]^2\,dx+D_2[1-\frac{L(F,\beta)}{L_{end}}]^2 \xrightarrow[F,\beta]{} \min, \qquad (17)$$

where D_1 and D_2 are the dimensionless weight coefficients subject to the constraints

1. $F \in [0, F_{\max}]$, (18)
2. $\beta \in [0\,, 90^\circ - \alpha]$, (19)
3. determination of the growing beam configuration (3)-(8). (20)

The problem was solved by applying the Hooke-Jeevs numerical optimisation method [8]. The calculation gave the following values of $F_{opt} \approx 40\ g$ and $\beta_{opt} \approx 7.3^\circ$.

Conclusions

In the present work we have developed a biomechanical model of children's palate cleft and proposed an algorithm for determining individual optimal loads developed by the orthopedic apparatus in the process of bringing down the palate fragments.

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Математическое моделирование ортопедической реконструкции врожденной верхнечелюстной аномалии у детей

А.Г.Масич, Ю.И.Няшин (Пермь, Россия)

Расщелины твердого неба и губ являются весьма частым проявлением детской врожденной патологии. По данным статистики различных стран число детей, рожденных с врожденной патологией зубочелюстной системы в среднем составляет от 1 на 500 до 1 на 1000 новорожденных. Такие дети требуют экстренной специализированной медицинской помощи. При расщелине твердого неба необходимо принять меры для исправления дефекта перегородки, разделяющей полости носа и рта. В 1978 году сотрудники Пермской государственной медицинской академии профессор Е.Ю.Симановская и профессор Т.В.Шарова разработали оригинальный метод пошаговой предоперационной ортопедической реконструкции дефектов верхней челюсти. Для этой цели был разработан сменный ортопедический аппарат, который создает механическое усилие для низведения порочно развитых фрагментов твердого неба. В данной работе строится биомеханическая модель, позволяющая рассчитать поведение фрагментов неба под нагрузкой. Модель фрагментированного твердого неба представляет собой изгибаемую балку, защемленную на одном конце и подверженную действию силы на другом. Для описания скорости деформации костной основы неба применяется модель, учитывающая ростовые деформации в зависимости от напряжения. Описывается алгоритм идентификации параметров модели с помощью отпечатков, сделанных у пациента. Дается постановка оптимизационной задачи получения в результате лечения заданной конфигурации фрагментированного неба. Решение позволило найти величину и направление силы, развиваемой ортопедическим аппаратом. Библ. 8.

Ключевые слова: расщелина твердого неба, ортопедическая реконструкция, биомеханическая модель, ростовая деформация, оптимальное решение

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