Russian Journal of Biomechanics, № 1-2: 89-97, 1998.

MECHANICS OF A SKI JUMP

R.N. Rudakov, A.R. Podgayets

Department of Theoretical Mechanics, Perm State Technical University, 29a, Komsomolsky Prospect, 614600, Perm, Russia

Abstract: A mathematical model of a ski jump is built. Aerodynamic coefficients as functions of the attack angle are found from numerical solution of the problem of turbulent airflow around the ski jumper. The wind, the push during break-off, mass and sizes of skier-on-skis system were taken into account. The detailed shape of this system was not considered as well as the processes of forming flight position and preparing to landing. Permeability of the skiing suit was not regarded also. Despite these simplifications the model behaves rather well and describes a range of experimental facts. Distance optimization problem is solved for V-style of jump to find maximum distance with constraint on landing velocity for given sizes and mass of skier-on-skis.

Key words: ski-jumping, aerodynamic coefficients, turbulence

Introduction

Ski-jumping was qualified as a sporting event 100 years ago in Scandinavia and northern regions of Russia, and from the first years scientists helped sportsmen to sharpen their technique. The first of them was R. Shtrauman, who created the "Norway" style of jump in 1924.

There have been several studies about ski jumping published in the last two decades. E.A. Grozin made an integrated study of jump styles of the 1950s and 1960s [1]. Aerodynamic coefficients are obtained from his experiment with figurines of ski jumpers in an aerodynamic tube. L.P. Remizov [2,3] used these coefficients in the optimization problem solved by Pontryagin's maximum principle. P.V. Komi et al. [4], I. Tani and M. Iuchi [5] used numerical methods on similar problems. They all were considering old techniques of jump and, surprisingly, none considered the wind or used constraint on the landing velocity in the case of optimization problems. The most recent paper of N.A. Bagin et al. [6] employs an original mathematical method based on the theory of complex variables and tries to add wind speed to movement equations but they have been written with an error.

The overview of the works above allows us to come to the conclusion that none of the following issues have been sufficiently studied: landing velocity, the affect of wind on a jump and the influence of the exact individual parameters of a fully equipped skier. So our study will take these points into account.

The ski jumpers use so-called "V-style" for the last decade or so. The ski jumper parts his skis and lies between them forming the triangle wing. The skier is continuously controlling his flight by adapting position of his skis, body, arms, legs, that is attack angle. Athlete's position (relative location of body parts) does not change. The problem is to find influence of an operating factor (attack angle) on the flight distance and to maximize it. There is also a constraint on the trajectory form, especially not far from the landing point, the trajectory must approach the landing slope with angle no more than 5°. After landing the speed component parallel to the surface is reducing during the further movement of the skier. The major danger is introduced by landing speed component normal to the surface, which is reduced in the squat by muscle force of the sportsman. The normal speed component must not exceed 7 m/s and usually is about 3-4 m/s. This condition was used as a constraint is the identification process and optimization problem statement. To identify the model the videorecord of a live broad casting from "Four Hills" World Cup Contest held on 4 January 1998 on K110 jumping hill in Innsbruck, Austria was used.

First let us introduce some definitions on the jumping hills' construction. The jumping hills are created for a certain flight distance, which is calculated as a distance between the break-off point and touch the ground point over the slope. There are special equations and standards for calculating the geometrical sizes of the hills to prevent traumas due to a too short or too long flight distance.

A jumping hill consists of an acceleration slope and the so-called break-off table, from where the flight begins. The break-off table is slightly turned to the horizon, and the angle is usually between -6° and -12°. The hill below the break-off point is called the landing hill. The distance is calculated using taut ribbon from the break-off point to touch the ground point. The landing hill consists of three regions - a zone with a height H and length N, a landing zone (from B - beginning of landing zone to K - end of landing zone also called maximum distance point, critical point and C-Point) and a damping zone. The landing zone is plane and has an angle to the horizon, usually between -25° and -40°. The height of the break-off point over the landing hill denoted by S usually is 2%-4% of maximum distance (Fig. 1).

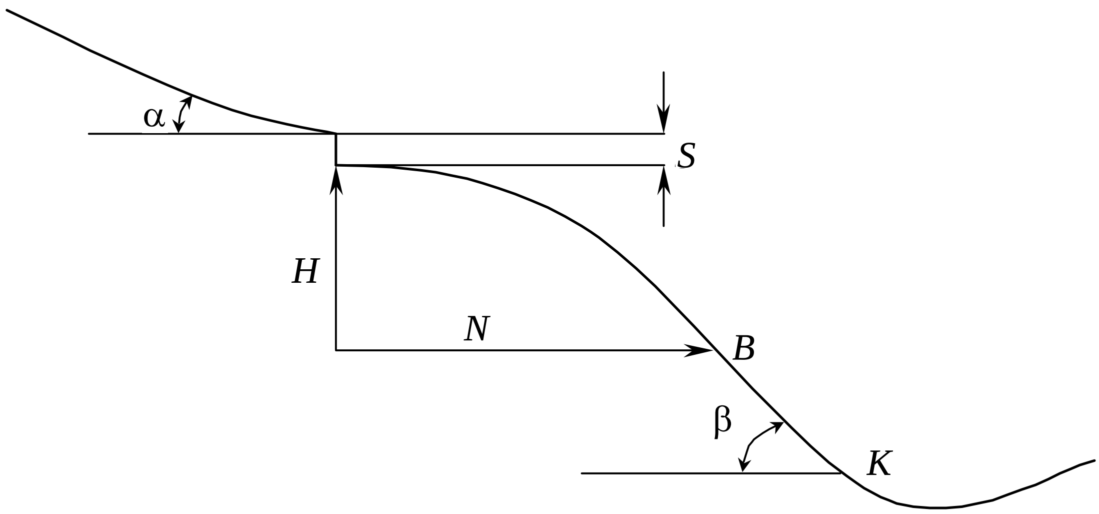

Fig. 1. Basic geometrical elements of a jumping hill.

Methods

The following factors are considered in this model: the wind, the push during break-off, the mass, length, width and height of the fully equipped skier. Let us consider the problem in a 2D statement. Only V-style of jump is regarded as noted above, i.e. the skis-to-horizon angle practically is constant and the skier is assumed to lie in the same plane with his skis.

Let's consider skier's jump in the stationary reference system $Oxyz$ with a center of the system in the break-off point (Fig. 2).

During the greater part of the flight phase the sportsman is moving translationally, thus it can described by a model of a moving material particle, affected by forces of gravity **P**, air resistance **R** and lift **Q**. The forces **R** and **Q** depend on speed of a skier relative to air **w**, which is equal to difference between speed of a skier relative to ground **v** and wind velocity **u** in the point where the skier is in a certain moment of time: **w** = **v**-**u**.

In the borders of hydraulic estimation

$$|\mathbf{R}| = \tfrac{1}{2} c_\tau \rho S w^2, \qquad |\mathbf{Q}| = \tfrac{1}{2} c_n \rho S w^2,$$

where $w = |\mathbf{w}|$, ρ is air density, c_τ and c_n are aerodynamic coefficients (they are usually denoted by c_x and c_y during straightforward movement). The air resistance **R** is directed opposite the relative velocity **w**:

$$\mathbf{R} = -kw\mathbf{w}, \quad \text{where} \quad k = \tfrac{1}{2} c_\tau \rho S\,;$$

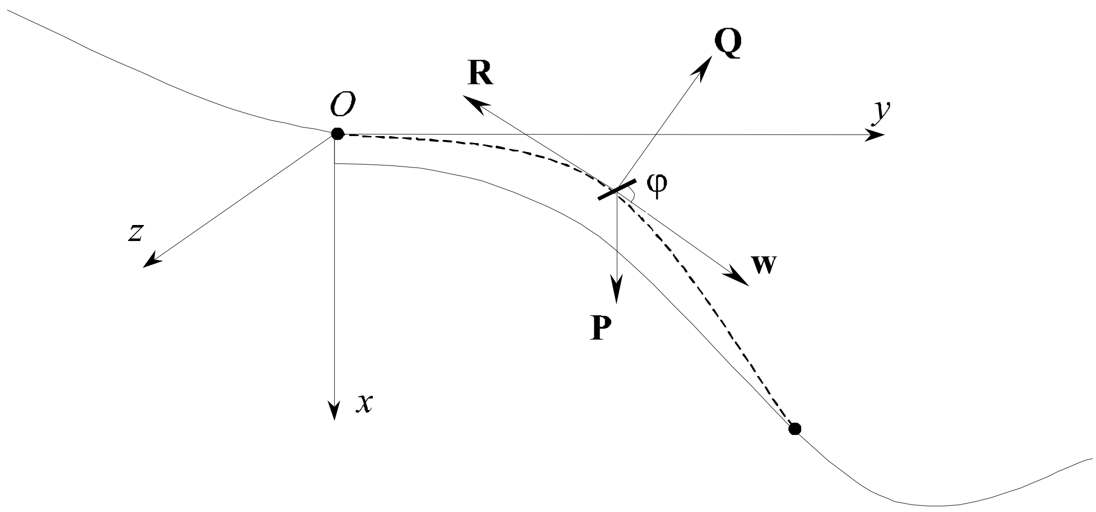

Fig. 2. Reference system for the problem of modeling skier's flight.

the direction of a lift **Q** could be found with the help of ort **e** of Oz axis:

$$\mathbf{Q} = f\,w(\mathbf{e}\times\mathbf{w}), \quad \text{where} \quad f = \tfrac{1}{2}c_n\rho S\,.$$

The second Newton's law is connecting the mass of a skier m and his acceleration **a** with applied forces:

$$m\mathbf{a} = \mathbf{P} - kw\mathbf{w} + f\,w(\mathbf{e}\times\mathbf{w})\,. \qquad (1)$$

Dividing (1) by m and projecting to coordinate axes, we come to a set of second order differential equations:

$$\begin{cases} \ddot{x} = g - \dfrac{k\,w(\dot{x}-u_x) + f\,w(\dot{y}-u_y)}{m}, \\ \ddot{y} = \dfrac{-k\,w(\dot{y}-u_y) + f\,w(\dot{x}-u_x)}{m} \end{cases} \qquad (2)$$

where g is gravity coefficient, $w = \sqrt{(\dot{x}-u_x)^2 + (\dot{y}-u_y)^2}$, u_x and u_y are projections of wind velocity on coordinate axes not affected by skier. The fourth order Runge-Kutta method is used to solve the set of equations (2).

Functions $u_x(t,x,y)$ and $u_y(t,x,y)$ are found from the problem of airflow around landing slope. The wind is measured only in one point during the competition, but in can change throughout the entire trajectory, thus it is necessary to predict the whole picture of airflow around the jumping hill knowing the wind exactly only in one point. Wind velocity is found from solution of boundary problem with no-slip condition on the surface of the hill. The wind is considered horizontal on the other boundaries and the air pressure difference is given.

Only the landing hill is considered because the acceleration slope is rather narrow and it does not play a big role in forming the airflow. The reference system is pictured in Fig. 1. Theory of border layer is used. The air is considered linear-viscous incompressible liquid. The wind velocity is considered rather low because jumping is forbidden if the wind is strong. Because of weak wind we ignore the nonlinear components of Navier-Stokes equations and assume the airflow is laminar. Thus plane stationary laminar flow of air is considered in a rather big almost rectangular area so that inflow and outflow could be assumed strictly horizontal. Experimental data state that the wind does not change much after the height of 500 m, so the border layer height is confined to this value. Two Navier-Stokes equations and the equation of incompressibility describe the 2D flow of viscous compressible liquid. The problem has been solved by Galerkin's method in terms of speed-pressure.

The finite element method (FEM) was used as most convenient for describing the complex borders. The problem was being solved in natural variables for simplicity of satisfaction the border equation. The FEM program utilized linear approximation for speeds and constant approximation for pressure. Quadrangular finite element was used for pressure, which was split into 2 triangles for speeds.

The aerodynamic coefficients c_τ and c_n used in the equations (2) are found from the solution of 2D boundary problem of airflow around the plane of finite width with no-slip condition on its surface. The other boundary conditions were gotten from the previous problem. The problem was also solved by FEM, but here nonlinear components of Navier-Stokes equations were taken into consideration. The program was made for stationary airflow calculation. The theory of quazylaminarity was used to regard the turbulence. The iterational process was used for linearization, and velocities from previous iteration were introduced in rigidity matrix instead of unknown velocities.

Thus, parameters of the model are the following: parameters of the jumping hill; the mass, length, width, height of the skier-on-skis, the skis-to-horizon angle during the flight, the starting velocity and the additional velocity of the push during the break-off; the wind around the jumping hill.

The model was identified using the live videorecord of a World Cup Competition “Four Hills” held on K110 jumping hill in Innsbruck on 4 January 1998. One parameter (the air resistance coefficient in the beginning of the flight) was found from the solution of optimization problem, where the difference between experimental and calculated distances was minimized for all the jumpers.

Another optimization problem was solved to find optimal skis-to-horizon angles to achieve the maximum distance for different parameters of the athletes and their equipment.

Results and discussion

Identification of the model

Parameters of K110 jumping hill in Innsbruck are the following: α = -12°, β = -36°; maximum flight distance K =110 m, minimum flight distance (the distance between break-off point and the beginning of landing zone) B = 90 m, height of break-off table above jumping hill surface S = 4 m; parameters of hill’s slope H = 63.2 m, N = 87.0 m. The results of the identification are the following: 1/6 of the jumpers’ flights distances were predicted with an error less than 0.5 m, 50% of the flights – with the error of 1-3 m and the others – with the error of 3-8 m.

Airflow problems

The sample results obtained from solution of the problem of airflow around jumping hill are presented in Fig. 3. Rectangle is an area presented in Fig. 4. The results are obtained for horizontal wind against the jump. The lines are the vectors of wind velocity drawn from corresponding mesh points. The length of lines presents the value of wind velocity. Fig. 4 is an example of airflow around the ski jumper. Turbulent whirlwinds are clearly seen in Fig. 5. Length of lines in Figs. 3 - 5 represents velocity values.

Aerodynamic coefficients

Sample aerodynamic coefficients found from the airflow problem are presented in Fig. 6. They are qualitatively similar to known experimental data. It is necessary to know the attack angle to solve equations (2). The attack angle as the skis-to-velocity angle is the sum of two angles – the angle between skis and horizontal line and the angle between velocity and horizontal line. The first is assumed constant in this model and the second could be easily found in any point of the trajectory. The calculated attack angle belongs to the interval from 12° to 40°, which corresponds to experimental data.

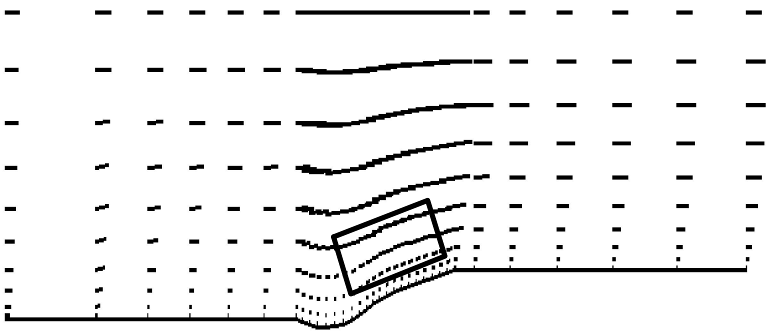

Fig. 3. The field of air velocities around the jumping hill. Rectangle is the area presented in Fig. 4.

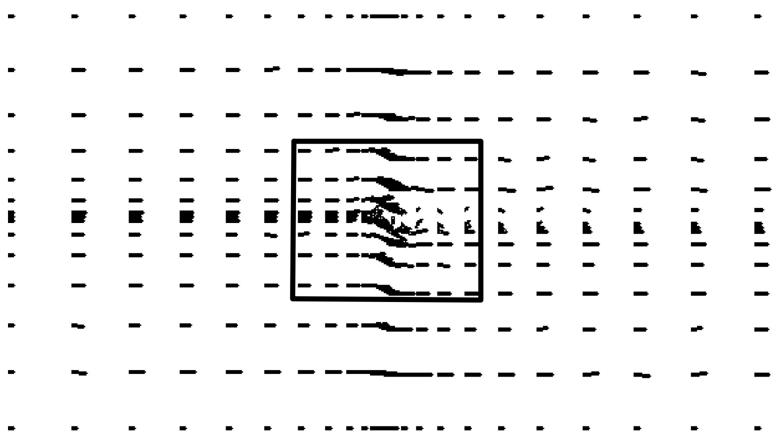

Fig. 4. The field of air velocities around the ski jumper. Rectangle is the area presented in Fig. 5.

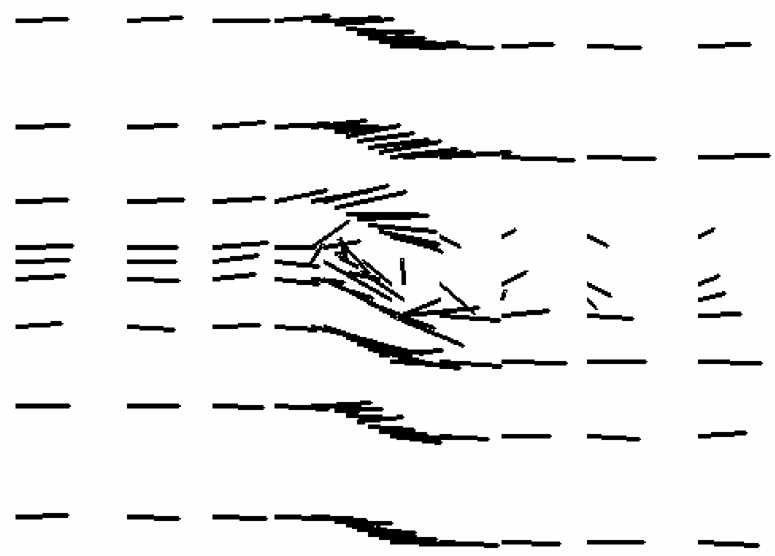

Fig. 5. The field of wind velocities near the ski jumper.

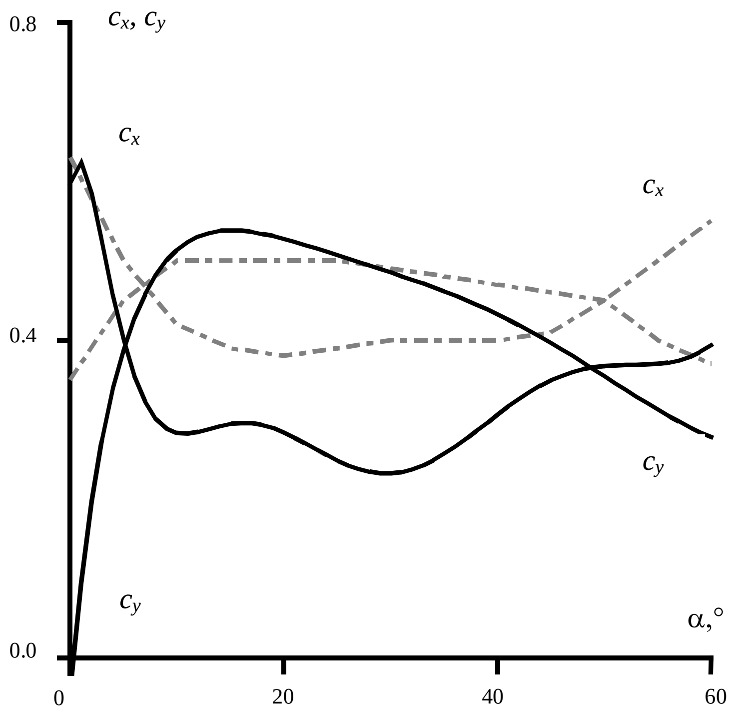

Fig. 6. Aerodynamic coefficients of lift and air resistance calculated in the problem of airflow around skier-on-skis model (solid line) and found from the experiments with figurines of jumpers using jumping styles of 1960s in aerodynamic tube (Grozin) (dotted line).

Optimization problem

The optimal angles between skis and horizontal line were found for jumpers of different height and weight to achieve maximum flight distance. The results are presented in Figs. 7-9. It is clearly seen that optimal attack angle is about 0°, which is confirmed by experimental data: videorecord of world leading ski jumpers shows that they keep practically horizontal position during the main phase of the flight. It was found that the heavier the ski jumper and his equipment and the shorter are his skis, the bigger is his optimal skis-to-horizon angle i.e. the higher he must rise his toes.

There are Olympic standards for ski length. The length of skis must not exceed the sportsman height by more than 0.8 m. It is obvious that athletes use the largest skis he is fit for. For example, a man of 1.85 m height can use skis of length 2.65 m. These are the largest skis size available.

Fig. 9 shows isolines of distance. It is obvious that the lighter and higher the sportsman, the longer is his largest jump. But in terms of “struggle for distance” mass is more important that length: e.g. decrease of mass by 6.5 kg (7.7 % of 84 kg) is equal to increase of skis length by 25 sm (9.4 % of 2.65 m). It is worth mentioning that these results are valid for sportsmen whose aerodynamic coefficients are close to the coefficients used in this mathematical model. The heavier jumpers will take advantage for higher air resistance.

The push during the break-off does not affect the flight distance very much as was stated by this investigation. Probably, a good push speeds up the process of forming the optimal flight position and thus greatly increases the distance.

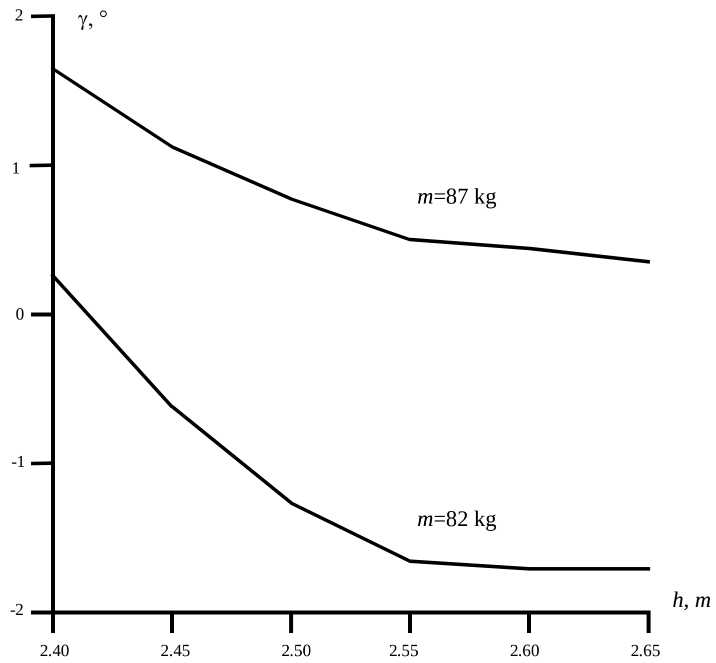

Fig. 7. Optimal skis-to-horizon angles for different masses of the ski jumper in his equipment and fixed skis length. The results are obtained for initial speed 25 m/s, push during break-off 3 m/s and no wind.

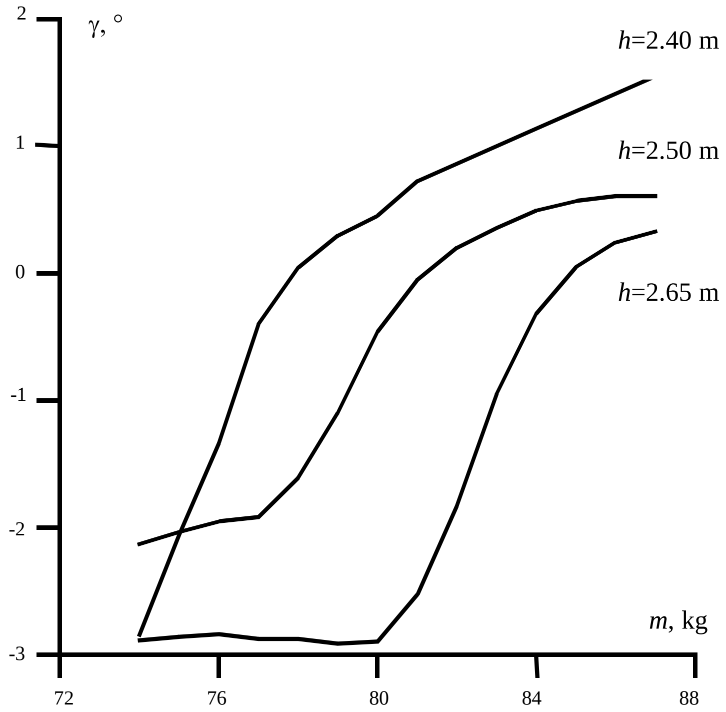

Fig. 8. Optimal skis-to-horizon angles for different skis length and fixed mass of skier-on-skis system. The results are obtained for initial speed 25 m/s, push during break-off 3 m/s and no wind.

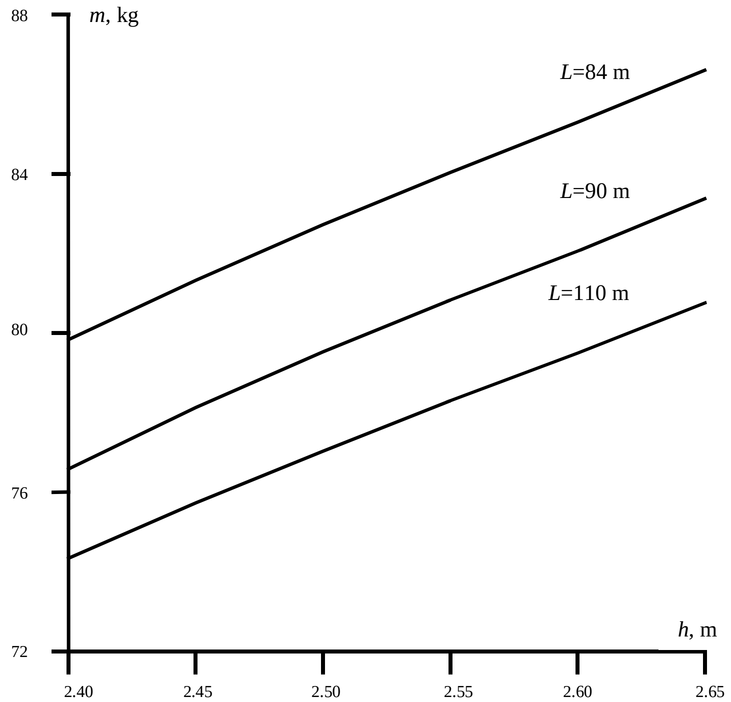

Fig. 9. Lines of equal distance in coordinates mass-length of skier-on-skis system. The results are obtained for initial speed 25 m/s, push during break-off 3 m/s and no wind.

Conclusions

The identification process included measuring of several jumping hill parameters, which had its errors. For example, the error of angle finding is about 1°. The mass and height of sportsmen were taken in average with only the most general adjustments (e.g. Japanese tend to be shorter and lighter than Europeans). Permeability of a skiing suit was not taken into account. Aerodynamic coefficients were obtained from mathematical model of 2D airflow while it is known that 2D and 3D turbulence are described by qualitatively different laws. Theory of quazylaminarity is the simplest of all turbulence theories. Aerodynamic coefficients were calculated for the triangle plane, not for the human figurine.

Considering this long list of weak points of which we are fully aware, the verification of the model has shown rather a high precision. It reproduces a range of experimental facts, which testify that even such a simple model of a ski jump could describe the qualitative relations between the parameters.

References

1. Grozin E.A. **Ski jumping**. Physical culture and sports, Moscow, 1994 (in Russian).
2. Remizov L.P. Maximum skis jump distance. **Theory and practice of physical culture**, 3: 73-75, 1973 (in Russian).
3. Remizov L.P. Biomechanics of optimal ski jump. **J Biomechanics**, 17: 167-171, 1984.
4. Tani I., Iuchi M. Flight-mechanics investigation. In: S**cientific study of skiing in Japan**, Hitachi, Tokyo, 1971, 34-52.
5. Komi P.V., Nelson R.S., Pully M. **Biomechanics in ski jumping**, Jivaskyla, 1974.
6. Bagin N.A., Voloshin Y.I., Evteev V.P. On the theory of a ski jumper’s flight. **Theory and practice of physical culture**, 2: 9-11, 1997 (in Russian).

Механика прыжка на лыжах с трамплина

Р.Н. Рудаков, А.Р. Подгаец

Построена математическая модель полета лыжника. Учтена полученная на основе решения задачи турбулентного обтекания лыжника воздухом зависимость аэродинамических коэффициентов лобового сопротивления и подъемной силы от угла атаки системы лыжник-лыжи. Учтено также влияние на полет ветра, отталкивания от стола отрыва, массы и размера системы лыжник-лыжи. Детально форма этой системы, процесс формирования полетной позы и подготовка к приземлению не учитывались. Несмотря на эти допущения, модель достаточно хорошо описывает ряд экспериментальных фактов. Решена задача оптимизации положения лыжника в полете при V-стиле прыжка, найдено влияние массы и размера системы на дальность полета и скорость приземления. Библ. 6.

Ключевые слова: прыжки с трамплина, аэродинамические коэффициенты, турбулентность

Received 6 May 1998