THE ROLE OF PRINCIPAL STRAINS WITHIN THE PERIODONTAL LIGAMENT OF A TOOTH DURING LONG-TERM INTRUSION

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Abstract. This paper is a preliminary theoretical attempt to derive original quantitative formulas in orthodontics. Based on previously derived analytical formulas of stress and strains within the periodontal ligament (PDL) for the particular case of a central incisor during intrusion, the strains are directly related to the motion of the interface between the alveolar bone and the PDL, called bone surface. It is rigorously shown that both normal and shear strains within the PDL are of the same importance for bone surface. In line to that, both ‘simple average’ and ‘geometrical average’ of principal strains within the periodontal ligament play a significant role in the bone remodelling process, as they contribute in the same degree of importance. In summary, the proposed formulas differ from previous ones that had been successfully applied to describe remodelling within long bones. The proposed theory is also sustained by a finite element analysis.

Key words: orthodontics, dental biomechanics, periodontal ligament, principal strain, bone remodelling, finite elements, applied mechanics

Introduction

It has been known since at least Galileo’s time that bone’s form is based in part on its structural function [1]. In 1838, the English anatomist Ward was studying the human femur and “recognised the resemblance to a crane” [2]. In 1866, G.H. von Meyer gave a lecture on the architecture of the cancellous bone with accompanying sketches of the cancellous structure of various bones. The mathematician C. Culman was in attendance at the lecture and noted that the drawings of the cancellous structure bore a strong resemblance to the principal stress trajectories [3]. In 1884, J. Wolff coined the phrase “law of bone transformation”, which was based on Culman’s crane [4]. Significant work presented by Roux, between 1880-1881, publications of Wolff from 1869 plus the ideas of Culman, von Meyer, and Roux were published in Wolff’s book [5] in 1892. In the 1900’s significant research work has been presented by Kummer [6], Frost [7], Gjelsvik [8], Cowin [9], Hart [4], Carter [10], Huiskes [11], and many others. An overview on net bone remodeling in long bones, has been reported by Hart [12]. In summary, the literature suggests that the remodelling rate is a function of either stress or strain. In general, bone responds to a change in the state of stress/strain by trying to alter its form in some manner [13] but the strain seems to be the main contributor to the stimulating mechanism [7,14]. In spite of many existing qualitative theories, only little quantitative findings are available [13]. Briefly, the aforementioned literature suggests that in orthopaedic theories, in general, the remodelling occurs perpendicular to the surface of the bone.
With respect to long-term orthodontic movement, some numerical calculations using the finite element method (FEM) have been presented within the last decade [15-23]. It has been discussed in detail by Middleton et al. [16] that the significance of the PDL is particularly important in orthodontics, as the rate of resorption and addition of bone far exceeds the rates found in other areas of the body, unless major trauma has occurred [7]. Concretely, it was reported by Middleton et al. [16] that the strain within the PDL is of the order 0.1 (10%), i.e. about fifty times larger than the well-known ‘minimum effective strain’, $\varepsilon_{eff} = 0.002$ (0.2%), a threshold which was introduced by Frost [7]. A characteristic difference from long bones is that on the compressive side the osteoclasts eat away at the socket wall (bone absorption), while on the tensile side the osteoblasts add to the bone (bone apposition). In the tooth socket the osteoclasts are most active on the compressive side causing resorption of bony tissue in the region [24].

With respect to computational models in long-term orthodontics, i.e. considering bone remodelling, the following reports have been made:

1. Middleton and associates [15, 16] applied the Hayes’ [25] ideas, according to which the internal reorientation of the trabecular fibres is in line with the direction of the principal stresses. Later, Hickman et al. [23] extended their work by developing a validated three-dimensional model.

2. Tanne and Sakuda [26] have applied Wolff’s law at the initial stage only.


4. The abovementioned work was extended by Schneider et al. [21, 22] in three dimensions. Upon the application of Frost’s law, different bone remodelling rates were applied to tensile and compressive areas.

5. Bourauel et al. [20] applied Frost’s theory within the alveolar bone to control tooth movement.

6. Provatis did applied an extremely fast FEM model that considers only the action of the PDL to control tooth movement [18, 19]. Preliminary bone remodelling aspects with respect to normal and tangential strains within the PDL have been also reported [27-29].

In spite of the above-mentioned works, due to the fact that not many long-term clinical data (tooth movement and loading histories) of teeth under treatment are available, it seems that it is still difficult to establish a commonly accepted algorithm in orthodontics. Usually, a specific law is assumed and later the participating constants are back calculated by assuming, for example, that the tooth tip is moved at a constant rate known from literature (e.g. Berkovitz [30]) or by trying to achieve the horizontal displacement at the bracket. But even if a tooth is considered as a rigid body, there are six rigid-body displacements to be controlled, three translations and three rotations! Things become more complicated when considering time-dependent (viscoelastic) phenomena and/or anisotropy due to the appearance of collagen fibres within the PDL [31, 36-37].

In this context, this paper contributes in deriving a new formula for the case of an axisymmetric tooth of paraboloidal shape during intrusion, under orthodontic treatment. In this particular case, the stress and strain fields within the PDL are analytically known [27]. This fact permits the derivation of a relationship between the movement of bone surface (external remodelling) due the existence of the PDL and the induced strains or stresses. The proposed formula is based on the assumption that both the shape and the material properties of the PDL do not alter during the bone remodelling process. In addition, the case of uniform enlargement, usually accepted, may be also considered.
Equations of elasticity within the periodontal ligament

For reasons that have been previously explained \[31-33\] on the basis of mechanical properties of the involved tissues given by Tanne et al. \[34\], it is assumed that the tooth is a rigid body supported into the rigid socket through a linearly-elastic foundation, due to the nearly-incompressible periodontal ligament (PDL). By considering an elementary volume with respect to curvilinear axes \((n, t, \theta)\) as shown in Fig. 1, the Hooke’s law within the PDL becomes \[35\]:

\[
\begin{align*}
\sigma_n &= \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_n + \nu(\varepsilon_\theta + \varepsilon_t)], \\
\sigma_t &= \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_t + \nu(\varepsilon_n + \varepsilon_\theta)], \\
\sigma_\theta &= \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_\theta + \nu(\varepsilon_n + \varepsilon_t)], \\
\tau_{nt} &= \frac{E}{(1+\nu)}\varepsilon_{nt} = \frac{E}{(1+\nu)} \frac{\gamma_{nt}}{2}, \\
\tau_{t\theta} &= \frac{E}{(1+\nu)}\varepsilon_{t\theta} = \frac{E}{(1+\nu)} \frac{\gamma_{t\theta}}{2}, \\
\tau_{\theta n} &= \frac{E}{(1+\nu)}\varepsilon_{\theta n} = \frac{E}{(1+\nu)} \frac{\gamma_{\theta n}}{2}.
\end{align*}
\]  

(1)

In equation (1), \(\sigma\) and \(\tau\) denote normal and shear stresses, respectively. Also, \(E\) and \(\nu\) denote the elastic modulus and the Poisson’s ratio of the periodontal ligament, respectively.

The above equations may be simplified by considering the axisymmetric shape of the tooth root and the small uniform thickness \(\delta\) of the PDL, as follows:

i) due to the symmetry of tooth root, the related to \(\theta\)-direction shear strains vanish:

\[
\gamma_{n\theta} = \gamma_{t\theta} = 0,
\]  

(2)

ii) due to the symmetry and the small thickness \(\delta\) of the PDL, the average diameter of a cross section does not change, so that the circumferential strain practically vanishes:

\[
\varepsilon_\theta = 0,
\]  

(3)

iii) due to the rigid tooth, if it is assumed that the thickness \(\delta\) of the PDL is extremely thin, the normal strain along the tangential direction vanishes:

\[
\varepsilon_t = 0.
\]  

(4)

All the above-mentioned assumptions and associated conclusions given by equations (2)-(4) have been numerically validated using detailed finite element analysis \[27\]. Taking into consideration the above-mentioned relations, we finally conclude that

\[
\begin{align*}
\sigma_n &\approx \frac{E}{(1+\nu)(1-2\nu)}(1-\nu)\varepsilon_n , \\
\sigma_t &\approx \frac{E}{(1+\nu)(1-2\nu)}\nu\varepsilon_n , \\
\sigma_\theta &\approx \frac{E}{(1+\nu)(1-2\nu)}\nu\varepsilon_n = \sigma_t , \\
\tau_{nt} &\approx \frac{E}{2(1+\nu)}\gamma_{nt} ,
\end{align*}
\]  

(5)
Based on the configuration of Fig. 1, the vertical tooth displacement (downwards) is analysed into two curvilinear components:

\[ V_{\text{tooth}}^n = -V \cos \alpha > 0 \]  \hspace{1cm} (6)

and

\[ V_{\text{tooth}}^t = V \sin \alpha < 0, \]  \hspace{1cm} (7)

where \( V \) is the vertical displacement downwards (negative quantity) and \( \alpha \) is the angle defined by the tangent at the tooth surface with the horizontal axis. In this convention, the sign of tooth displacement components is taken to be positive when is directed towards the positive direction of the corresponding curvilinear axis \( n \) or \( t \).

By considering one elementary element of continuum along the thickness of the PDL, an assumption adequately validated in [27], the strains \( \varepsilon_n \) and \( \gamma_n \) are the only non-vanishing strains, which are approximated as:

\[ \varepsilon_n = -\frac{V_{\text{tooth}}^n}{\delta} = -\frac{V \cos \alpha}{\delta} < 0 \]  \hspace{1cm} (8)

and

\[ \gamma_n = -\frac{V_{\text{tooth}}^t}{\delta} = -\frac{V \sin \alpha}{\delta} > 0. \]  \hspace{1cm} (9)

Fig. 1. Schematic representation of a tooth (T) subjected to an intrusive force \( F \) causing a displacement \( V \). At each point of tooth surface, a local coordinate system (\( n \)-normal, \( t \)-tangential) is defined. The root of length \( h \) is characterized by a uniform thickness \( \delta \). The alveolar bone (S-socket) is considered to be rigid.
Strain energy conservation

In virtue of equations (5) and (8)-(9), the strain energy due to normal and shear stresses are given as:

\[ U_{\text{normal}} = \frac{1}{2} \int_{\text{Vol}} \sigma_n \varepsilon_n d\text{Vol} = c_1^2 V^2 \]  \hspace{1cm} (10)

and

\[ U_{\text{shear}} = \frac{1}{2} \int_{\text{Vol}} \tau_{nt} \gamma_{nt} d\text{Vol} = c_2^2 V^2, \]  \hspace{1cm} (11)

where Vol is the volume of the PDL, again \( V \) is the vertical tooth displacement, and \( c_1, c_2 \) are constants that depend on the root length, \( h \), the root diameter, \( D \), and the uniform thickness of the PDL, \( \delta \), as shown in Fig. 1. For the case of an axisymmetric tooth of paraboloidal shape, the interested reader may found the analytical closed-form expressions of the aforementioned constants in Reference [27].

The application of energy conservation makes possible the determination of the value of the vertical displacement \( V \). In fact, the work of the applied intrusive (axial) force \( F \) should be undertaken by both the normal and shear stresses, so that

\[ \frac{1}{2} FV = U_{\text{normal}} + U_{\text{shear}} = c_1 V^2 + c_2 V^2, \]  \hspace{1cm} (12)

whence

\[ V = \frac{1}{2(c_1 + c_2)} F. \]  \hspace{1cm} (13)

After equation (13) is applied, strains can be calculated using eqs. (8)-(9) and stresses using eq. (5).

Principal strains

Starting from the strain tensor, which essentially corresponds to a two-dimensional problem:

\[
H = \begin{pmatrix}
\varepsilon_n & \varepsilon_{nt} & 0 \\
\varepsilon_{nt} & \varepsilon_{nt} & 0 \\
0 & 0 & 0
\end{pmatrix},
\]

it can be easily found that the principal strains are given by [35]:

\[ \varepsilon_1 = \frac{V}{2\delta} (\cos \alpha - 1) > 0, \]
\[ \varepsilon_2 = 0, \]  \hspace{1cm} (15)
\[ \varepsilon_3 = \frac{V}{2\delta} (\cos \alpha + 1) < 0. \]

It should be again clarified that in eq. (15) the sign of the intrusive displacement, \( V \), is negative, as it is opposite to the \( y \)-axis.

Now, by taking into consideration the vanishing terms (\( \varepsilon_1, \varepsilon_\theta, \gamma_1, \gamma_\theta \)), the first strain invariant becomes [35]:

\[ I_1 = \varepsilon_n = \varepsilon_1 + \varepsilon_2 < 0. \]  \hspace{1cm} (16)

Similarly, the second strain invariant becomes [35]:

\[ I_2 = -\varepsilon_{nt}^2 = -\frac{\gamma_{nt}^2}{4} = \varepsilon_1 \varepsilon_3 < 0. \]  \hspace{1cm} (17)
A hypothesis for bone remodeling

In this paper, the following assumptions are made during the tooth intrusion:

1. The PDL preserves its uniform thickness, $\delta$, around the tooth.

2. The PDL is a linear-elastic continuum possessing mechanical properties that do not alter during the remodelling process. In this paper, no viscoelastic phenomenon is considered.

3. Tooth is considered to be rigid.

4. At each instantaneous movement, the alveolar bone (socket) is considered to be rigid.

According to the first assumption, an instantaneous picture of the tooth root remains unaltered during the remodelling phase. Let us consider a point $P_{\text{tooth}}$ along the tooth surface. Let us also consider a straight line $(\varepsilon)$ passing through $P_{\text{tooth}}$, and also being parallel to the axis of symmetry $(y)$. The intersection of the line $(\varepsilon)$ with the bone surface determines a new point $P_{\text{bone}}$, as shown in Fig. 2. Obviously, in order to preserve the above-mentioned first assumption (i.e., uniform thickness, $\delta$, of the PDL), it is sufficient that both aforementioned points ($P_{\text{tooth}}$ and $P_{\text{bone}}$) have the same displacement:

$$V_{\text{bone}} = V_{\text{tooth}}.$$  \hspace{1cm} (18)

Using the normal $(n)$ and tangential $(t)$ components, eq. (18) is equivalently written as a system of the following two equations:

$$V^n_{\text{bone}} = V^n_{\text{tooth}}$$  \hspace{1cm} (19)

and
\[
V'_{\text{bone}} = V'_{\text{tooth}}. 
\]

(20)

Now, the right hand sides in eqs. (19, 20) are first substituted by eqs. (6, 7), leading to:

\[
V^n_{\text{bone}} = -V \cos \alpha 
\]

(21)

and

\[
V'_{\text{bone}} = V \sin \alpha. 
\]

(22)

Then, eqs. (8, 9) substitute the right hand sides of eqs. (21, 22), leading to:

\[
V^n_{\text{bone}} = -\delta \varepsilon_n 
\]

(23)

and

\[
V'_{\text{bone}} = -\delta \gamma_{nt}. 
\]

(24)

From eqs. (23, 24) it becomes clear that both normal and shear components contribute \textit{in the same degree}, since the same coefficient of proportionality, \(\delta\), appears.

In virtue of eqs. (16)-(17), equations (23)-(24) are written in terms of strain invariants as follows:

\[
V^n_{\text{bone}} = -\delta I_1 > 0 
\]

(25)

and

\[
V'_{\text{bone}} = -2\delta \sqrt{I_2} < 0. 
\]

(26)

Alternatively, the principal strains may be directly introduced as follows:

\[
V^n_{\text{bone}} = -\delta (\varepsilon_1 + \varepsilon_3) = -2\delta \bar{\varepsilon}_{\text{ma}} 
\]

(27)

and

\[
V'_{\text{bone}} = -2\delta \sqrt{\varepsilon_1 \varepsilon_3} = -2\delta \bar{\varepsilon}_{\text{ga}}, 
\]

(28)

where \(\bar{\varepsilon}_{\text{ma}}\) and \(\bar{\varepsilon}_{\text{ga}}\) denote the ‘simple average’ and the ‘geometrical average’ of the non-vanishing principal strains, respectively. The theoretical procedure that led to the bone remodelling formulas (23, 24), as well as the equivalent expressions (27, 28) in terms of principal strains constitutes the novel features of this paper.

\textbf{Remarks:}

1. The above formulas were obtained by assuming that the thickness of the PDL does not alter during the remodelling process. However, it is possible and accepted that PDL increases. In this case, the thickness \(\delta\) in eqs. (23, 24) should be replaced by a larger value \(\delta' (> \delta)\), while the strains \(\varepsilon_n\) and \(\gamma_{nt}\) are referenced to the initial situation (\(\delta\)).

2. By substituting eqs. (5) into (23) and (24), one receives

\[
V^n_{\text{bone}} = -\delta \frac{(1+v)(1-2v)}{E(1-v)} \sigma_n 
\]

(29)

and

\[
V'_{\text{bone}} = -\delta \frac{2(1+v)}{E} \tau_{nt}. 
\]

(30)

In other words, the bone remodelling formulas expressed in terms of normal and shear stresses are characterized by \textit{different} coefficients of proportionality, while in terms of strains the corresponding coefficients (\(-\delta\)) are \textit{identical}.

3. It should be noticed that equation (27) associated with \(n\)-direction is a \textit{linear combination} of principal strains, as it usually happens within the bone, for example, in Hayes’ model [25]. On the contrary, equation (28), associated with \(t\)-direction, is \textit{not}. Again, the proposed bone remodelling formulas given by eqs. (27)-(28) concern strains within the soft
periodontal ligament while previously established theories of long bones (e.g. Hayes’ model) concern strains within the bone.

**Materials and methods**

Using the above theoretical background, the following bone remodelling algorithm was applied.

**Bone remodelling algorithm**

It consists of the following steps:

- **Step 1.** The PDL around the tooth is divided into a certain number of finite elements. Its external surface, which is bonded with the bone surface, is fully restrained. The cartesian coordinates of the nodal points along the bone surface, \((x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)\) are recorded.

- **Step 2.** An intrusive force \(F\) is applied to the tooth. The displacements of the tooth as well as the strains within the PDL are calculated using the FEM.

- **Step 3.** Nodal points along the bone surface move according to eqs. (23, 24) or, equivalently, eqs. (27, 28), so as the updated cartesian coordinates are determined on the basis of \(x\)- and \(y\)-projections by:

\[
\begin{align*}
  x_i^{\text{new}} &= x_i^{\text{old}} + (V^n_{\text{bone}})_x + (V^t_{\text{bone}})_x, \\
  y_i^{\text{new}} &= y_i^{\text{old}} + (V^n_{\text{bone}})_y + (V^t_{\text{bone}})_y, \quad i = 1, 2, \ldots, N.
\end{align*}
\]

- **Step 4.** Tooth is located at the displaced position and a new mesh is generated within the PDL, between tooth and displaced bone surface according to eq. (31).

- **Step 5.** The old coordinates are substituted by the new ones:

\[
\begin{align*}
  x_i^{\text{old}} &= x_i^{\text{new}}, \\
  y_i^{\text{old}} &= y_i^{\text{new}}, \quad i = 1, 2, \ldots, N.
\end{align*}
\]

- **Step 6.** Go to Step 2 until a prescribed number of iterations are completed.

The above-mentioned procedure was applied to a maxillary central incisor with the following dimensions:

- Root length: \(h = 13.0\) mm,
- Root diameter: \(D = 7.8\) mm,
- Thickness of the PDL (uniform): \(\delta = 0.25\) mm.

The periodontal ligament has been assumed as an isotropic and linearly-elastic material with the following material data, taken from Tanne et al. [34]:

- Elastic modulus: \(E = 0.68\) MPa,
- Poisson’s ratio: \(\nu = 0.49\).

An ‘in-house’ FEM-code using eight-node three-dimensional finite elements and considering the tooth as a rigid body was implemented. This code is a bone-remodelling extension of a previous code of which details may be found in References [31-33]. Here, the principal strains are calculated in the middle of the PDL and eqs. (27, 28) are applied to determine the new locations of the interface alveolar-bone/PDL.

A special pre-processor was applied. The FEM-mesh was generated by 26 intersections perpendicular to the axis of symmetry at equal distances (per 0.5 mm), in combination with 16 equiangular subdivisions. The periodontal ligament was divided into two layers, so as 832 finite elements were used.
Results

Bone process is illustrated in Fig. 3. There, the initial situations of tooth and surrounding ligament as well as the updated bone surface after 1000 and 2000 iterations are shown. It can be noticed that the bone surface remains unaltered and it merely translated downwards, parallel to the direction of intrusive force.

Discussion

The developed theory is based on some considerations mainly concerned with the mechanical properties of the periodontal ligament. This model does not consider collagen fibres that may lead to stress concentrations instead of the smooth field used in this work. Also, the shape of the tooth surface was an ideal paraboloid and this could probably play a certain role. Finally, nonlinearities and viscoelastic phenomena have not been here considered.
Conclusions

In this paper, analytical formulas previously derived for the intrusion of an ideal axisymmetric tooth with parabolic root were used to establish an external bone-remodelling algorithm for the bone surface. It was found that both normal and shear strains contribute to the phenomenon in the same degree of importance; otherwise the PDL would obtain an intensively non-uniform shape. The bone-remodelling formula was also expressed in terms of principal strains, where it was found that both the first invariant and the square root of the second one equally contribute to the bone-remodelling process. The last constitutes the essential difference between teeth and long bones.

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РОЛЬ ГЛАВНЫХ ДЕФОРМАЦИЙ В ПЕРИОДОНАЛЬНОЙ СВЯЗКЕ ЗУБА В ХОДЕ ДОЛГОСРОЧНОЙ ИНТРУЗИИ

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Данная статья представляет собой предварительную теоретическую попытку вывести оригинальные количественные формулы в ортодонтии. Основываясь на ранее полученных аналитических формулах для напряжений и деформаций внутри периодонтальной связки (ПДС) для конкретного случая центрального резца в ходе интрузии, деформации были непосредственно связаны с движением поверхности раздела между альвеолярной костью и ПДС, называемой поверхностью кости. Строго
показано, что нормальные и сдвиговые деформации в ПДС имеют одинаковую значимость для поверхности кости. В соответствии с этим и среднее арифметическое и среднее геометрическое главных деформаций в ПДС играют ванную роль в процессе перестройки. В целом предложенные формулы отличаются от предыдущих, которые успешно применялись для описания перестройки в длинных костях. Предложенная теория также подтверждена конечно-элементным анализом.

Ключевые слова: ортодонтия, стоматологическая биомеханика, периодонтальная связка, главные деформации, перестройка кости, конечные элементы, прикладная механика

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