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CONTACT INTERACTION OF PLATES AND BEAMS UNDER THE INFLUENCE OF WHITE NOISE

T.V. Yakovleva[1, 2], V.G. Bazhenov[1], V.A. Krysko[2], E.Yu. Krylova[1, 3]

[1] Mechanics Research Institute of N.I. Lobachevsky State University of Nizhny Novgorod, Nizhny Novgorod, Russian Federation
[2] Yuri Gagarin State Technical University of Saratov, Saratov, Russian Federation
[3] Saratov State University, Saratov, Russian Federation

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ABSTRACT

This paper deals with contact interaction of a multi-layer structure in the form of a plate reinforced by beams. There are small gaps between the plate and beams. Such systems are integral elements of modern devices. The created mathematical model is based on the following hypothesis: the system has a multi-layer structure; materials are isotropic. We used the kinematic model of the first approximation (the Kirchhoff model for the plate, the Euler-Bernoulli model for beams). To solve the problem we used the finite difference method with approximation 0 (h2), 0 (h4) and the Faedo-Galerkin method in higher approximations of spatial coordinates, as well as the Runge-Kutta method 0 (h4), 0 (h6), 0 (h8) in time. When solving problems associated with chaotic vibrations, it is necessary to solve the issue of error and realness of chaos, so we need to use different numerical methods at each stage of the study to validate the results in order to distinguish chaos from the numerical error. For the analysis of chaotic dynamics we have applied all methods of qualitative analysis. We have investigated the spatiotemporal chaos based on wavelet analysis. We have studied the influence of white noise on the contact interaction of elements of the multi-layer structure. Also, the analysis of the complex vibrations of plates and beams depending on different intensities of noise and types of applied load has been made. It was found that by using an external additive white noise, it is possible to control chaotic oscillations and transfer the system from a chaotic state to a harmonious one and enable or disable the contact interaction.

© Tatyana V. Yakovleva – CSc in Physical and Mathematical Sciences, Junior Researcher, e-mail: yan-tan1987@mail.ru

Valentin G. Bazhenov – Doctor of Physics and Mathematics, Professor, e-mail: bazhenov@mech.unn.ru

Vadim A. Krysko – Doctor of Technical Sciences, Professor, e-mail: tak@san.ru

Catherine Y. Krylova – CSc in Physical and Mathematical Sciences, Junior Researcher, e-mail: kat.krylova@bk.ru

1. Introduction

The structures of modern devices and equipment are complex multi-layer systems of beams and plates with small gaps between them. Such structures are affected by extreme conditions caused by deterministic external influence of various kinds and random fluctuations in the environment properties which causes modification of modes of dynamic systems [1]. It is necessary to take into account the contact interaction of layers which is the reason of constructive nonlinearity [2]. A gap between the elements (plate-beam, plate-plate, beam-beam, shell-beam and other combinations), as well as the imperfection of the device, even if there are small deflections comparable with the gap between the elements, can cause the studied object to be in a state of chaotic oscillations [3-4]. Therefore, it is important to study influence of control parameters, and different kinds of external load on performance of the system. Such systems have a wide application to electronics, in particular, to gyroscopes (multi-layered flat micromechanical accelerometers (MMA)), described in articles [5-6], but these works do not deal with the contact interaction of layers. When modeling behavior of structures of modern devices, the type of the chaotic state is important. This phenomenon can be studied from the perspective of analysis of the variety of signs of Lyapunov exponents [7-9], Fourier analysis and wavelet analysis [10]. An important matter is the control of chaos and complex nonlinear oscillations that lead to various performance errors of sensors of measuring instruments. Such errors can be studied by taking into account the mathematical model of white noise. The problem of increasing the accuracy and structural strength of modern devices is currently relevant. This problem can be solved by using new technical solutions and new technologies, as well as by creating new mathematical models that describe nonlinear dynamics of distributed systems. At the moment, both foreign and Russian scientific schools have increased interest in effects associated with the influence of external noises on the behavior of dynamic systems. In such areas as physics, chemistry, and biology it has already been proved that random influences play a very significant role in the behavior of dynamic systems [11-12]. External noise can cause not only fluctuations in the properties of dynamic systems, but also a qualitative modification of their modes [13-15]. Using the example of the Anishchenko-Astakhov oscillator, T.E. Vadivasova [16] has proved that the effect of noise signal causes a shift of doubling bifurcations towards the increase of the control parameter. In the work by V.D. Potapov [17] it is shown that the deterministic parametric system which is unstable according to Lyapunov, can be stabilized by applying random noise to the parametric load. The work [18] reports on the results of the sample study of a beam-impact system under Gaussian noise disturbances. It is proved that when taking into account several forms of oscillations, the reaction of a nonlinear model differs greatly from that of the calculation model with one degree of freedom. The article [19] shows that by taking into account the number of degrees of freedom we can significantly affect reliability of the results. The work [20] presents calculations of vibrations and radiation of sound of reinforced plates covered with a damping layer under harmonic load and load of the white noise type. The solution is obtained by natural decomposition of oscillations. S.I. Denisov [21] examines noise-induced transitions in one-dimensional systems that cause their stationary distribution functions to change qualitatively with the change of noise intensity. When fluctuations are modeled by Gaussian white noise, the necessary condition for such transitions is multiplicity of noise. The authors of the article [22] suggest a theoretical static model of equilibrium of a liquid drop on a rough flat surface. The surface is described using a random stationary function of the white noise type in a limited frequency range. A generalization of the model for surface topography in the form of a set of random functions was carried out. However, in the literature that we know about, there are no works devoted to the matter of external influence of the environment on nonlinear oscillations of multi-layer systems in the form of beams and plates as systems with a variety of degrees of freedom.

2. Setting the Problem

The authors of this work have created a mathematical model of complex vibrations of a multi-layer system consisting of a plate and three beams, which is subjected to external load of different types (Fig. 1). The upper layer is a plate described by equation of the Germain-Lagrange type, and the lower layer is a set of parallel beams. Each beam is described by the Euler-Bernoulli equation. The contact interaction is taken into account by the Winkler model. The plate and beams are isotropic and connected by the boundary conditions, there is a small gap between them. The mathematical model is described by a following system of equations:

$$\begin{cases} \frac{1}{12(1-\mu^2)}\left(\frac{1}{\lambda^2}\frac{\partial^4 w_1}{\partial x^4}+\lambda^2\frac{\partial^4 w_1}{\partial y^4}+2\frac{\partial^4 w_1}{\partial x^2 \partial y^2}\right)+ \\ +\frac{\partial^2 w_1}{\partial t^2}+\varepsilon\frac{\partial w_1}{\partial t}-q_1(x,y,t)-K\left(w_1-w_2-h_k\right)\Psi_1- \\ -K\left(w_1-w_3-h_k\right)\Psi_2-K\left(w_1-w_4-h_k\right)\Psi_3=0, \\ \frac{1}{12}\frac{\partial^4 w_2}{\partial x^4}+\frac{\partial^2 w_2}{\partial t^2}+\varepsilon\frac{\partial w_2}{\partial t}-q_2(x,t)+ \\ +K\left(w_1-w_2-h_k\right)\Psi_1=0, \\ \frac{1}{12}\frac{\partial^4 w_3}{\partial x^4}+\frac{\partial^2 w_3}{\partial t^2}+\varepsilon\frac{\partial w_3}{\partial t}-q_3(x,t)+ \\ +K(w_1-w_3-h_k)\Psi_2=0, \\ \frac{1}{12}\frac{\partial^4 w_4}{\partial x^4}+\frac{\partial^2 w_4}{\partial t^2}+\varepsilon\frac{\partial w_4}{\partial t}-q_4(x,t)+ \\ +K\left(w_1-w_4-h_k\right)\Psi_3=0, \end{cases} \qquad (2.1)$$

where functions $\psi_1 = \frac{1}{2}\left[1 + sign\left(w_1 - h_k - w_2\right)\right]$, $\psi_2 = \frac{1}{2}\left[1 + sign\left(w_1 - h_k - w_3\right)\right]$, and $\psi_3 = = \frac{1}{2}\left[1 + sign\left(w_1 - h_k - w_4\right)\right]$.

The ratio $K(w_1 - w_i - h_k)\Psi_{i-1}$ (where i = 2; 3; 4 is the beam number) is the contact pressure between the layers. In the contact problems of the theory of plates and beams this ratio is the Winkler connection between compression and contact pressure.

$\nabla_\lambda^4 = \frac{1}{\lambda^2}\frac{\partial^4}{\partial x^4} + \lambda^2\frac{\partial^4}{\partial y^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2}$; $\Psi_{i-1} = 1$, if $w_1 > w_i + h_k$ is the contact between the plate and the beam, otherwise $\Psi_{i-1} = 0$; w_1, w_i are functions describing deflections of the plate and beams, respectively, K is the stiffness coefficient of transversal compression of the structure in the contact zone, h_k is the gap between the layers. Adhesion zones are unlikely to occur, since the contact pressure between the layers is low. The conditions of contact between the layers may depend on coordinates and include all kinds of the imperfect one-side contact [23].

The system of equations (1.1) has the following dimensionless form: $x = a\bar{x}$, $y = a\bar{y}$; $q = \bar{q}\frac{E(2h)^4}{a^2 b^2}$, $\tau = \frac{ab}{2h}\sqrt{\frac{\gamma}{Eg}}$, $\lambda = \frac{a}{b}$, where a, b are dimensions of the plate regarding x and y, respectively, t is the time, ε is the damping coefficient, w is the deflection function, $2h$ is the thickness of the plate, $\mu = 0.3$ is the Poisson's ratio, g is the gravity acceleration, E is the modulus of elasticity, $q_1(x, y, t)$ is the transverse load applied to the plate, $q_i(x, t)$ are transverse loads applied to the beams, γ is the specific weight of the material.

The multi-layer system can be subjected to different external loads: transverse loads with or without additive noise.

The general form of transverse loads can be written as follows:

$$q_1(x, y, t) = q_0 \sin(\omega_p t) + a^0 \times \frac{2rand}{rand\_\max + 1} - 1; \quad (2.2)$$

$$q_i(x, t) = q_i^0 \sin(\omega_p t) + a_i^0 \times \frac{2rand}{rand\_\max + 1} - 1,$$

$$(i = 2, ..., 4); \quad (2.3)$$

The additive noise is a form of deterministic input, whereby the noise is used only by applying the external load and expressed by the formula $a^0 \times \frac{2rand}{rand\_\max + 1} - 1$, where a^0 is the noise intensity, the function $rand$ is the random number generator of a random value.

For simplicity sake, bars over dimensionless parameters in the system of equations (1.1) can be omitted.

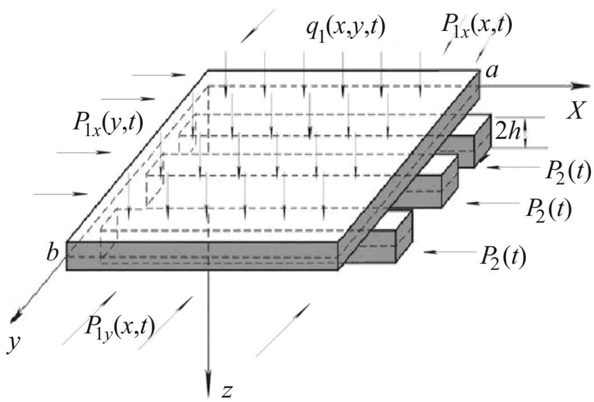

Fig. 1. Schematic view of a multi-layer structure consisting of a plate and three beams

The boundary conditions (hinge support along the contour and zero initial conditions) should be added to the initial equations:

$$w_m = 0;\ w''_m|_x = 0;\ \text{at } x = 0;1;$$

$$w_m = 0;\ w''_1|_y = 0;\ \text{at } y = 0;1; \quad (2.4)$$

m = 1, 2, 3, 4 is the index corresponding to the plate and beams,

$$w_1(x, y)|_{t=0} = 0,\ w_2(x)|_{t=0} = 0,\ w_3(x)|_{t=0} = 0,$$

$$w_4(x)|_{t=0} = 0,\ \dot{w}_m|_t = 0,\ m = 1, 2, 3, 4. \quad (2.5)$$

These conditions and conditions of nonpenetration of one system into the body of another one should be attached. Obtained system of nonlinear PDEs (1.1) is reduced to second order ODEs (1-3) by the Faedo-Galerkin higher approximation order method [24]. Functions w_1 and w_i $(i = 2,..,4)$, being solutions to (1.1), are approximated by the following functions that depend on time and coordinates:

$$w_1 = \sum_{k=1}^{N}\sum_{j=1}^{N} A_{kj}(t)\varphi_{kj}(x, y),,$$

$$w_i = \sum_{k=1}^{N} A^k{}_i(t)\varphi^k{}_i(x) \quad (i = 2,..,4),\ N = 6.$$

The functions $\kappa_{kj}(x, y)$ and $\varphi^k{}_i(x)$ are chosen in such a way that they are linearly independent, continuous together with their differential derivatives up to fourth order, and satisfy boundary and initial conditions. In this regard the equation is written as follows:

$$\varphi_{kj}(x, y) = \sin(k\pi x)\sin(j\pi y),\ \varphi^k{}_i(x) = \sin(k\pi x),$$
$$(i = 2,..,4).$$

The coefficients $A_{kj}(t)$ и $A_i^k(t)$ are the required functions of time. The system of second order with respect to functions of time is obtained by the Faedo-Galerkin method. The number of equations in the system depends on the number of beams. The system of equations of second order is reduced to first order system by the variable replacement method. Cauchy problems for the nonlinear system of first-order equations is solved by the Runge-Kutta method in time.

When solving problems associated with chaotic vibrations, it is necessary to solve the issue of error, so we need to use different numerical methods to validate the results. For this purpose, we reduced the obtained differential equations to the Cauchy problems by the finite difference method with approximation 0(h2) and 0(h4) and the Faedo-Galerkin method in higher approximations, studied convergence of the method for different numbers of terms of the order $N = 1,...,6$, and checked the accuracy by the Runge rule. Obtained ODEs were solved by the 4th, 6th, 8th order Runge-Kutta method in time, a comparative analysis of the results was carried out [1, 4]. Since the system of equations is nonlinear, it is not possible to solve it analytically.

Based on this algorithm, a complex solution was created that allows to study a multi-layer system of a plate and three beams described by a system of equations (1.1). The obtained results are analyzed using the methods of nonlinear dynamics and qualitative theory of differential equations: each layer is studied using such techniques as construction of signals, phase portraits, Poincaré maps, Fourier spectra, use of wavelet transforms as well as analysis of the Lyapunov exponents. Various wavelets are used: Morlet, Mexican hat, and Gaussian derivatives wavelets (from 1st to 8th order inclusive). 8th order Gaussian wavelet and the Morlet wavelet give similar results, however, the Morlet wavelet is preferable, because it allows to better localize the frequency in time, i. e. it is more informative [25-26].

3. Numerical Experiment

Let us study complex vibrations of a multi-layer system consisting of a plate and three parallel beams positioned at an asymmetric distance from the center of the plate ($y = 0.2$, $y = 0.4$ and $y = 0.7$), the gap between the plate and each of the beams is $h_k = 0.01$. Let us study the contact interaction depending on three types of load:

1) external distributed transverse load $q_1(x, y, t) = q_0 \sin(\omega_p t)$ is applied only to the upper plate, the beams are at standstill, i. e. $q_i(x,t) = 0$;

2) external distributed transverse load with the noise component $q_1(x, y, t) = q_0 \sin(\omega_p t) + a^0 \times \frac{2rand}{rand\_\max+1} - 1$, is applied only to the upper plate, the beams are at standstill $\left(q_i(x,t) = 0\right)$;

3) external distributed transverse load $q_1(x, y, t) = = q_0 \sin\left(\omega_p t\right)$ is applied to the upper plate, all three beams are subjected to the additive white noise, i. e. the load is defined as $q_i(x,t) = a_i^0 \times \frac{2rand}{rand\_\max+1} - 1$.

Let us discuss the first type of the load. The upper plate is subjected to an external distributed transverse load $q_1(x, y, t) = q_0 \sin\left(\omega_p t\right)$ with the intensity $q_0 < 0.065$ and excitation frequency $\omega_p = 5$, close to the natural-vibration frequency of the plate. The plate executes harmonic vibrations, and the beams are at standstill. When the external load amplitude is $q_0 = 0.065$, there is a contact between the plate and beams, which causes short time damped vibrations of the beams. When $q_0 = 0.07$, vibration behavior of the plate and beams changes. the system goes into chaos state, the period triples (see Fig. 2, b1, b2), while the second and third beams are at standstill. Figure 2 shows the graphs of the Fourier power spectra (b1, b2), 2D Morlet wavelet-spectra (c1, c2) of the plate and the first beam, respectively, as well as the graph of coupled vibrations of the plate and the first beam $(y = 0.2)$ (a), a continuous line is the plate, a dotted line is the beam.

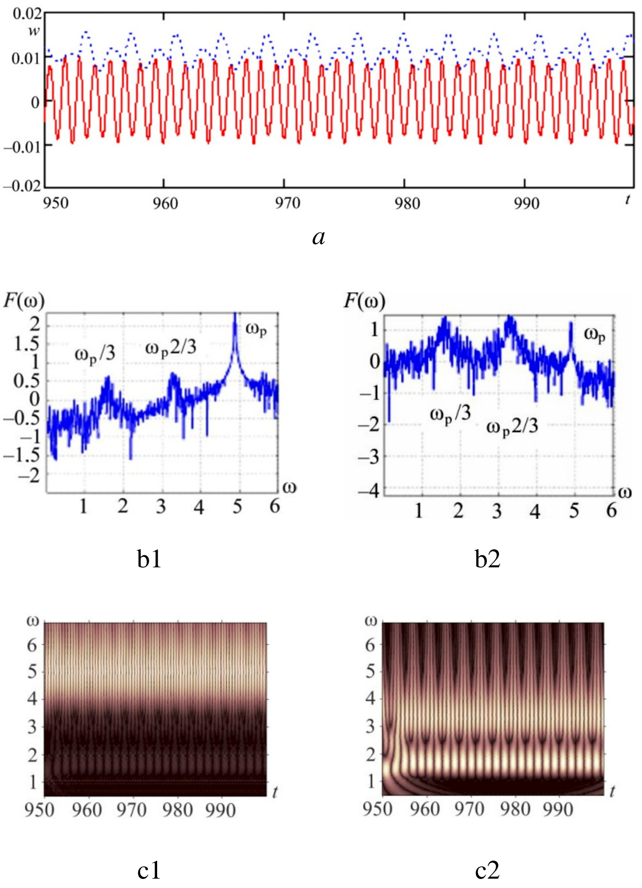

Fig. 2. Contact interaction of the plate and three asymmetric beams at the load amplitude $q_0 = 0.07$ with the gap $h_k = 0.01$

With an increase of the external load amplitude, the plate executes chaotic vibrations during period tripling with each of the end beams at a time. When the load amplitude is in the range $q_0 \in [0.074; 0.09]$, each element of the two-layer beam-plate system (plate and all three beams) executes vibrations. The graphs of 2D Morlet wavelet-spectra show that the vibrations are executed at different frequencies at different time intervals (see Fig. 3 b1-b4), unlike the Fourier power spectra (see Fig. 3, a1-a4), which show frequencies of vibrations for the entire period of time. This analysis allows us to draw a conclusion about the presence of intermittency zones and areas of enabling/disabling frequencies. With increase of the load, the plate once again executes vibrations with each of the beams at a time. And finally, starting with the load amplitude $q_0 = 0.31$, the plate interacts simultaneously with two end beams ($y = 0.2$, $y = 0.7$), the vibration behavior is chaotic.

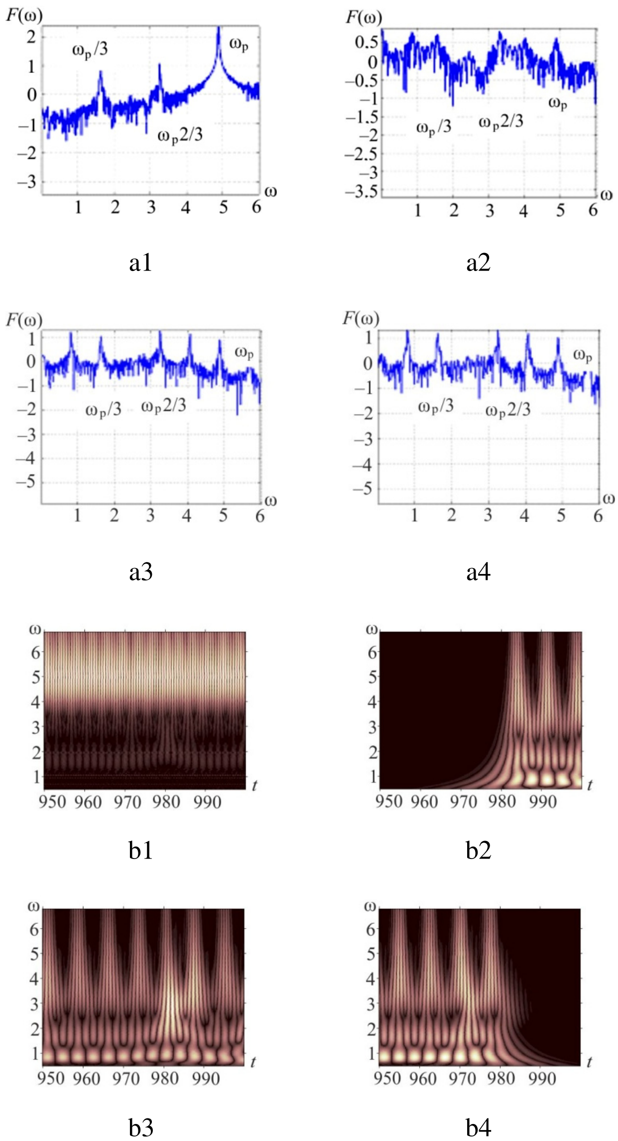

Fig. 3. Contact interaction of the plate and three asymmetric beams at the load amplitude $q_0 = 0.077$ with the gap $h_k = 0.01$

Let us discuss the second type of the load. The upper plate is subjected to an external distributed transverse load with the amplitude $q_0 = 0.07$. Let us study the influence of additive noise added to the external transverse load applied to the plate, on the complex vibrations of the multi-layer system. The noise intensity from the range of $a^0 \in [1\cdot 10^{-6};\ 6.6\cdot 10^{-3}]$ has absolutely no influence on the vibration behavior (see Fig. 4 a1, b1, c1, d1). The system is still deterministic, chaotic vibrations are captured during period tripling of the plate and the first beam $(y = 0.2)$. Figure 4 shows graphs of Fourier power spectra and 2D Morlet wavelet-spectra of the plate $w_1(0.5; 0.5, t)$ (a1-a4 and b1-b4), first beam $w_2(0.5; t)$ (c1-c4 and d1-d4), second beam $w_3(0.5; t)$ (e3-e4 and f3-f4) and third beam $w_4(0.5; t)$ (g3-g4 and h3-h4), respectively, for different noise intensities. When increase of the noise intensity $a^0 = 6.7\cdot 10^{-3}$, the vibration behavior of the entire system has changed. The vibrations of the plate became harmonic at the excitation frequency of the external load $\omega_p = 5$, and the beams are at standstill. Thus, there is no contact interaction with the presence of additive noise $a^0 = 6.7\cdot 10^{-3}$ in the external load (see Fig. 4, a2, b2). With the noise intensity of $a^0 \in [6.7\cdot 10^{-3}; 5\cdot 10^{-2}]$ and amplitude of the external transverse load of $q_0 = 0.07$, the vibrations of the system are harmonic, the phase portrait has the form of a ring. When the noise intensity is increased up to $a^0 = 6\cdot 10^{-2}$ at the previous values of other control parameters, the system changes and tripling of orbits is shown in the phase plate portrait. The beams execute vibrations due to the contact as a result of the contact interaction of layers (see Fig. 4, c3, d3, e3, f3, g3, h3). The Fourier power spectrum of the plate shows localization of frequencies around a frequency, which will occur with a further increase of the noise intensity (see Fig. 4, a3, b3).

When the noise intensity is $a^0 = 7\cdot 10^{-2}$, the system executes chaotic vibrations during period tripling. When the noise intensity is increased up to $a^0 = 1\cdot 10^{-1}$, the multi-layer system executes complex chaotic vibrations during period tripling. At different time intervals, the plate interacts with each of the beams at a time. The contact interaction of the plate with the second $(y = 0.4)$ and third $(y = 0.7)$ beams occurs only for a short period of time (see Fig. 4, a4, b4, c4, d4, e4, f4, g4, h4).

Now, let us discuss the third type of the load. The upper plate is subjected to an external distributed transverse load with the amplitude $q_0 = 0.07$ and excitation frequency $\omega_p = 5$. Let us study the contact interaction under the influence of additive noise applied to all three beams, i. e. the load on the beams is defined as $q_i(x,t) = a_i^0 * \dfrac{2rand}{rand\_\max + 1} - 1$.

When there is no external white noise $\left(a_i^0 = 0\right)$, there is a contact between the plate and the first beam $(y = 0.2)$, the

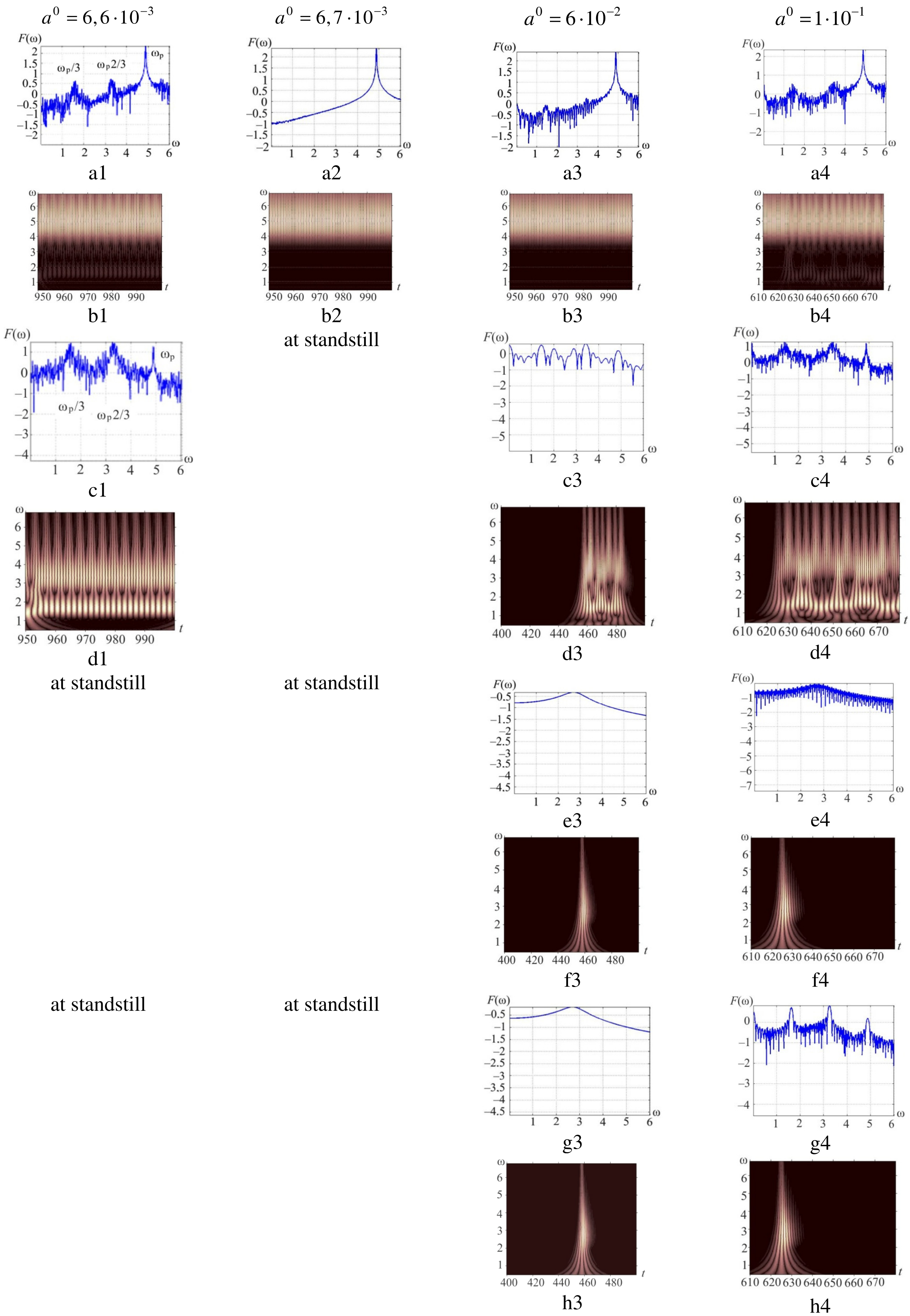

Fig. 4. Study of the influence of white noise on vibrations of a multi-layer system using the Fourier method and wavelet analysis

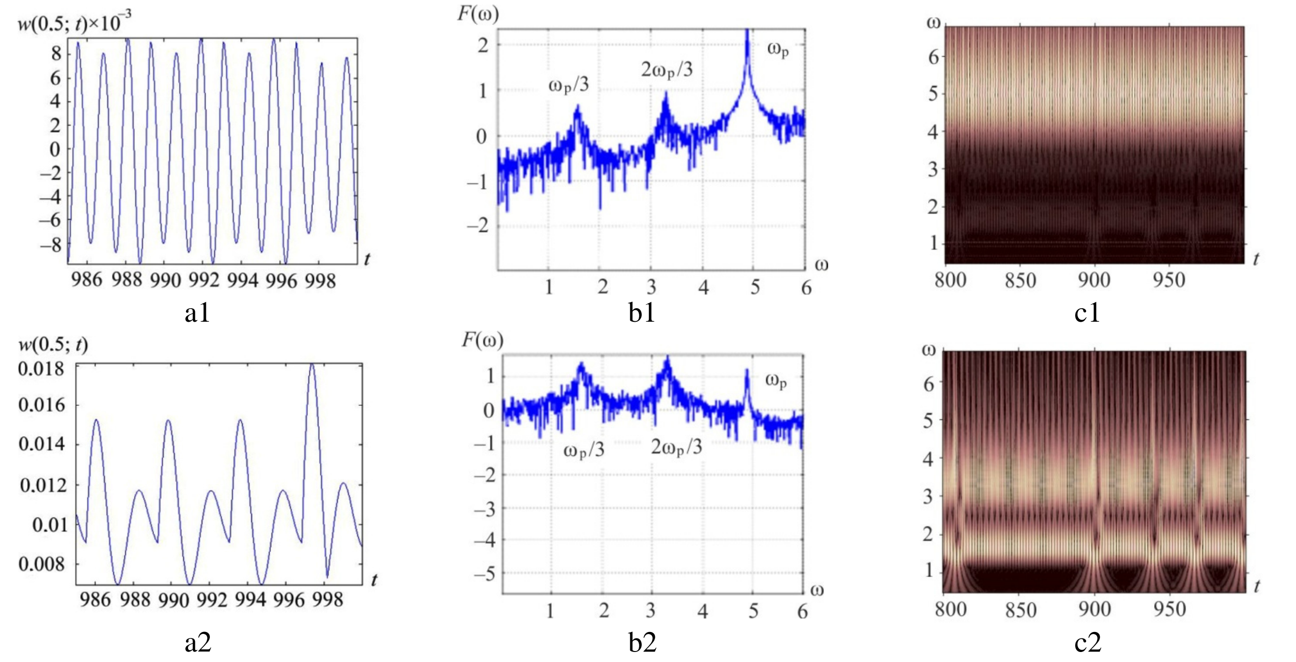

Fig. 5. Contact interaction of the plate $w_1(0.5;0.5,t)$ and the first beam $w_2(0.5;t)$ under the influence of white noise $a_i^0 = 0.001$

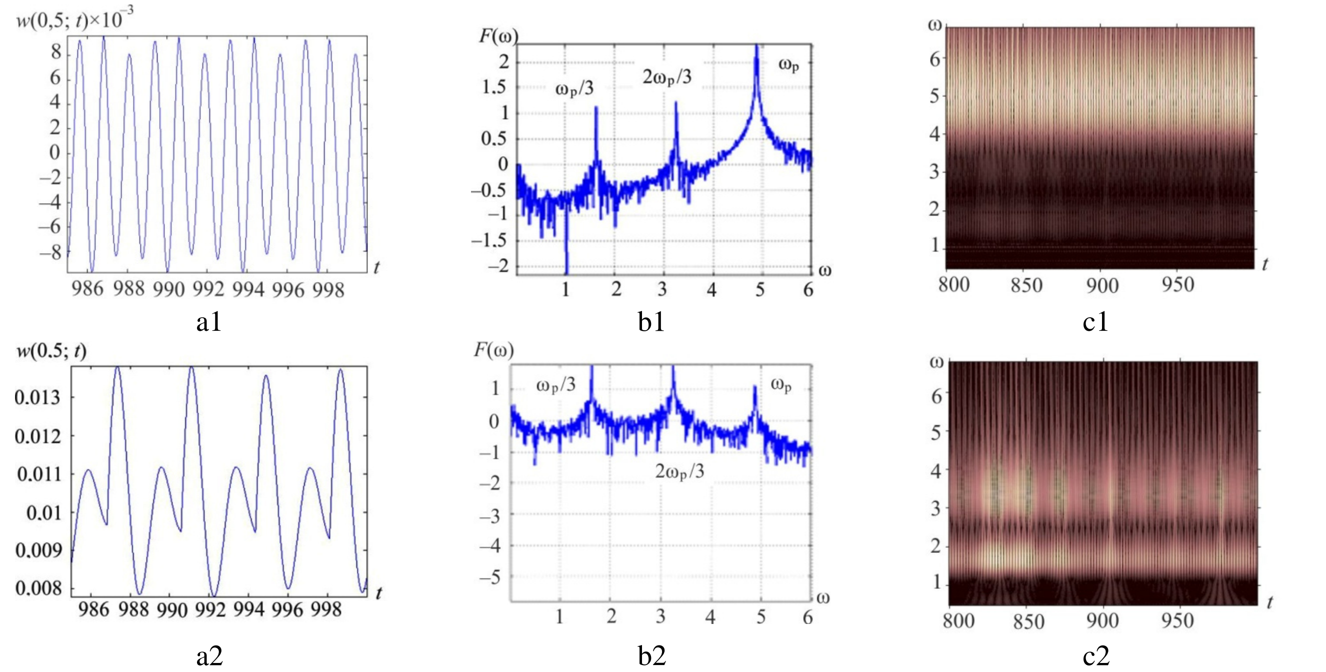

Fig. 6. Contact interaction of the plate $w_1(0.5; 0.5,t)$ and the third beam $w_4(0.5; t)$ under the influence of white noise $a_i^0 = 0.031$

plate and the beam execute vibrations during period tripling, while the second ($y = 0.4$) and third ($y = 0.7$) beams are at standstill. The system changes with increase of the additive noise intensity up to $a_i^0 = 0.001$. The second and third beams execute chaotic vibrations with a small amplitude compared to the gap between the plate and beams, phase portraits have the form of a solid spot, the Fourier power spectra and wavelet-spectra show a deep chaos. The contact interaction occurs between the plate and the first beam during period tripling (see Fig. 5, b1, b2), but there is intermittency of frequencies of these elements (see Fig. 5, c1, c2). Fig. 5 shows the graphs of the signal (a1, a2), the Fourier power spectrum (b1, b2) and 2D Morlet wavelet spectra (c1, c2) of the plate and the first beam, respectively.

When the noise intensity is $a_i^0 = [0.002; 0.029]$, the plate and the third beam come into contact interaction during period tripling, the first and second beams make chaotic oscillations, but there is no contact with the plate. When $a_i^0 = 0.002$, the Fourier power spectrum of the plate is cleared, the plate and the third beam are synchronized with small noise components. The vibrations are executed during period tripling, but a dominating frequency of the plate is $\omega_p = 5$, and dominating frequencies of the beam are $\omega_p / 3 = 1.6$ and $2\omega_p / 3 = 3.3$. When the white noise intensity is $a_i^0 = 0.03$, the plate executes harmonic vibrations due to the fact that there is no contact with any of the beams, although they are in a state of deep chaos. With increase of the noise intensity up to $a_i^0 = 0.031$, the plate once again touches the third beam with a tripling of the frequency period, i. e. these two elements of the beam-plate structure are synchronized (see Fig. 6).

4. Conclusion

This work presents a mathematical model of the contact interaction of a plate reinforced by ribs with gaps under the influence of white noise. As a part of the study, the authors have studied behavior of complex vibrations of a multi-layer system depending on three types of load under the influence of white noise. We can draw a conclusion that as soon as elements of the system come into contact, regardless of the presence of white noise, their vibrations become chaotic at linearly dependent frequencies $\omega_p = 5$, $\omega_p / 3 = 1.6$ and $2\omega_p / 3 = 3.3$.

The external additive white noise can affect the vibration behavior (presence or absence of the contact interaction of elements of a multi-layer system). The absence of contact interaction causes the system to execute harmonic vibrations.

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