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ON NEW METHOD OF PROCESSING OF SPORTSMEN AND SPORTS PROJECTILES MOVEMENTS

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Abstract. In previous papers [1], [2] we showed how to determine aerodynamic coefficients of a volleyball and a ski-jumper by means of the video technology. Dependences of coordinates on time are obtained according to video records. Velocities and accelerations are found by coordinates numerical differentiation, and aerodynamic coefficients – by movement equations. The applied method of defining the velocities and accelerations led to a low accuracy of the found values, it is evidenced by the large scatter of aerodynamic coefficients in those studies. The currently suggested method of processing the video records is based on solving the boundary value problems of sports movements. It allows to keep the accuracy which took place in experiments. Usage of this method is illustrated by an example of a three-dimensional movement of a ball. Also it is shown that its aerodynamic coefficients may be defined without coordinates differentiation but by solving the boundary value problem with regular differentiated equations. This raises significantly the accuracy of the treatment of the videotape recording the sports motions as compared with applied methods of the aerodynamic coefficients determining.

Key words: sports projectile, aerodynamic coefficients, boundary value problem.

Introduction

While moving in continuous media, bodies are affected by pressure forces of these media. Aerodynamic forces are the pressure forces caused by body movements in the air. Aerodynamic forces resultant is usually decomposed into two forces: the drag force **R** and the lifting force **Q**. The drag force is directed opposite to the movement velocity **V** of the body centre of mass. The lifting force direction depends on the body geometry, and in the case of a spherical body it depends on the angular velocity vector $\boldsymbol{\omega}$ direction in compliance with the so-called Magnus' effect. Numeric values of aerodynamic forces are calculated as follows:

$$R = \frac{1}{2}\rho S c_D V^2, \quad Q = \frac{1}{2}\rho S c_L V^2, \tag{1}$$

where ρ is the air density, S is the middle section area – the area of a maximum body section perpendicular to the velocity of the centre of mass movement, c_D and c_L are the coefficients of the drag force and the lifting force.

To perform a mathematical description of the body movement by means of mechanics equations we need to know aerodynamic coefficients of those bodies. They are commonly found out by blowing the body in a wind tunnel. We know about experimental dependences of aerodynamic coefficients of a moving sphere on the velocity of the centre of mass movement and the angular velocity [3-6].

Currently due to the specificity and uniqueness of the wind tunnel experiments, as well as their dearness, it is fairly difficult to meet all the needs of sports sciences by performing the experiments in wind tunnels. Especially as each of sports projectiles and sportsmen has its or his own individual particular features so it may require a lot of experiments. However there is much more available and efficient way of aerodynamic coefficient determining.

During last years videorecording of moving bodies was becoming more and more popular as a mean of aerodynamic coefficient determining. Specially designed video systems allow defining the dependence of some body point coordinates on time. The major drawback of the existing methods of videorecords processing is that aerodynamic coefficients are evaluated through velocities and accelerations which are determined by the coordinate numerical differentiation. This is shown in [1], [2], where there is a large scatter of aerodynamic coefficients determined by means of these methods.

Investigations performed in Perm Biomechanics school have demonstrated that there is an alternative way to process video records of moving bodies, and the drawback mentioned above does not belong to this way. The way includes multiple solutions of a boundary value problem of the movement of the body centre of mass and determination of calculated path coincident with the experimental one. Here at each small time period the body aerodynamic coefficients and the velocity of its centre of mass movement are found. In the suggested method the coordinate numerical differentiation leading to low accuracies is replaced by integration of movement differential equations which can be performed with a high accuracy, much higher than the accuracy of experimental determination of centre of mass coordinates. The new method introduces no extra inaccuracy into determination of moving body aerodynamic coefficients.

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Some conditions of experiment performing

The centre of mass movement of a moving body is to be determined by means of video recording determining aerodynamic coefficients. In the case of translational movement of the body it is enough to put one mark on it and make a video camera keep track of this mark. For instance, during the major part of his fly phase the ski-jumper's movement is close to the translational one. The mark may be put on any part of sports equipment within the video camera view, for instance, on the ski-jumper's hat.

In the case when the body's movement is three-dimensional (composite) one there should be more than one mark on it. For recording a broad jump sample, for instance, the marks are to be put on all the main segments of the sportsman's body. Centre of mass coordinates are to be determined according to the coordinates of these marks and masses of the corresponding segments of the body.

It is not so easy to keep track of a sports ball's centre of mass when it makes three-dimensional (composite) movement. If the ball is small, the video camera may perceive it as one mass point. If the ball is bigger it will be recognized as an extensive body where the centre of mass should be determined.

For the boundary value problem solution it is important to determine the initial conditions of the studied subject movement. The initial coordinates of the centre of mass are recorded by the video camera, and the initial velocity can be determined by the first frames of the record or by means of special measure sets which were used already at big tennis competitions, in ski-jumping and other sports.

So video records help us to figure out dependences between coordinates of the sports body centre of mass and time: $x(t)$, $y(t)$, $z(t)$, as well as initial conditions of its movement

$$t = 0: x = x_0,\ y = y_0, z = z_0, \dot{x} = \dot{x}_0, \dot{y} = \dot{y}_0, \dot{z} = \dot{z}_0, \tag{2}$$

where the above dot means the time derivative.

And, besides, considering the sports ball flight, at the start time its angular velocity $\boldsymbol{\omega}$ has to be determined due to its dimensions and direction.

Differential equations of the spherical sports projectile (ball) centre of mass movement

Let us consider arbitrary movement of the ball with a radius *r* and mass *m* in Cartesian rectangular coordinates *Oxyz* (with ***y*** axis directed vertically upwards) within a homogeneous gravity field in some resistant medium. Besides the constant gravity $P = mg$, the ball is affected by the drag force $\mathbf{R}$, and in the case of its rotation also by the lifting force $\mathbf{Q}$. Aerodynamic forces are defined as follows [7]:

$$\mathbf{R} = -R\frac{\mathbf{V}}{V}, \quad \mathbf{Q} = Q\frac{\boldsymbol{\omega}\times\mathbf{V}}{|\boldsymbol{\omega}\times\mathbf{V}|}, \quad \omega \neq 0, \tag{3}$$

where R and Q are taken from (1). The lifting force is equal to zero ($Q = 0$) when $\omega = 0$.

Considering (1), (5), the differential equations of the sports ball centre of mass movement have a form:

$$\begin{aligned}
m\frac{du}{dt} &= -\frac{1}{2}\rho S c_D V u + \frac{1}{2}\rho S c_L \frac{\omega_y w - \omega_z v}{|\boldsymbol{\omega}\times\mathbf{V}|},\\
m\frac{dv}{dt} &= -mg - \frac{1}{2}\rho S c_D V v + \frac{1}{2}\rho S c_L \frac{\omega_z u - \omega_x w}{|\boldsymbol{\omega}\times\mathbf{V}|},\\
m\frac{dw}{dt} &= -\frac{1}{2}\rho S c_D V w + \frac{1}{2}\rho S c_L \frac{\omega_x v - \omega_y u}{|\boldsymbol{\omega}\times\mathbf{V}|},\\
\frac{dx}{dt} &= u,\ \frac{dy}{dt} = v,\ \frac{dz}{dt} = w,\ S = \pi r^2, V = \sqrt{u^2 + v^2 + w^2},\ \omega \neq 0,
\end{aligned} \tag{4}$$

where *u*, *v*, *w* are projections of the centre of mass movement velocity **V**, and ω_x, ω_y, ω_z are projections of the angular velocity vector onto axes x, y, z correspondingly (here $\boldsymbol{\omega}$ is the ball angular velocity).

Differential equations (6) integration assumes the angular velocity vector to remain constant during the ball flight ($\boldsymbol{\omega}$ = const). This assumption is defended by the fact that viscous friction forces, generating in the air an angular momentum, are too small so they cannot affect the ball angular velocity distinctly. The assumption of the ball angular velocity as constant makes the solution of equations (4) significantly simpler. The direction of the vector $\boldsymbol{\omega}$ does not matter, as its module ω is not included into the differential equations (4), but aerodynamic coefficients c_D and c_L are dependent on the value of the angular velocity ω.

The differential equations (4) should be completed by the initial conditions of the sports ball centre of mass movement

$$t = 0 : x = x_0,\ \ y = y_0,\ \ z = z_0,\ \ u = u_0,\ \ v = v_0,\ \ w = w_0, \tag{5}$$

which have to be determined experimentally.

With the preassigned values of aerodynamic coefficients the Cauchy problem (4), (5) may be solved by the step-by-step integration methods.

The mentioned method of video record processing demands the accuracy of a numerical solution to be rather high, so we suggest that Runge-Kutta methods of the fourth order of accuracy are to be used. Aerodynamic coefficients c_D and c_L obscurity allows us to add boundary conditions determined experimentally to the equations (4), (5) and develop a boundary value problem which solutions may result in the aerodynamic coefficients.

The case of the centre of mass plane trajectory

Let the initial velocity vector $\mathbf{V_0}$ of the ball centre of mass lays in the coordinate plane *Oxy*, and the angular velocity vector $\boldsymbol{\omega}$ is parallel to *Oz* axis (Fig. 1).

In this case all the external forces affecting the ball locate in the coordinate plane *Oxy*, and the centre of mass C will be moving within this plane. The differential equations of centre of mass (6) become a bit simpler:

$$\begin{aligned} & m\frac{du}{dt} = -\frac{1}{2}\rho S c_D V u \mp \frac{1}{2}\rho S c_L V v, \\ & m\frac{dv}{dt} = -mg - \frac{1}{2}\rho S c_D V v \pm \frac{1}{2}\rho S c_L V u, \\ & \frac{dx}{dt} = u, \ \ \frac{dy}{dt} = v, \quad V = \sqrt{u^2 + v^2}, \end{aligned} \tag{6}$$

where the upper sign in the lifting force relates to the vector $\boldsymbol{\omega}$ direction shown in Fig. 1 (reverse rotation) and the lower sign means the case with the rotation in the opposite direction (direct rotation). The rotation direction is determined by the kick on the ball video record. The initial conditions of the ball centre of mass movement are considered to be unknown

$$t = 0 : x = 0,\ \ y = 0,\ \ u = V\cos\alpha,\ \ v = V\sin\alpha. \tag{7}$$

The Cauchy problem would be solvable if we knew the aerodynamic coefficients c_D and c_L. To find these let us define a boundary value problem. According to the video frame frequency let us define a time interval Δt between two sequential shots of a ball location. The time axis is to be splitted into the intervals of this amount, and the moments of shooting can be defined as follows:

$$t_k = k\,\Delta t, \quad k = 0, 1, 2, \ldots . \tag{8}$$

Let us assume that according to video frames we defined coordinates of the centre of mass at these time points

$$t = t_k : x = x_k,\ \ y = y_k,\ \ k = 0, 1, 2, \ldots . \tag{9}$$

First, let us use the boundary condition in the end of the first step

$$t = \Delta t : x = x_1,\ \ y = y_1\ . \tag{10}$$

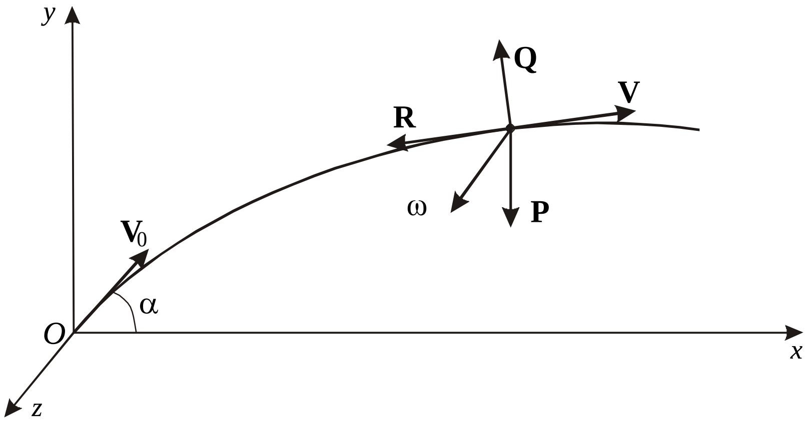

Fig. 1. The system of forces affecting a ball under $\boldsymbol{\omega} \| Oz$.

The task (6), (7), (10) is multipoint or boundary value problem with redundant boundary conditions (10). We are to find the aerodynamic coefficients c_D and c_L values with which the conditions (10) will be satisfied. The boundary value problem can be solved by a well-known way [8], which may be regarded as a modified Newton's method.

The Cauchy problem solution (6), (7) depends on the independent parameters c_D and c_L. In particular, at the time instant $t = \Delta t$ the coordinates may be considered as functions of these parameters

$$t = \Delta t : x = x(c_D, c_L), \;\; y = y(c_D, c_L) . \tag{11}$$

A null approximation c_D^*, c_L^* is chosen so that the functions (11) were close enough to the values of coordinates within the boundary conditions (10).

The Cauchy problem (6), (7) is solved at the time interval $[0, \Delta t]$ with $c_D = c_D^*, c_L = c_L^*$. By the end of integration the coordinate values x_1^*, y_1^* become a null approximation to the given boundary value condition (12). It should be noted that the integration step h has to be much less than the time interval Δt and has to obtain a high accuracy of the equations integration (6).

The desired solution is expanded into the Taylor’s series satisfying the boundary condition close to a null approximation correct to first order:

$$\begin{aligned} x_1 &= x_1^* + \left(\frac{\partial x_1}{\partial c_D}\right)^* \Delta c_D + \left(\frac{\partial x_1}{\partial c_L}\right)^* \Delta c_L , \\ y_1 &= y_1^* + \left(\frac{\partial y_1}{\partial c_D}\right)^* \Delta c_D + \left(\frac{\partial y_1}{\partial c_L}\right)^* \Delta c_L . \end{aligned} \tag{12}$$

Partial derivatives are defined approximately by using two complementary solutions of the Cauchy problem with new aerodynamic coefficient values

$$\begin{aligned} &1)\, c_D = c_D^* + dc_D, \;\; c_L = c_L^*, \\ &2)\, c_D = c_D^*, \;\; c_L = c_L^* + dc_L, \end{aligned} \tag{13}$$

where dc_D and dc_L are small but finite additions to the null approximation.

The inhomogeneous algebraic equations set (12) determines corrections Δc_D and Δc_L to the null approximation. A new approximation $c_D^* + \Delta c_D$**,** $c_L + \Delta c_L$ is considered as a null one at the next iteration of the boundary value problem solution (6), (7), (10) and the iterative process goes on until the boundary condition satisfies the preset accuracy, i.e. modules Δc_D and Δc_L do not become less than some preassigned positive number.

The boundary value problem solution gives mean values of the aerodynamic coefficients c_{D_1} and c_{L_1} at the first time interval Δt. Besides, the boundary value problem solution gives velocity projections u_1 and v_1 at the end of the time interval Δt.

At the next time interval $[\Delta t, 2\Delta t]$ the boundary value problem is similarly developed. The initial conditions are determined by the solution at the first step

$$t = \Delta t : x = x_1,\ \ y = y_1,\ \ u = u_1,\ \ v = v_1, \tag{14}$$

and the complementary boundary conditions have the following form:

$$t = 2\Delta t : x = x_2,\ \ y = y_2. \tag{15}$$

The boundary value problem (6), (14), (15) is solved in the same way as it has been already at the first time step. This solution gives the values c_{D_2} and c_{L_2} at the second time interval of the video record. Solving the sequence of boundary value problems for all the time points recorded by a video camera it is possible to obtain a discrete dependence between aerodynamic coefficients and time

$$c_D = c_D(t),\ \ c_L = c_L(t). \tag{16}$$

As there is the centre of mass velocity at each time interval, so it is not difficult to find the dependences

$$c_D = c_D(V),\ \ c_L = c_L(V), \tag{17}$$

which are usually obtained in wind tunnels and which are used in mathematical modeling of solid bodies moving in a resisting medium. It should be noted that these results were obtained under some specified value of the ball rotation angular velocity ω determined experimentally. Recording the ball movement with different angular velocity values and processing the experiment results as it was described above it is possible to reveal the dependence of aerodynamic coefficients on the angular velocity ω.

At the end of the general algorithm of the current problem solution it should be noted that the boundary value problem solution does not introduce any extra inaccuracy into aerodynamic coefficients because the accuracy of movement differential equations numerical integration is much higher than the accuracy of experimental defining of the ball centre of mass coordinates. The algorithm does not contain the co-called “sign fall down” which is observed in the coordinates numerical differentiation in previous ways of processing the sport projectiles movement video records.

In the special case when a ball does not rotate ($\omega = 0$) the same boundary value problem is to be solved as in the case when $\omega \neq 0$ with complete differential equations (6). As a result of its solution the value of the aerodynamic coefficient c_L has to be close to zero. It is not possible to presuppose in advance that $c_L = 0$, and this makes the boundary value problem setting more complicated. The number of redundant boundary conditions appears to be more than the number variable parameters. The way of overcoming this sort of trouble is considered in the next section of the paper.

Spatial trajectory case

Let us consider the movement of the ball centre of mass along a spatial trajectory by an example of a corner kick with a goal hit to the rival's gate. Fig. 2 demonstrates projections of the ball centre of mass trajectories onto the plane Oxy (curve 1) and plane Oxz (curve 2). The projections of the ball centre of mass initial velocity onto these planes are denoted $\mathbf{V}_0'$ and $\mathbf{V}_0''$ correspondingly. The total initial velocity vector $\mathbf{V}_0$ lays in the vertical plane $Ox'y$ making the angle β_0 with the plane Oxy and angle α_0 with the axis Ox'.

The kick makes the ball rotate so that the angular velocity vector $\boldsymbol{\omega}$ also lays in the plane $Ox'y$ making the angle α_0 with the axis Oy. Such a direction of the angular velocity vector is quite natural, as it is perpendicular to the football meridional plane; both a boot kick point and a vector of a boot velocity before the kick lies in this plane; the "lifting force" perpendicular to the vectors $\boldsymbol{\omega}$ and **V** (formula 3)) contorts the trajectory so that the ball gets into the gate (point *M* in Fig. 2).

Initial conditions of the ball centre of mass movement have the form:

$$\begin{gathered} t=0: x=0,\ \ y=0,\ \ z=0,\ \ u=V_0\cos\alpha_0\cos\beta_0, \\ v=V_0\sin\alpha_0, \quad w=V_0\cos\alpha_0\sin\beta_0. \end{gathered} \tag{18}$$

Initial conditions and dependences of the centre of mass on time

$$x = x(t),\ y = y(t),\ z = z(t) \tag{19}$$

have to be determined experimentally.

Similarly to the boundary value problem development in the case with a plane trajectory let us put down the boundary condition at the end of the first time interval Δt of the video record

$$t=\Delta t: x=x_1,\ \ y=y_1,\ \ z=z_1, \tag{20}$$

where x_1, y_1, z_1 are the coordinates determined according to the video frame.

The boundary value problem (4), (18), (20) has three redundant boundary conditions (20), while in the equations (4) there are only two variable parameters c_D and c_L. So one more parameter has to be introduced artificially, and it is convenient to take a parameter

$$c_\beta = \cos\beta, \tag{21}$$

included into the initial conditions (18). It should be noted beforehand that velocity components *u* and *w* in the initial conditions (18) have to get corrected after each iteration during the boundary value problem solution.

$$u=V_0\cos\alpha_0 c_\beta, \quad \omega=V_0\cos\alpha_0\sqrt{1-c_\beta^2}. \tag{22}$$

These changes in the initial conditions must not make the accuracy of aerodynamic coefficients determining lower; vice verca it brings one of the initial conditions into accord with a form obtained by the video record of the trajectory.

With $t=\Delta t$ the Cauchy problem solution (4), (18) has a form of a three-parameter function

$$t=\Delta t:\ x=x(c_D,c_L,c_\beta),\ y=y(c_D,c_L,c_\beta),\ z=z(c_D,c_L,c_\beta). \tag{23}$$

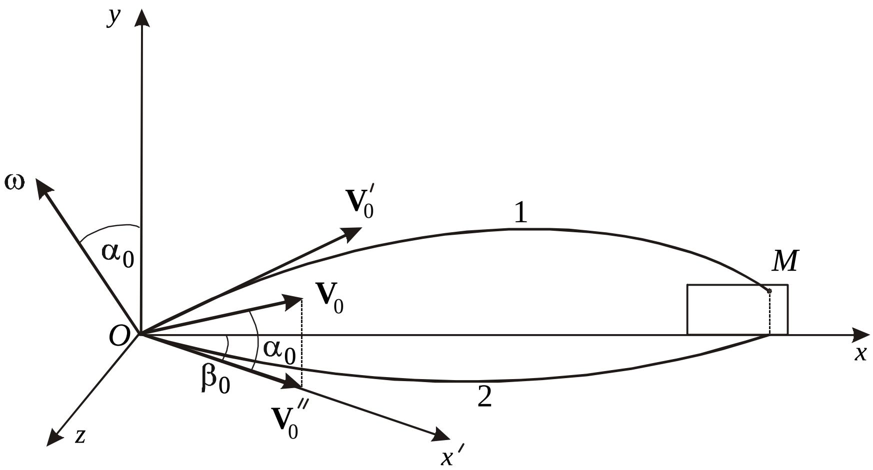

Fig. 2. Spatial trajectory projections of the football centre of mass onto planes *Oxy* (curve 1) and *Oxz* (curve 2) under the corner kick.

As a null approximation for c_β let us take its value $c_\beta^* = \cos\beta_0$ at the initial time point. Null approximations c_D^* and c_L^* are taken whether considering available information about them, or by means of computer analysis of the trajectory. The Cauchy problem solved at the time period [0, Δt] with parameters c_D^*, c_L^*, c_β^*, results in numerical coordinates of the football centre of mass at the end of this time interval

$$t = \Delta t : x = x_1^*, y = y_1^*, z = z_1^*. \tag{24}$$

The desired solution corresponding to the boundary conditions (20) are expanded into the series close to the null approximation correct to first order

$$\begin{aligned}
x_1 &= x_1^* + \left(\frac{\partial x_1}{\partial c_D}\right)^* \Delta c_D + \left(\frac{\partial x_1}{\partial c_L}\right)^* \Delta c_L + \left(\frac{\partial x_1}{\partial c_\beta}\right)^* \Delta c_\beta, \\
y_1 &= y_1^* + \left(\frac{\partial y_1}{\partial c_D}\right)^* \Delta c_D + \left(\frac{\partial y_1}{\partial c_L}\right)^* \Delta c_L + \left(\frac{\partial y_1}{\partial c_\beta}\right)^* \Delta c_\beta, \\
z_1 &= z_1^* + \left(\frac{\partial z_1}{\partial c_D}\right)^* \Delta c_D + \left(\frac{\partial z_1}{\partial c_L}\right)^* \Delta c_L + \left(\frac{\partial z_1}{\partial c_\beta}\right)^* \Delta c_\beta.
\end{aligned} \tag{25}$$

The same as in the plane trajectory case, here partial derivatives are determined approximately by means of three complementary solutions of the Cauchy problem (4), (18) with small additions into the null approximation of each of variable parameters. Corrections to the null approximation are determined by the heterogeneous algebraic system (25)

$$\Delta c_D,\ \Delta c_L,\ \Delta c_\beta.$$

After the first iteration a new approximation for the variables was obtained by the solution of the boundary value problem

$$c_D' = c_D^* + \Delta c_D, \quad c_L' = c_L^* + \Delta c_L, \quad c_\varphi' = c_\varphi^* + \Delta c_\varphi. \tag{26}$$

Initial conditions of the second iteration have the form:

$$t = 0 : x = 0,\ \ y = 0,\ \ z = 0,\ \ u = V_0 \cos\alpha_0 c'_\varphi,$$
$$v = V_0 \sin\alpha_0, \quad w = V_0 \cos\alpha_0 \sqrt{1 - c'^2_\varphi}. \qquad (27)$$

The approximation (26) is refined by the boundary value problem solution according to the algorithm considered for the first iteration. The iterations go on until the modules of corrections to the aerodynamic coefficients Δc_D, Δc_L and to the direction cosine Δc_β become smaller than some preassigned small positive value ε. In this case the boundary condition (20) is satisfied correct to ε according to expansions (25).

So we obtained the aerodynamic coefficients c_{D_1}, c_{L_1} at the first time interval [0, Δt]. Beyond that point the centre of mass velocities u_1, v_1, w_1 were obtained at the time point $t = \Delta t$. It allows us to come to determining of aerodynamic coefficients at the next time interval [Δt, $2\Delta t$].

Let us formulate a boundary value problem for this interval. The initial conditions are to be kept like (18) so that the direction cosine c_β would remain as a variable parameter. New values of the angles α и β determining the direction of the velocity vector $\mathbf{V}_1$ at the end of the first time interval can be obtained according to the formulas analogous to the boundary conditions (18)

$$V_1 = \sqrt{u_1^2 + v_1^2 + w_1^2}, \quad \cos\alpha_1 = \sqrt{1 - \frac{v_1^2}{V_1^2}}, \quad \cos\beta_1 = \frac{u_1}{V_1 \cos\alpha_1}. \qquad (28)$$

Then the movement initial conditions at the next time interval will have the form:

$$t = \Delta t : x = x_1,\ \ y = y_1,\ \ z = z_1,\ \ u = V_1 \cos\alpha_1 \cos\beta_1,$$
$$v = V_1 \sin\alpha_1, \quad w = V_1 \cos\alpha_1 \sin\beta_1. \qquad (29)$$

The boundary condition at the end of the new time interval, corresponding to the experimental data, looks as follows:

$$t = 2\Delta t : x = x_2,\ \ y = y_2,\ \ z = z_2. \qquad (30)$$

Let us choose the following values for the variables as the null approximation for the boundary value problem solution (4), (29), (30)

$$c_D^* = c_{D_1}, \quad c_L^* = c_{L_1}, \quad c_\beta^* = c_{\beta_1}. \qquad (31)$$

The boundary value problem solution analogous to the one given above concerning the first time interval gives new values of the aerodynamic coefficients c_{D_2}, c_{L_2} at the second time interval and components of the centre of mass velocity at the end of this interval u_2, v_2, w_2.

Boundary value problems are solved in the same way at all the time intervals between the video points for the whole trajectory shown in Fig. 2. So we obtain the discrete dependence of aerodynamic coefficients and the components of the football centre of mass movement velocity on time

$$c_D = c_D(t),\ \ c_L = c_L(t),\ \ u = u(t),\ \ v = v(t),\ \ w = w(t), \qquad (32)$$

Aerodynamic coefficients dependences on the centre of mass movement velocity are found in direct experiments. Formulas (32) allow us to follow this relation:

$$c_D = c_D(V),\ \ c_L = c_L(V). \qquad (33)$$

So those are the final results of the football flight video processing. The Cauchy problem solutions (4), (18) taking into account dependences (33) have to plot the trajectory of the football centre of mass movement close to that one obtained by the video record. Some difference may be connected with the third parameter introduced artificially.

It should be noted that dependences (33) relate to a certain value of the angular velocity of a football rotation ω which has been recorded by video record. Varying the ball angular velocity during the experiment it is possible to get the aerodynamic coefficients dependence on the angular velocity ω as well.

Conclusions

The new method of processing of sports projectiles and sportsmen movement video data in a resisting medium is developed. The method is based on solving the boundary value problems sequence which boundary conditions are found experimentally. This method allows to determine aerodynamic coefficients of a moving body and the movement velocity of its centre of mass by its coordinates found according to a video data. The suggested method is resistant to failures committed in the course of experiments. From the point of view of mathematics, the advantage of this method is that the numerical differentiation of coordinates applied earlier is replaced here by the integration of the regular differential equations set performed with a high accuracy.

The method is illustrated by the examples of a plane and spatial movement of a spherical sports projectile (ball) considering its rotation.

The algorithm for solving the boundary value problem with redundant boundary conditions determining the ball aerodynamic coefficients is suggested. The paper shows the way of finding dependences between aerodynamic coefficients and the centre of mass movement velocity of a football.

Accurate determination of aerodynamic coefficients allows us to use them in mathematical modeling of sports projectiles and sportsmen movement and in solving optimization problems.

The method disclosed in the paper is supposed to be applied in investigations of some other sports projectiles (javelin, discus) and sportsmen (ski-jumper) movement.

The proposed algorithm of the aerodynamic coefficients determination is only first step to the solution of this fundamental problem of sports biomechanics. Because of the nonlinearity of the movement equations the many-valuedness of the obtained solution is possible. The problem of the selection of a correct solution appears. The problem has a unique solution when only aerodynamic coefficient is searched. If two unknown variables are considered the aerodynamic coefficients can be determined unambiguously by means of complication of the algorithm and utilizing the subsidiary boundary conditions in boundary-value problem.

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НОВЫЙ МЕТОД ОБРАБОТКИ ВИДЕОЗАПИСЕЙ ДВИЖЕНИЯ СПОРТСМЕНОВ И СПОРТИВНЫХ СНАРЯДОВ

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Предлагаемый метод обработки видеозаписей основан на решении краевых задач спортивных движений. Он позволяет сохранить ту точность, которая имела место при проведении эксперимента. Применение указанного метода проиллюстрировано на примере сложного движения мяча. Показано, что его аэродинамические коэффициенты можно определить путем решения краевой задачи с обыкновенными дифференцированными уравнениями. Это существенно повышает точность обработки результатов видеозаписи спортивных движений по сравнению с прежними методами определения аэродинамических коэффициентов.

Ключевые слова: спортивный снаряд, аэродинамические коэффициенты, краевая задача.

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