In order to construct a theory that adequately describes the effects of cyclic loadings, it is initially necessary to analyze the experimental plastic loop of a hysteresis of stainless steel SS304; and three types of backstresses responsible for the displacement of the center of the surface of loading are specified on this steel. For each type of backstresses, we have formulated evolution equations on basis of the equations of the theory of plastic flow under combined hardening. We have allocated the material functions which close the theory, and formulated the basic experiment and method of the material functions identification.

Evaluating the work of different types of backstresses on the field of plastic deformations under cyclic loadings with various magnitudes of the deformation up to the experimental values of the number of cycles before failure, it has been obtained that the work of backstresses of the second type is a universal characteristic of the material. This result made it possible to formulate the kinetic equation of damage accumulation, based on which we have considered the processes of nonlinear damage accumulation. To determine the material functions, responsible for the destruction, we have formulated the basic experiment and identification method. The authors have given material functions for stainless steel SS304.

We have investigated the processes of elastic-plastic deformation of stainless steel SS304 with non-stationary hard cyclic loading under block changes of amplitude and mean deformation of the cycle. Also the processes of soft non-stationary and non-symmetric cyclic loading (ratcheting) under block changes of amplitude and mean stress cycle have been examined. The results of calculations are compared with the experimental results. The results of calculations are compared with the experimental results.

Computational research of nonlinear processes of damage accumulation and low cycle fatigue of stainless steel SS304 are conducted under symmetric hard cyclic loading both at the constant amplitude of strain and block change of the amplitude of strain. The calculation results show that the scope of deformation reduction leads to increase of the nonlinearity of damage accumulation, while the increase of the deformation scale results in the fact that the accumulation of damages tends to be linear. There is a significant deviation from the rule of linear summation of damages under a satisfactory conformity of calculation results with the experiments.

In order to construct a theory that adequately describes the effects of cyclic loadings, it is initially necessary to analyze the experimental plastic loop of a hysteresis of stainless steel SS304; and three types of backstresses responsible for the displacement of the center of the surface of loading are specified on this steel. For each type of backstresses, we have formulated evolution equations on basis of the equations of the theory of plastic flow under combined hardening. We have allocated the material functions which close the theory, and formulated the basic experiment and method of the material functions identification.

Evaluating the work of different types of backstresses on the field of plastic deformations under cyclic loadings with various magnitudes of the deformation up to the experimental values of the number of cycles before failure, it has been obtained that the work of backstresses of the second type is a universal characteristic of the material. This result made it possible to formulate the kinetic equation of damage accumulation, based on which we have considered the processes of nonlinear damage accumulation. To determine the material functions, responsible for the destruction, we have formulated the basic experiment and identification method. The authors have given material functions for stainless steel SS304.

We have investigated the processes of elastic-plastic deformation of stainless steel SS304 with non-stationary hard cyclic loading under block changes of amplitude and mean deformation of the cycle. Also the processes of soft non-stationary and non-symmetric cyclic loading (ratcheting) under block changes of amplitude and mean stress cycle have been examined. The results of calculations are compared with the experimental results. The results of calculations are compared with the experimental results.

Computational research of nonlinear processes of damage accumulation and low cycle fatigue of stainless steel SS304 are conducted under symmetric hard cyclic loading both at the constant amplitude of strain and block change of the amplitude of strain. The calculation results show that the scope of deformation reduction leads to increase of the nonlinearity of damage accumulation, while the increase of the deformation scale results in the fact that the accumulation of damages tends to be linear. There is a significant deviation from the rule of linear summation of damages under a satisfactory conformity of calculation results with the experiments.

The paper presents such new results as:

- specifying three types of backstresses responsible for kinematic hardening analyzing the experimental loops of plastic hysteresis;
- establishing the work universality of the second type backstresses under low-cycle and high-cycle fatigue on basis of experimental results analysis;
- constructing the theory of plastic flow under combined hardening and kinetic equations of damage accumulation on basis of the evolution equations for three types of backstresses;
- identifying the material parameters and verifying the proposed theory.
Introduction

Mathematical modelling of deformation under cyclic loadings is largely built based on variants of the plastic flow theory under combined hardening, and their review and analysis can be found in [1–13]. The main problem related to construction of these variants is the formulation of quite sufficient evolution equations for displacement of the centre of the loading surface (tensor of backstresses). To describe the displacement of the surface loading, the model of Novozhilov-Chaboche [14, 15], which entails that the total displacement is the sum of displacements, for which there is a certain evolution equation. Mainly, the evolution equations of Ishlinskii-Pruger, Armstrong-Frederick-Kadashevich [18, 19], as well as Ohno-Vang [20] are taken as such evolution equations. The verification of different variants of the flow theory under combined hardening is given in [2, 4–13].

In order to describe the process of damage accumulation, the following kinetic equations of damage accumulation are formulated [1–4, 21], where the work of backstresses (the displacement tensor) on the field of plastic deformations is taken as the energy spent on creating damages in the material. The responsibility for the process of damage accumulation comes from the hypothesis of Novozhilov-Rybakina [22] about the proportionality of the damage accumulation rate of the intensity of backstresses. The experimental justification of this statement can be found in [23], and the kinetic equation based on the work of backstresses on the field of plastic deformations (the work criterion of backstresses) was first considered in [24], when studying the low-cycle integrity of conical shells during thermal cycling. In this paper, the criterion for the work of backstresses was first tested under complex nonisothermal loading.

In this paper, based on the analysis of the experimental results under cyclic loadings, three types of backstresses responsible for the kinematic hardening are distinguished, and the universality of the value of the work of backstresses of the second type at low-cycle and high-cycle fatigue (up to to $10^6 \div 10^7$ cycles) is obtained. Based on these results it is possible to formulate the variant of the theory of plastic flow under combined hardening and kinetic equations for nonlinear processes of damage accumulation. The reference experiment and the identification procedure of material parameters are formulated. The version of the theory is verified under nonstationary and nonsymmetric cyclic loads of SS304 stainless steel for both rigid and soft uniaxial cyclic loads.

1. Main Principles and Equations of the Theory

The material is homogeneous and initially isotropic. Only polycrystalline structural steels and alloys are considered. In the process of elastoplastic deformation, only plastic deformation anisotropy can take place in the material. The process of cyclic deformation can take place under the conditions of soft, rigid or mixed loading modes, be stationary or nonstationary, symmetric or nonsymmetric.

The stretching tensor is represented in the form of a sum of strain rate tensors of elastic and plastic deformations,

$$
\dot{\varepsilon}_i^p = \dot{\varepsilon}_i^e + \dot{\varepsilon}_i^p.
$$

(1)

Elastic deformations follow the generalized Hooke's law

$$
\varepsilon_i^p = \frac{1}{E} [\sigma_{ij} - \nu (3\sigma_{ij} - \sigma_{kk})] \quad (\sigma_{kk} = \sigma_i/3),
$$

(2)

where $E, \nu$ is Young's modulus and Poisson ratio, correspondingly.

It is assumed that in the space of stress tensor components there is a loading surface separating the domains of the elastic and elastoplastic strains,

$$
f(\sigma_{kk}) = \frac{3}{2} (s_{ij} - a_{ij}) (s_{ij} - a_{ij}) - \left[ C_p (\varepsilon_{ps})^p \right]^2 = 0,
$$

(3)

$$
\left\{ s_{ij} = s_{ij} - a_{ij}, \quad \sigma_{kk} = \frac{3}{2} s_{ij} s_{ij}^* \right\}, \quad \varepsilon_{ps}^p = \left[ \frac{2}{3} \varepsilon_{ps} \varepsilon_{ij} \right]^{1/2}.
$$

Here $s_{ij}, s_{ij}^*, a_{ij}$ are the stress deviators, active stresses, backstresses (additional stresses, residual backstresses); $\varepsilon_{ps}^p$, is the length of the curve of plastic deformation (accumulated plastic deformation, Odqvist parameter). Tensor $a_{ij}$ characterizes the displacement of the loading surface (anisotropic hardening), and the scalar $C_p$ corresponds to the size (radius) of the loading surface and characterizes isotropic hardening. In case when of increases of $C_p (\varepsilon_{ps}^p)$, the material is cyclically hardened, in case of decreases, it is cyclically softened, and at the constant value, it is cyclically stable.

The displacement of the loading surface is described based on the model of Novozhilov-Chaboche [14, 15], entailing that a complete displacement is a sum of displacements, and for each of them has its own evolution equation,

$$
a_{ij} = \sum_{m=1}^M a_{ij}^{(m)}.
$$

(4)

To analyze the behavior of backstresses and the choice of evolution equations, we consider a stabilized plastic hysteresis loop in the following coordinates: stress $\sigma$, plastic strain $\varepsilon^p$. Fig.1. shows such a loop (cyclic diagram) for stainless steel SS304 [25, 26]. Then half-cycle from the proportionality limit (the beginning of the deviation from the vertical), a coordinate system is introduced: backstresses $a$, plastic deformation $\varepsilon^p$. The curve in the coordinates $a, \varepsilon^p$ characterizes the relieve and formation of backstresses.
during cyclic loading. The range of plastic deformation here is about 0.012, which makes it possible to come to the linear asymptote of the curve of backstresses’ formation.

After it, we calculate derivative \( da/d\varepsilon^p \) and build the curve in the coordinates \( da/d\varepsilon^p \) and \( \varepsilon \) (Fig. 2). Here the calculation of \( da/d\varepsilon^p \) derivative is made numerically based on averaged differences. Three domains, characterizing a different behavior of backstresses, can be allocated on the obtained curve. In the first domain, the derivative has a practically constant value, and here for the first type of backstresses, the evolution equation of Ishlinskii-Pruger [16, 17] is effective, in the second domain, the derivative varies linearly, and here the evolution equation of Armstrong-Frederick-Kadashevich [18, 19] is effective. Then, in the third domain, the derivative varies according to the linear law, which can be described with a series of evolution equations of Ohno-Vang [20].

Thus, the equation of Ishlinskii-Pruger [16, 17] is taken as the first evolution equation for backstresses of the first type.

\[
\alpha_1^{(1)} = \frac{2}{3} g^{(1)} \beta_{ij},
\]

The equation of Armstrong-Frederick-Kadashevich [18, 19] is taken as the second evolution equation for backstresses of the second type.

\[
\alpha_2^{(2)} = \frac{2}{3} g^{(2)} \beta_{ij} + g_{a} \alpha_1^{(2)} \beta_{ij}.
\]

The upcoming evolution equations for backstresses of the third type correspond to the simplest analog [3, 21] of equations of Ohno-Vang [20]

\[
\alpha_3^{(m)} = \frac{2}{3} g^{(m)} \beta_{ij}, \quad (m = 3, ..., M).
\]

The defining functions included into equations (5)-(7) are expressed via material parameters in the following way:

\[
g^{(1)} = E_a, \quad g^{(2)} = \beta \cdot \sigma_a, \quad g_a^{(2)} = -\beta
\]

\[
g^{(m)} = \begin{cases} \beta^{(m)} \sigma^{(m)} \\ 0, \text{ or if } a^{(m)} \geq \sigma^{(m)} \cap \alpha_3^{(m)} s_j > 0 \end{cases}
\]

\[
a_3^{(m)} = \frac{3}{2} \alpha_3^{(m)} \alpha_3^{(m)} \quad m = 3, ..., M
\]

Here \( E_a, \sigma_a, \beta, \sigma^{(m)}, \beta^{(m)} \) are the material parameters.

The introduction of additional equations (7) to the previously proposed and sufficiently approved equation with a three-term structure [1-3], equivalent to equations (4)-(6), makes it possible to describe finer effects of cyclic loading that arise during nonstationary and asymmetric cyclic loadings. Such effects include the effect of a small cycle in a large one[5], consisting in the fact that the loop of the first small cycle practically returns to the same initial point, from which a small cycle began. The final equation for displacement of the loading surface considering (4)-(9) will have the following form

\[
\alpha_1 = \frac{2}{3} g^{(1)} \beta_{ij} + \left( -\frac{2}{3} g^{(1)} \beta_{ij} + g_{a} \alpha_1 \right) \beta_{ij}.
\]

\[
g = \sum_{m=1}^{M} g^{(m)} = E_a + \beta \sigma_a + \sum_{m=3}^{M} g^{(m)}
\]

\[
g_e = \beta E_a, \quad g_{a} = -\beta, \quad a_1 = \alpha_1^{(1)} + \alpha_1^{(2)}.
\]

For nonsymmetric cyclic, both proportional soft and disproportionate soft and mixed loading modes, one-sided accumulation of deformation takes place (ratcheting of the plastic hysteresis loop), which intensity increases with an increasing asymmetry of the loading process. Work [27] shows that the ratcheting effect is caused by the symmetry principle of materials’ cyclic properties.

The description of the ratcheting phenomenon in the framework of the considered version of the theory of plastic flow is that the parameter \( E_a \), which is included into the first evolution equation for backstresses of the first type, is
assumed to depend on the accumulated plastic strain as follows:

$$E_u = E_{uo} / \left[ 1 + K_E \left( b_{uo}^p \right)^{n_u+1} \right].$$  \hspace{1cm} (11)

Here $K_E$, $n_E$ are the material parameters.

Plastic strains are determined based on the theory of flow, associated with the surface (3) in the following way:

$$\dot{\varepsilon}^p_{ij} = \frac{\partial f}{\partial \sigma_{ij}} = \frac{3}{2} \sigma_{ij} \gamma^p_{ij}.$$  \hspace{1cm} (12)

For the rate of the accumulated plastic strain, for soft and rigid loadings respectively, the following equations can be obtained [1-3]:

$$\dot{\varepsilon}^p_{ij} = \frac{3}{2 E} \gamma^p_{ij},$$  \hspace{1cm} (13)

$$\dot{\varepsilon}^p_{ij} = \frac{3 G}{E} \gamma^p_{ij},$$  \hspace{1cm} (14)

$$E_s = q_e + g_s \gamma^p_{ij} + g_u a^*.$$

Here $G$ is the shear modulus. For mixed modes of loading, the equations for $\varepsilon^p_{ij}$ are given in [1-3].

The conditions of elastic and elastoplastic state have the following form

$$\sigma^e < C_p \cup b^p_{uo} \leq 0 \text{elasticity},$$

$$\sigma^e > C_p \cap b^p_{uo} > 0 \text{elastoplasticity.}$$  \hspace{1cm} (15)

Here, the rate of the accumulated plastic deformation is set by expressions (13) or (14) or by any other expression that relates the rate of the accumulated plastic deformation and the rates of stresses and strains (mixed loading modes).

The results of the experimental studies of nonlinear damage accumulation processes are given in [4], where the relative volume fraction of defects determined on the basis of ultrasonic and metallographic studies is taken as a measure of damage. In order to describe the nonlinear processes of damage accumulation, the following kinetic equation of damage accumulation is introduced, which is based on the energy principle, where the energy equal to the work of backstresses of the second type on the field of plastic deformations is taken as the energy spent to create damages in the material. The kinetic equation which describes nonlinear process of damage accumulation has the following form:

$$\phi = \alpha \omega \frac{\gamma^p_{ij}}{a_{ij}^{(2)}} \frac{b^p_{uo}}{W_a}.$$  \hspace{1cm} (16)

$$\alpha = \left( \frac{\sigma_a/I_{a}^{(2)}}{n_u} \right).$$  \hspace{1cm} (17)

Here $\omega$ is the measure of damage; $W_a$ is the fracture energy; $\alpha$ and $n_u$ is the function and parameter of the nonlinearity of the damage accumulation process (at $n_u = 0$ the damage accumulation is linear); $\gamma^p_{ij}$ is the intensity of backstresses of the second type.

During nonsymmetric soft cyclic loading, the ratcheting of the plastic hysteresis loop takes place. In this case, the work of backstresses of the first type will differ from the zero value, in contrast to the symmetric cyclic loading, when this work is zero. As follows from the experimental results of [28], the nonsymmetry of the soft cyclic loading significantly affects the low-cycle fatigue. In addition, the damage caused by the one-sided increase of strain is determined by the ratio of one-sided deformation to the ultimate deformation of the material, i.e. the so-called quasi-static damage [28]. Therefore, the kinetic equation for damage caused by the work of backstresses of the first type will have the following form:

$$\dot{\phi}_1 = \frac{dW_1}{W_s},$$  \hspace{1cm} (18)

where $W_s$ is the fracture energy by means of ratcheting. Then the complete damage will be equal to the sum of damages of the first and second types

$$\phi = \phi_1 + \phi_2.$$  \hspace{1cm} (19)

The criterion of fracture (appearance of macrocracks with a length of about 1 mm) is the achievement by the damage of the limiting value, usually taken equal to one.

The equation is similar to (16) can be found in [29]. But in contrast to work [29], as well as [30], in which the fracture energy $W_a$ depends on the level of microstresses achieved ($\rho_{max}$), here the nonlinearity index of the process ($\rho_{max}$) is determined according to (17) by the level of backstresses of the second type. Kinetic equations in (16) and (17) properly describe the processes of nonlinear damage accumulation, which is proved in the work [31].

The responsibility for the damage accumulation of the work of backstresses of the second type is demonstrated with the results, given in Fig.3, which show the changes of the work of backstresses of the second (curve 2) and third (curve 3) types, corresponding to the experimental [32] values of the number of cycles before fracture for stainless steel 304.

The results, given in Fig. 3, demonstrate that the fracture energy, equal to the work of backstresses of the second type (backstresses of Armstrong-Frederick-Kadashevich), is a constant value at the considered range of the number of cycles until fracture from $10^2$ to $10^5$, i.e. the fracture energy for backstresses of the second type is a universal characteristic of material’s fracture. Perhaps, this characteristic will be universal up to high-cycle fatigue, i.e.
to $10^6$-$10^7$ cycles. The comparison of the calculated and experimental results until $10^6$ cycles is given in [2, 27].

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{The change of works of backstresses of the second and third types}
\end{figure}

Thus, this variant of the theory of plastic flow under combined hardening close the following material functions to be experimentally determined:

- elastic parameters, $E$, $\nu$;
- hardening modules, $E_m$, $\sigma_a$, $\beta$;
- ratcheting modules, $K_E$, $n_E$;
- coordinates of the plastic flow in the stress space:
  \[ \sigma_m^{(m)}(m=3,...M) \]

Then, the derivative $\frac{d\hat{a}}{d\hat{e}_p}$ is calculated, and the curve in the coordinates $d\hat{a}d\hat{e}_p$ is built (Fig. 2). Thus, this variant of the theory of plastic flow under combined hardening is extended the following material functions:

- $E$, $\nu$ are elastic parameters;
- $E_m$, $\sigma_a$ are the modules of anisotropic hardening;
- $K_E$, $n_E$ are the ratcheting modules;
- $\sigma_m^{(m)}(m=3,...M)$ are the modules of anisotropic hardening, corresponding to the analog of Ohno-Vang model;
- $C_p(\varepsilon_p^0)$ is the function of isotropic hardening;
- $W_a$ is the fracture energy;
- $n_a$ is the parameter of nonlinearity of the damage accumulation process;
- $W_f$ is the fracture energy during ratcheting.

### 2. Computational and Experimental Method Aimed at Determining Material Functions

Material functions are determined based on the testing results in the conditions of the elastoplastic uniaxial stressed state. The basic experiment includes the following set of data:

- elastic parameters, which are determined by the traditional methods;
- the diagram of plastic strain under tensile test up to strain value 0.05-0.1;
- cyclic diagrams under the symmetric tension and compression at a constant range of strain 0.015-0.02;
- cyclic diagrams under nonsymmetric tension and compression at a constant range of strain 0.005-0.01 and average deformation of the cycle 0.05-0.1;
- data on the low-cycle fatigue under single-block rigid symmetric cyclic loading;
- data on the low-cycle fatigue during double-block rigid symmetric cyclic loading;
- data on the low-cycle fatigue during single-block soft nonsymmetric cyclic loading.

To determine the moduli of anisotropic hardening, the stabilized plastic hysteresis loop (cyclic diagram) is reconstructed in the coordinates: stress $\sigma$, plastic deformation $\varepsilon_p$ (Fig. 1). Then, on the cyclic diagram, a curve is identified that is responsible for the displacement of the loading surface in coordinates: displacement (backstress) $a$, plastic deformation $\varepsilon_p$ (Fig. 1). The origin of the coordinate system is determined by the proportionality limit (the origin of the deviation from the vertical).

Then the derivative $d\hat{a}d\hat{e}_p$ is calculated, and the curve in the coordinates $d\hat{a}d\hat{e}_p$ is built (Fig. 2). Three domains, characterizing different behaviors of backstresses, can be allocated on the obtained curve, in Fig. 2. In the first domain, the derivative has a practically constant value, and here for the backstress of the first type, the Ishlinskii-Pruger evolution equation takes place and the derivative value is equal to the value of parameter $a_m$. Then the curve in Fig. 2 is reconstructed in the coordinates:

\[ \frac{d\hat{a}}{d\hat{e}_p} = \frac{d\hat{a}}{d\hat{e}_p - E_m}, \quad \hat{a} = (a - E_m \varepsilon_p), \]

so the effect of the backstress of the first type is excluded. The obtained linear dependence at the second domain makes it possible (Fig. 4) to find the parameters of $\sigma_a$ and $\beta$ based on the formula

\[ \beta = \frac{d\hat{e}_M}{\hat{a} - \hat{a}_M} \quad (20) \]

\[ \sigma_a = \frac{\hat{a}_M - \hat{a}_M}{2 \exp(-\beta \hat{e}_M)} \quad (21) \]

Here $\varepsilon_p^0$ is the plastic deformation of the completion of the third domain (completion of changes of backstresses of the third type, corresponding to the evolution equations of Ohno-Vang).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{The changes of works of backstresses of the second and third types}
\end{figure}
After it, the curve which is responsible for the displacement of the loading surface (Fig. 1) is reconstructed in the coordinates \( \bar{a} = a - E_m \varepsilon_p^{(0)} - 2 \sigma_a^{(0)} \left[ 1 - \exp \left( - \beta \varepsilon_p^{(0)} \right) \right] \).

The obtained curve (Fig. 5) is responsible for the backstresses of only the third type. The length of the third interval is quite small \( \varepsilon_p^{(0)} \approx 0.001-0.003 \), i.e. the backstresses of the third type (Ohno-Vang) occur almost within the tolerance of the residual strain \( \varepsilon_p^{(0)} = 0.002 \), when determining the conventional yield limit. After it, the domain \([0; \bar{a}_{\infty}]\) is divided into \((M - 2)\) parts and the parameters of the anisotropic hardening, corresponding to Ohno-Vang models, are calculated based on the following formula:

\[
\beta^{(m)} = 2 \alpha \varepsilon_p^{(0)} (m = 3, ..., M) \\
\sigma_a^{(M)} = \bar{a}_M - \bar{a}_{M-1} \left[ \frac{1}{\beta^{(M)}} \right] \\
\sigma_a^{(m)} = \frac{1}{\beta^{(m)}} \left[ \bar{a}_m - \bar{a}_{m-1} - \bar{a}_{m+1} - \bar{a}_m \right] \\
(m = M - 1, ..., 3; \bar{a}_2 = 0; \varepsilon_p^{(0)} = 0).
\]

Fig. 5. The behavior of backstresses of the third type

The modules of ratcheting are determined based on the results of uniaxial testing under rigid nonsymmetric cyclic loading. The gained experimental dependence between the mean stress of the cycle \( \sigma_m \) and the number of cycles \( N \) is built in the logarithmic coordinates:

\[
y = \lg \left( E_m \varepsilon_p^{(0)}/\sigma_m - 1 \right) \\
x = \lg \left( \varepsilon_p^{(0)} + 4 \varepsilon_p^{(0)} N \right),
\]

where \( \varepsilon_p^{(0)} \) is the mean plastic deformation of the cycle; \( \varepsilon_p^{(0)} \) is the amplitude of plastic deformation at the cycle. This experimental dependence is linear [27], which makes it possible to determine the ratcheting moduli according to the formula, based on the relationship of line \( \alpha \) and \( \gamma_0 \) intersecting point of a line with the ordinate axis

\[
\lg K_n = \gamma_0, n_k + 1 = \tan \alpha.
\]

Having obtained the parameters of anisotropic hardening and ratcheting moduli, we can calculate the displacement (backstresses) value for both uniaxial tension and cyclic loading. Then, by subtracting the obtained backstresses from the stresses at the corresponding values of the accumulated plastic deformation, one can obtain the isotropic hardening function \( C_p \left( \varepsilon_p^{(0)} \right) \).

The fracture energy \( W_u \) is determined by calculating the work of backstresses of the second type under cyclic loadings to experimental values of the number of cycles until fracture (Fig. 3).

The parameter of the nonlinearity of the damage accumulation process is determined by the selection until the compliance of the calculated and experimental results at double-block cyclic loading.

Fracture energy during ratcheting \( W_u \) is also determined by the selection during the comparison of the calculated and experimental results at the nonsymmetric soft cyclic loading.

The authors have given the material functions for stainless steel SS304, obtained based on the experimental results [25, 26, 32].

\[
E = 2 \cdot 10^5 \text{MPa, } \nu = 0.3 \\
E_m = 5000 \text{MPa, } \beta = 344, \sigma_m = 120 \text{MPa} \\
\beta^{(1)} = 50000, \sigma_u^{(1)} = 8.3 \text{MPa,} \\
\beta^{(2)} = 20000, \sigma_u^{(2)} = 16.7 \text{MPa,} \\
\beta^{(3)} = 10000, \sigma_u^{(3)} = 33.3 \text{MPa,} \\
\beta^{(4)} = 4000, \sigma_u^{(4)} = 26.0 \text{MPa,} \\
\beta^{(5)} = 2222, \sigma_u^{(5)} = 16.9 \text{MPa,} \\
\beta^{(6)} = 1333, \sigma_u^{(6)} = 16.9 \text{MPa,} \\
\beta^{(7)} = 870, \sigma_u^{(7)} = 1.23 \text{MPa,} \\
\beta^{(8)} = 666, \sigma_u^{(8)} = 2.2 \text{MPa,} \\
K_e = 0.45, n_k = -0.65, C_p = 100 \text{MPa,} \\
W_u = 1110 \text{J/cm}^3, n_u = 1.5.
\]

3. Plasticity Under Nonstationary and Nonsymmetric Modes of Cyclic Loading

The calculation results, made according to the proposed variant of the plasticity theory, processes of nonstationary cyclic loading of stainless steel SS304 under uniaxial
stretching and compression are given in Fig. 6-9. In these figures, the solid curves correspond to the calculation, and the light circles correspond to the experiment [25, 26]. First, we consider rigid cyclic loadings under the conditions of a block change in the strain amplitude in the case of symmetric loading (Fig. 6), as well as the block change in the average strain of the cycle in the case of nonsymmetric loading (Fig. 7). Fig. 6b and Fig. 7b show the calculated cyclic diagrams that almost coincide with the experimental ones [25, 26]. The presentation of the experimental cyclic diagrams in Fig. 6b and 7b will give the compliance of the calculated and experimental curves. A good agreement between the calculated and experimental results is illustrated in Fig. 6a and 7a, where the behavior of the stress amplitudes during the cyclic loading is given.

Further we consider soft cyclic loadings under block change of stress amplitude at the constant of the mean stress cycle (Fig. 8), as well as a block change of mean stress cycle at constant stress amplitude (Fig. 9). The considered modes are nonsymmetric, while in the loading process, the nonsymmetry both increases and decreases. Fig. 6, b and Fig. 7, b show the calculated cyclic diagrams that illustrate the ratcheting process of the loop of plastic hysteresis. The computational cyclic diagrams correspond to the experimental ones [25, 26].
The increase in the difference (up to 20 %) in the numerical and experimental results under soft loading (Fig. 8, a), in comparison with the rigid one, is explained by the greater sensitivity of the deformation behavior from the error in setting the stresses in the experiment under soft loading, i.e. a big range.

4. Low-cycle Integrity Under Nonstationary Modes of Cyclic Loadings

The computational studies of nonlinear processes of damage accumulation and low cycle fatigue of stainless steel SS304 are conducted under symmetric hard cyclic loading both at the constant amplitude of strain and block change of the amplitude of strain. The nonlinear process of damage accumulation under different strain ranges of single-block cyclic loading is given in Fig. 10, a. The calculation results show that the range of deformation reduction leads to an increase of the nonlinearity of damage accumulation, while the increase of the deformation scale results in the fact that the accumulation of damages tends to be linear. In Fig. 10, b the solid line shows the calculated curve of low-cycle fatigue, and the light circles indicate the experimental data of [32] for single-block cyclic loading.

Fig. 11 presents the research results of the nonlinear damage cumulation at the double-block change of the strain range and different levels of preliminary cycling in the first, two first and four first blocks, respectively, at a two-block, three-block and five-block change of the strain range are shown in Fig. 11, and vice versa (0.015 → 0.005) (Fig. 11, b). The calculation results demonstrate, that, as soon as the strain range increases, the cumulative durability grows, and as soon as it decreases, the latter reduces. Violations of the rule of the linear summation of damages in case of a multi-block change in the strain range and different levels of preliminary cycling in the first, two first and four first blocks, respectively, at a two-block, three-block and five-block change of the strain range are shown in Fig. 12. The calculation results in these figures are shown with solid lines, and the results of experiments [32] are shown with dark circles in case of the strain rate increase (0.005 → 0.015; 0.005 → 0.01 → 0.015; 0.005 → 0.008 → 0.01 → 0.012 → 0.015), and the light circles indicate the decrease of the strain range (0.015 → 0.005; 0.015 → 0.01 → 0.005; 0.015 → 0.012 → 0.01 → 0.008 → 0.005).

4. Low-cycle Integrity Under Nonstationary Modes of Cyclic Loadings

The computational studies of nonlinear processes of damage accumulation and low cycle fatigue of stainless steel SS304 are conducted under symmetric hard cyclic loading both at the constant amplitude of strain and block change of the amplitude of strain. The nonlinear process of damage accumulation under different strain ranges of single-block cyclic loading is given in Fig. 10, a. The calculation results show that the range of deformation reduction leads to an increase of the nonlinearity of damage accumulation, while the increase of the deformation scale results in the fact that the accumulation of damages tends to be linear. In Fig. 10, b the solid line shows the calculated curve of low-cycle fatigue, and the light circles indicate the experimental data of [32] for single-block cyclic loading.

There is a significant deviation from the rule of the linear summation of damages at a satisfactory conformity of the calculation results with the experiments [32]. Also, it is worth mentioning that as soon as the strain range increases, the accumulative durability may increase by almost 40 %, and as soon as the strain range decreases, the cumulative durability may decrease by almost 40 %, compared to the
accumulated durability, which is predicted according to the rule of the linear damage accumulation.

Fig. 11. The dependence of the damage accumulation on the number of cycles at a double-block cyclic loading: 
(a) is the transition from the smaller amplitude to the bigger one; 
(b) is the transition from the bigger amplitude to the smaller one.

Fig. 12. Damage accumulation at a double-block (a), tripple-block (b) and penta-block (c) change of strain amplitude

Conclusions

A proper description of the processes of elastoplastic deformation and fracture of structural steels and alloys under nonstationary and nonsymmetric cyclic loadings is an obvious advantage of the considered version of the theory of plasticity. While the basic experiment and the method of the material functions identification, which close the theory, is quite simple and easily implemented. The comparison of the results of calculations and experiments proves their reliable compliance. It is also important to note the importance of the results obtained, when processing and analyzing the calculated and experimental data, not only for steel SS304, but also for steels 16, 1026, 1070, 12X18H9, 12X18H10T, 45 [2, 21, 27, 31]:

- the displacement of the loading surface is determined with displacements (backstresses) of three types, and evolution equations are formulated for each of them;
- the work of backstresses of the second type is a universal characteristic of material’s fracture under cyclic loadings;
- when building up the loading surface, there is no clear boundary between the elastic and elastoplastic states, but there is a layer, where the relieve and formation of backstresses of the third type takes place (a small loop of hysteresis comes back into its starting point). The thickness of this strain layer corresponds to the deformation, where the change of backstresses of the third type takes place. The presence of this layer is shown in work [33];
- the loading surface can be shrinked into a point, and the elastic state can be related to the backstresses of the third type. This will help to formulate the variant of the theory of elastoplastic processes under proportional and unproportional (complex) modes of cyclic loads;
- the number of evolution equations for the third type of backstresses can be decreased up to two or three, which will lead to an error in stresses of not more than 5 %. Apart from it, the backstresses of the third type do not influence the damage accumulation process.

The work was made with the support of the Joint Institute for High Temperatures of the Russian Academy of Sciences.

References


22. Novozhilov V.V., Rybakina O.G. O perspektivah postroenija kriterijia prochnosti pri slozhnom nagruzhenii [About the prospects of building the strength criterion under complex loading]. *Prochnost' pri malom chisle nagruzhenij* [Strength at low-loading], Moscow, Nauka, 1969, pp. 71-80.


30. Bondar V.S., Danilin V.V., Semenov P.V. Nelinejnye processy nakoplenija povrezhdennij pri nestacionarnyh ciklicheskih nagruzhenijah [Nonlinear processes of damage accumulation in unsteady cyclic loads]. *Problemy prochnosti i plasticnosti – Problems of strength and plasticity*, 2012, Iss. 75, Ч.2, pp. 96-104.
