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DYNAMICS OF A GYMNAST'S TOUCHDOWN ON ELASTO-PLASTIC MATS

Y.V. Kalachnikov, Y.I. Nyashin, R.O. Saratchev

Department of Theoretical Mechanics, Perm State Technical University, 29a, Komsomolsky Prospect, 614600, Perm, Russia

Abstract: This paper considers questions of determination of dynamic forces applied to gymnast's body parts at the moment of his touchdown on elastic or elasto-plastic mats. The influence of mat mechanical properties on magnitude of dynamic reaction acting on a human foot is investigated. Some restrictions on a touchdown kinematics were taken into consideration and the optimization problem of gymnast's movement was solved. The obtained results were in good agreement with known both theoretical and experimental data at walking and run.

Key words: touchdown, mats, trauma, and optimization of movement

Introduction

Forces applied to elements of human osteomuscular system during his touchdown after sports exercises or circus tricks, have significant values. These forces may cause microtraumas of anatomical structures such as bones, muscles, tendons and ligaments. The accumulated microtraumas result in destruction or disease of the damaged tissues. To predict accumulation of possible damages it is necessary to know or to be able to calculate these loadings, which depend on velocity of a touchdown, mat's amortization characteristics, geometry and gymnast's weight.

The reduction of loadings is possible if to increase number of mats, to replace a mat by an elastic insurance net and to run movement during a touchdown (for example, squatting with the subsequent tumbling). However these ways sometimes result in unacceptable restrictions on performance technique of exercise.

With other things being equal, the most dangerous one is the touchdown on a foot from a vertical position with the subsequent straightening. The sportsmen use it very often at the end of many sport exercises. In this paper, the finishing phase of sport exercises is considered. The optimization problem of gymnast's movement during touchdown is formulated.

The model

Consider a gymnast touchdown after his vertical falling from height *h* downwards, i.e. the vector of his center of mass velocity is perpendicular to a mat's horizontal surface. The mechanical properties of the surface are known. The contact occurs by all foot simultaneously. In simulation the gymnast is modeled, usually as a system of rigid bodies. The right and the left sides of a body move synchronously, forming a uniform element. There are five rods in this system: foot, shin, femur, torso and head, hand. The geometrical and inertial characteristics of all these parts are known.

The character of movement downward and rather small touchdown velocity allow to neglect aerodynamic resistance force. At vertical movement downward of straightened gymnast the general theorems of dynamics are equivalent to a task of dynamics of a point. From these theorems the velocity of foot's center of mass can be calculated at the moment of a contact with the mat's surface:

$$V_1 = \sqrt{2gh}. \quad (1)$$

It is the initial condition for the further investigations.

The gymnast's movement after his contact with the surface is described by system of the second order differential equations. The number of the equations is defined by number of freedom degrees. These equations can be derived from the Lagrange's ones:

$$\psi_k(q_j, \dot{q}_j, \ddot{q}_j, R_X, R_Y, m_i) = 0, \quad j,k = \overline{1,S}, \; i = \overline{1,4}, \tag{2}$$

where q_j are generalized coordinates,

R_X, R_Y are horizontal and vertical forces applied to feet (ground reaction force of the mat's surface),

m_j are moments of muscle forces reduced to joints,

S is the number of freedom degrees.

The concrete form of the equations (2) depends on accepted simplifications of a gymnast's model and concrete kind of gymnast's movement. The initial conditions are added to the equations system (2). Then this system is solved analytically for simple cases or numerically for complicated ones.

The influence of mat's elasto-plastic properties on a touchdown dynamics

Several test problems were solved to investigate influence of mat's elasto-plastic characteristics on forces applied to feet at the touchdown moment. The touchdown of a gymnast in the straightened position (Fig.1) was considered, i.e. the movement was simulated by single-rod system.

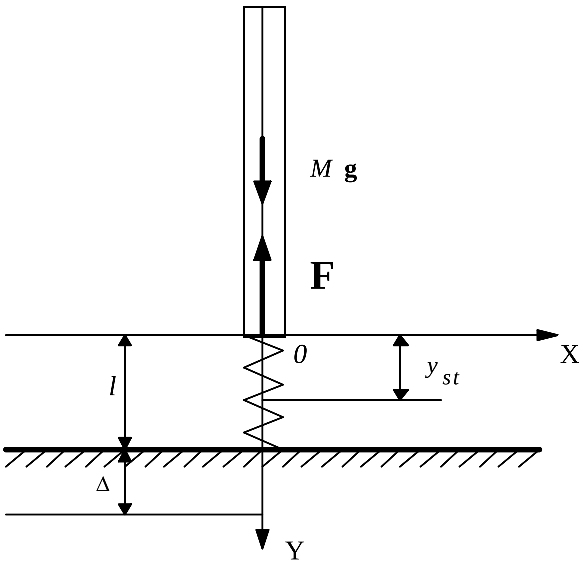

Fig. 1. Dynamics of the interaction between gymnast and elasto-plastic mats.

Let mats be elastic and their thickness decrease by the value ℓ after the gymnast's touchdown (the first phase). Then the movement of a gymnast can be described by the following equation.

$$\ddot{Y} + \frac{C}{M} Y = g, \tag{3}$$

and the initial conditions

$$Y(0) = 0, \; \dot{Y}(0) = V_1, \tag{4}$$

where M is the mass of a gymnast,

$C = tg\alpha$ is the elastic stiffness of mats (Fig.2).

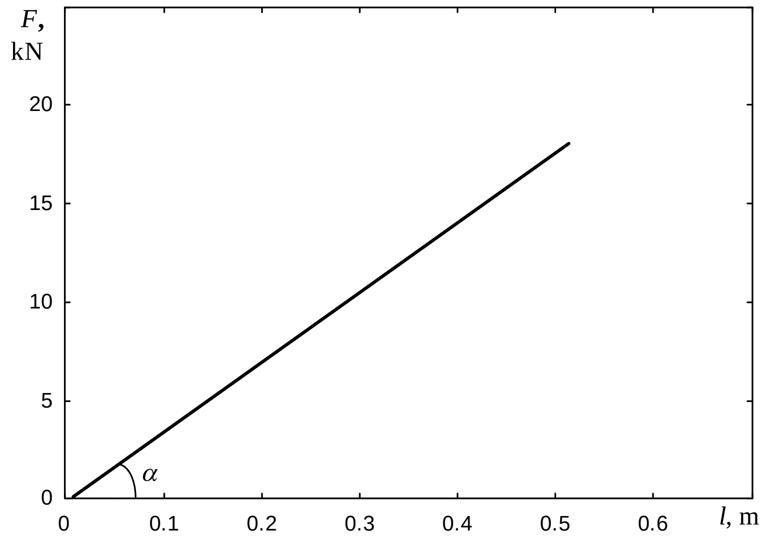

Fig. 2. Dependence of the mat's elastic force on reduction of their thickness.

Plastic deformation of the surface (the second phase) on size Δ measured in units of length is a sum of mats non-self-restorable deformation and the surface deformation, on which mats are spread (sawdust, sand). It is supposed that the elastic and plastic phases of the process are divided in time because of the highly inertial characteristics of a sand or sawdust layer. The equations of movement during the plastic phase of the process are obtained from the kinetic energy variation theorem.

$$\frac{M\dot{Y}^2(\tau)}{2} = \int_0^\Delta (R_Y - Mg)dy, \tag{5}$$

where $\dot{Y}(\tau)$ is the gymnast's velocity at the beginning of a surface plastic compression phase determined from (3),

R_Y is the time averaged ground reaction force.

The numerical results of the movement theoretical modeling are presented in Table 1. The elastic characteristic (Fig.2) was obtained by processing of the experiment with standard gymnastic mats. We measured deformations under the different static loads, and then averaged them on the set of experiments. The plastic characteristic, i.e. maximum deformation of sandy layer, was only evaluated because the exact experiment was not possible.

The obtained results of the touchdown on elastic mats with the unlimited pliability (Item A, Table 1) are in agreement with the papers [1, 2]. It only concerns the first phase for several mats with identical elastic properties. The ground reaction force is 1-8kN (for different kinds of the surface) during the gymnast's touchdown after his falling from 0.3-0.4m height. But the thickness compression of one mat on 0.51m is not possible; i.e. it cannot reduce the accumulated kinetic energy of the falling from height *h*=5m.

The touchdown on the real plastic (only the second phase, Tables 1, Item B) or elasto-plastic (Item C) surface gives results, which are outside of the human organism possibilities. Even the falling from smaller height can lead to appreciable traumas, and the obtained results confirm a hypothesis that the main shock absorber is the human locomotor system.

Table 1.
The obtained results of influence of mat's elasto-plastic properties on a touchdown dynamics

Type of a touchdown	Weight of a gymnast, kN	Elastic stiffness of mats C, kN/m	Elastic deformation ℓ, m	Plastic deformation Δ, m	Ground reaction force R_Y, kN	Falling height h, m
A: elastic mats with the unlimited pliability	0.78	32	0.51	0	16.4	5
B: real plastic surface with limited pliability	0.78	-	-	0.1	39.8	5
C: elasto-plastic mats	0.78	32	0.12	0.1	38.4	5

Optimization of a gymnast's movement during touchdown on a motionless undeformable surface

The above research shows that gymnastic mats cannot prevent a trauma, if the velocity of the touchdown is rather fast. Therefore it is necessary to study dynamics of the gymnast's touchdown, when the velocity decreases because of the human locomotor system amortization.

Consider the worst case of a touchdown from height *h*, the surface being absolute rigid. On the one hand this case is the supremum of forces magnitudes for any surface, on the other there are exercises, in which the deformation of a surface can be neglected (free-style exercises, art gymnastics etc.).

The generalized coordinates (q_j, $j = \overline{1,7}$) defining the position of the human model are: the coordinates X_H, Y_H of the hip joint, the angles $\varphi_i, i = \overline{1,5}$ between the vertical axis and the parts of the rod system (Fig.3). Let us define $\overline{q} = (\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, X_H, Y_H)$. Using (2) the optimization problem of movement can be written in the following brief form:

to determine
$$R_Y^0 = \min_{\overline{q} \in U}(\max_t R_Y)\,, \tag{6}$$

where
$$U = \left\{\overline{q} \in C^2 \middle| \psi_k(\overline{q}(t)) = 0, f_i(\overline{q}(t)) \le 0, k = \overline{1,7}, i = \overline{1,5}\right\}. \tag{7}$$

The inequalities (7) restrict the values of the generalized coordinates. The human physiology imposes these restrictions - it means, the maximum values of rotation angles are limited for all joints.

Using the half-inverse synergy method [3], it is possible to reduce *U* at the expense of generalized coordinates $\overline{q}(t)$ selection only from the part of C^2, if we are given the concrete form of functions $\overline{q}(t)$ and defining numerical coefficient of these functions. Thus the number of the generalized coordinates decreases and the superfluous equations (2) are used for the determination of the muscle moments m_j.

The analysis of a gymnast's movement kinematics during the touchdown shows that the

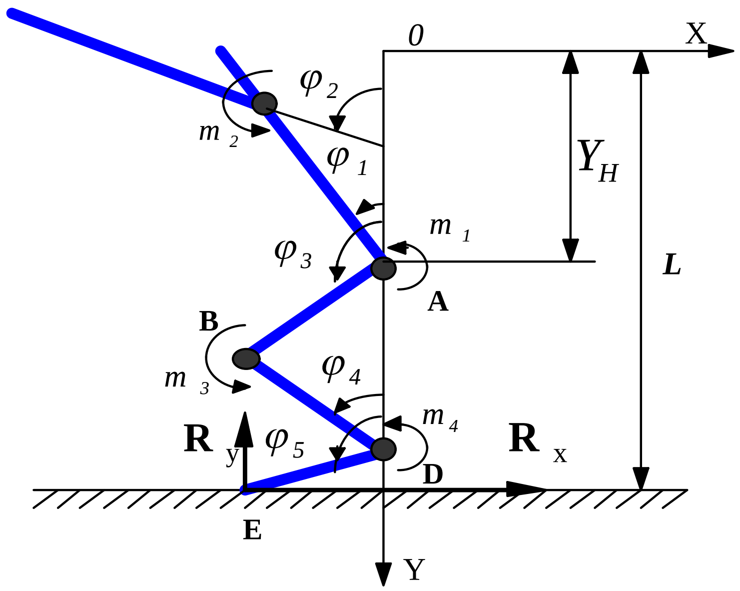

Fig.3 The model of the gymnast's touchdown on the motionless surface.

movement law of the point *A* (Fig.3) in the vertical direction is close to harmonic, considering the phase of squat and straightening is a half cycle. If the vertical coordinate is given as

$$Y_H(t) = Y_H = A_1 \sin(\omega t + \alpha) + B_1, \tag{8}$$

then A_1, B_1, ω, α can be determined from obvious conditions

$$Y(0) = 0, \quad \dot{Y}(0) = V_1, \quad \ddot{Y}(0) = g, \tag{9}$$

$$Y(t_2) = Y_2, \quad \dot{Y}(t_2) = 0,$$

where t_2 is the time which determines maximum coming together of points *A* and *D*.

The limiting value of $\overline{Y_2} \geq Y_2$ can be easily found for any concrete man. It can be measured, when the man is in the position of complete squat.

If we the more strict conditions on kinematics, namely:

- the points A and D during all time of the squatting phase move vertically;
- the lengths of the thigh and shank are almost equal *(AB=BD=* L_S*)*;
- the contact of the surface occurs by all foot;

then from kinematics of the flat mechanism when $\dot{Y}_D = 0, \dot{Y}_H = \dot{Y}_A$ we will derive

$$\cos\varphi_3 = -\cos\varphi_4 = -\frac{A_1}{L_S}(\sin\alpha - \sin(\omega t + \alpha)), \tag{10}$$

$$\cos\varphi_5 = 0, \tag{11}$$

where L_S is the length of the shank, $A_1 = Y_2(V_1^2 + gY_2)/\left(V_1^2 + 2gY_2\right)$.

Assuming, that φ_1 and φ_2 vary synchronously with the others we can write

$$\cos\varphi_1 = D_1 - C_1 \sin(\omega t + \alpha), \tag{12}$$

$$\cos\varphi_2 = D_2 - C_2 \sin(\omega t + \alpha), \tag{13}$$

where and C_2 would be found from the solution of the optimization problem (6) as control parameters, and D_2 will disappear during the derivation on time, then the equation (2) can be written in the form

$$R_Y = Mg + \omega^2\left(MA_1 + M_1C_1 + M_2C_2 - \frac{M_3A_1}{L_S} - \frac{M_4A_1}{L_S}\right)\sin(\omega t + \alpha) = \tag{14}$$

$$= Mg + A_0\omega^2 \sin(\omega t + \alpha),$$

where $M_i > 0,\ i = \overline{1,4}$, are the coefficients depending on the mass-inertial characteristics of the elements of a human body.

Now the optimization problem (6) will have the form:

to determine
$$R_Y^0 = \inf_{Y_2, C_1, C_2 \in U_0}\left(\max_t R_Y\right), \tag{15}$$

where
$$U_0 = \left\{Y_2, C_1, C_2 \in R \,\middle|\, 0 \le Y_2 \le \overline{Y_2},\quad |C_1| \le \overline{C_1}, |C_2| \le \overline{C_2}\right\}. \tag{16}$$

The extreme values of coefficients $\overline{C_1}$ and $\overline{C_2}$ are determined from expressions (12) and (13) using minimum and maximum angular deviations of the human body parts:

$$\varphi_1(0) = 0,\quad \varphi_1(t_2) = \pi/3,\quad \varphi_2(0) = 0,\quad \varphi_2(t_2) = 4\pi/3.$$

These coefficients are $\overline{C_1}$ =0.5 and $\overline{C_2}$ =1.5.

The unique solution $t = t_2$ of the equation obtained from the condition of the extremum on time

$$\dot{R}_Y = \omega^3 A_0 \cos(\omega t_2 + \alpha) = 0 \tag{17}$$

delivers the maximum value of the function $R_Y(t)$, if

$$\ddot{R}_Y(t_2) = -\omega^4 A_0 < 0, \tag{18}$$

where $A_0(Y_2, C_1, C_2)$.

The values of coefficients $C_1 = -\overline{C_1}$, and $C_2 = -\overline{C_2}$, which deliver the minimum of $R_Y(t)$, were defined from expression (14). We can rewrite that expression in the following form (using *ω*, *α* at $t = t_2$)

$$R_Y = Mg + g\left(M - \frac{M_3 + M_4}{2L_S}\right)\frac{2h + Y_2}{Y_2} - 2g\left(M_1\overline{C}_1 + M_2\overline{C}_2\right)\frac{h + Y_2}{Y_2^2}. \tag{19}$$

We obtain

$$Y_2 = \frac{h\left(M_1\overline{C}_1 + M_2\overline{C}_2\right)}{g\left[h\left(M - \dfrac{M + M}{2L_S}\right) - M_1\overline{C}_1 - M_2\overline{C}_2\right]} = \nu \tag{20}$$

using (19) and the next condition

$$\frac{\partial R_Y}{\partial Y_2} = 0. \tag{21}$$

The second derivative

$$\left.\frac{\partial^2 R_Y(t_2)}{\partial Y_2^2}\right|_{Y_2 = \nu} = -\frac{4gh}{\nu^4}\left(M_1\overline{C}_1 + M_2\overline{C}_2\right) < 0 \tag{22}$$

specifies the minimum of function in the point, therefore maximum ground reaction force is defined as

$$R_Y^0 = Mg + \frac{hg\left[M - \left(M_3 + M_4\right)/2L_S\right]}{2\left(M_1\overline{C}_1 + M_2\overline{C}_2\right)} + \frac{g\left(M_1\overline{C}_1 + M_2\overline{C}_2\right)}{2h}. \tag{23}$$

The numerical value of the ground reaction force R_Y^0 equals 9.48kN, if M_1 =1.68kg·m, M_2 =0.264kg·m, M_3 =0.939kg·m, M_4 =0.122kg·m the (If *h* = 2m, then force R_Y is equal to 4.28kN; if *h*=0.5m, then force R_Y is 1.77kN).

The horizontal component of the ground reaction force and moments in joints at this moment of time were found from (2) using the data received for the optimization problem:

R_X = 0.86 kN,

m_1 = -11.77 kN·m (hip joint),

m_2 = -1.195 kN·m (humeral joint),

m_3 = 11.56 kN·m (knee joint),

m_4 = -14.09 kN·m (ankle).

The signs of the moments show, that all of them oppose to joints inflections during the squatting. It is necessary to remember that all values need to be divided in two to refer them to the left or to the right human body side.

Discussion

The gymnast's touchdown assumes his contact with a motionless surface by the whole foot's square of a foot and the appropriate restrictions are imposed on a kinematics of his movement (V_D =0). In practice, the real touchdown ensures the best way to preserve equilibrium (jump through sport vaulting horse or executing some salto).

The problem solution is in the area of movement amplitudes restrictions. Therefore the more amplitude values the more kinetic energy of falling will be extinguished. It will occur because of relative movement of the human body parts. But the joints of a real man differ from ideal hinges therefore turning on any angle is not possible. The change of the sign of coefficients C_i indicates, that the gymnast has to move the torso and hands upwards to reduce vertical ground reaction force, but this will increase the moments in the appropriate joints.

The forces applied to the human foot much more exceed his own weight. The estimation of these forces is more exact in comparison with results of a single-rod model. It is more realistic. It is necessary to remember that stress in muscles and bones decreases, if the point of observation is moved from foot to head. Besides the load on the bone decreases, since the forces in joints are decomposed on component because of angles φ_i are increased.

The solution of problem can be specified using real kinematics and elasto-plastic characteristics of the surface. However it was beyond the scope of the discussed question.

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Динамика приземления гимнаста на упруго-пластические маты

Ю.В. Калашников, Ю.И. Няшин, Р.О. Сарачев

Во время приземления гимнаста после выполнения спортивных упражнений или цирковых трюков усилия, действующие на элементы его костно-мышечной системы, достигают значительных величин. В костях, мышцах, сухожилиях и связках появляются микротравмы, накопление которых приводит к разрушению или заболеванию поврежденных органов. Чтобы прогнозировать накопление возможных повреждений механического характера необходимо знать или уметь рассчитывать действующие нагрузки, которые зависят от скорости приземления, амортизационных характеристик поверхности приземления и характеристик тела человека.

Уменьшение нагрузок возможно за счет увеличения числа матов, замены их упругой страховочной сеткой, управлением движением при приземлении. Но эти способы приводят порой к неприемлемым ограничениям на технику выполнения упражнения.

В работе рассматриваются вопросы определения динамических усилий, действующих на элементы тела гимнаста при его приземлении со значительной высоты на упругие и упругопластические маты. Исследовано влияние механических свойств матов на величину динамической реакции, действующей на стопу человека. Поставлена и решена задача оптимального движения гимнаста при некоторых ограничениях на кинематику фазы приземления, целью которой было уменьшение усилий, действующих на элементы тела человека. Полученные результаты косвенно согласуются с теоретическими и экспериментальными данными при ходьбе и беге. Библ. 3.

Ключевые слова: приземление, маты, травма, оптимизация движения

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