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**MODELLING THE GENERATION OF NEW MATERIAL SURFACES IN A COMPOSITE WITH AN ADHESION LAYER UNDER COHESIVE DESTRUCTION**

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**ABSTRACT**

This paper considers the subcritical elastic-plastic deformation of a three-layer composite and the layer separation accompanied by the fracture of an adhesive layer. The problem is reduced to the system of two variational equilibrium conditions with respect to the bonded layers' velocity fields by means of averaging a stress component in the adhesive layer across its thickness. When we solve an elastic-plastic problem in terms of subcritical deformation, the \( \delta \)-area is distinguished where the fracture criterion is reached. The distribution of load (node forces) that affects a body from the \( \delta \)-area is determined by resolving a pre-critical deformation problem with the known motion law of the \( \delta \)-area boundary. As the next step, we consider changes in the body's stress-strain state (SSS) during the fracture of the \( \delta \)-area. We solve the elastic-plastic problem under simple unloading of the body's \( \delta \)-surface and remaining an external load that corresponds to the beginning of the fracture process. During the \( \delta \)-unloading, the formation of new plastic areas, partial unloading and reaching the fracture criterion are possible. As a result, the body's SSS at the moment when local unloading begins differs from its state when the \( \delta \)-unloading ends. This constitutes a principal distinction from the common procedure of "killing the elements" when the element rigidity (after reaching the fracture criterion) is supposed to be close to null. Herewith the body state outside a removed element is considered to be unchanged; and the generation of unloading and additional loading zones (after the element is excluded) is not considered. In case of linear elasticity, the solution of the problem with a removed area under fixed external load coincides with the \( \delta \)-unloading solution by virtue of the solution uniqueness and the superposition principle. However, the solution of the elastic-plastic problem for the body with the removed area under simple loading will not coincide with the \( \delta \)-unloading solution. The paper presents the solutions of composite delamination problems that illustrate the simple \( \delta \)-unloading method both in linear elastic and in elastic-plastic formulations.

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Introduction

Modelling the generation of new material surfaces provides for a description of two stages of the body's loading: subcritical and postcritical. The subcritical stage can be described within the framework of known equations of mechanics of a deformed solid body, both without taking into account nonlinear effects and in a nonlinear (elastic-plastic) formulation. And the main problem is to formulate criteria of a transition to the postcritical stage. A general formulation of the postcritical behavior problem under elastic-plastic deformation is far from being complete. This is due, firstly, to the choice of conditions for the termination of interaction between surfaces and, secondly, to the necessity of formulation of boundary conditions on newly formed surfaces. If a fracture is within the material surface of a zero measure, then the classical model of a mathematical cut takes place [1-3]. If the interaction terminates as a result of destruction of a layer of finite thickness between the surfaces, then the physical cut concept is applied [4, 5]. For the time being, subcritical deformation models are the most studied and reasonable ones. Since in this case no generation of new material surfaces takes place, and any approach being in line with a limit load experiment can be considered as a predestruction model. Corresponding models apply to evaluate crack strength in both structures of uniform bodies, bimetal [6], and composite materials where adhesive layers are considered as a probable fracture area [7, 8]. Let us distinguish the models where a medium structure parameter is present [5, 8-12]. When performing this or that fracture criterion, the phase of formation of new material surfaces begins. The principal moments of this stage are the discreteness of increment of discontinuity surfaces [13-15] and the localization of a destruction process. In the modeling of destruction within the material surface of the zero measure, cohesive elements [16-19] of a finite element package [20] or system contact elements [21] are used. The widely used bilinear law of the distribution of interaction within the cohesive zone from the opening of a crack with a falling section was considered in the works [4-5]. The main disadvantage of this approach is that cohesive elements must be located in the fracture trajectory that, in general, depends on new material surfaces. In addition, various cohesive interaction laws and material characteristics of cohesive elements considerably affect the distribution of stress-strain state [25] and require experimental confirmation.

A difficult description of destruction of a material volume when using the tensile diagram's falling section [26, 28] is connected with the construction of defining relations of the unstable Drucker deformation and their confirmation in experiments. The main approach to finite element modeling of the destruction process is the "kill element" procedure described in detail in [21]. In this case, when the criterion characteristic of destruction is reached in a final element, an isolated material volume is excluded from consideration by multiplying the element's local stiffness matrix by a number close to zero. Let us note that this procedure is correct for elastic deformation when the load and unloading are determined by the same modules in defining relations. In case of elastic-plastic deformation of a body with an element excluded in this way, it is necessary to take into account a possible unloading.

This work suggests considering the destruction of δ-element as local unloading from stresses that affect the body from δ-element under unchanged external loading. The discretization of a problem by the finite element method implies the description of interaction among the body's particles by means of nodal forces. When the material volume, in the general case interacting with several finite elements, is destroyed, we assume that the nodal interaction forces will be unloaded up to zero within a simple process, with unchanged external load reached at the time of destruction. Thus, the unstable Drucker deformation is excluded from consideration. It is important to note that the determination of nodal forces of interaction with the excluded material volume, at the deformed body's internal points, is a separate problem. The article suggests a procedure for the body's repeated load without a destructible volume, but with a displacement field found along its boundary, in order to find the corresponding nodal forces.

1. Problem Setting

Fig. 1 shows the body that consists of three parts with different material properties in a general case where area No. 3 is associated with an adhesive base of thickness δ0 which size is small, as compared to the thickness of bodies 1 and 2. We consider the loading process to be quasistatic and isothermic. An equilibrium flow condition of the process [31] with small deformations and rotations of material fibers is taken as follows:

\[
\int_\delta \sigma \cdot \delta w dt = \int_L p \cdot \delta v dl,
\]

where \( v \) is the velocity field; \( w = 0.5(\nabla v \cdot \nabla v) \) is the strain rate tensor; \( \sigma = e \cdot \frac{\partial}{\partial x} \); \( p = \frac{\partial P}{\partial t} \) is the external load rate on the circuit \( L \); \( e = \frac{\partial \sigma}{\partial t} \) is the rate of the stress tensor; \( t \) is the time-like parameter. \( S = S_1 + S_2 + S_3 \) is the inner region of the compound body.

In area 3, we define average stress rates along thickness \( \delta_0 \) as follows:

\[
\hat{\sigma}_{21}(x_i) = \frac{1}{\delta_0} \int_{S_{1/2}}^{S_{3/2}} \sigma_{21}(x_1, x_2) dx_2,
\]

\[
\hat{\sigma}_{12}(x_i) = \frac{1}{\delta_0} \int_{S_{1/2}}^{S_{3/2}} \sigma_{12}(x_1, x_2) dx_2,
\]
\[ \tilde{\sigma}_{22}(x_i) = \frac{\delta_0}{\delta_0} \int \frac{\delta_0}{\delta_0} \sigma_{22}(x_i, x_2) \, dx_2, \]
\[ \tilde{\sigma}_{11}(x_i) = \frac{\delta_0}{\delta_0} \int \frac{\delta_0}{\delta_0} \sigma_{11}(x_i, x_2) \, dx_2, \]

but average velocities and strain rates are found through their boundary values:
\[ w_{22}(x_i) = \frac{\nu_i'(x_i) - \nu_i(x_i)}{\delta_0}, \quad (2) \]
\[ w_{11}(x_i) = 0.5 \left( \frac{\partial v_i'(x_i)}{\partial x_1} + \frac{\partial v_i'(x_i)}{\partial x_1} \right), \]
\[ \tilde{v}_i(x_i) = 0.5 \left( \nu_i'(x_i) + v_i'(x_i) \right), \]
\[ \tilde{v}_2(x_i) = 0.5 \left( v_i'(x_i) + v_i'(x_i) \right), \quad (3) \]

where \( v^+, v^- \) are velocity vectors of the upper and lower boundaries of area 3.

We assume that the traction rates on the conjugate boundaries of layer 3 are equal and opposite to the traction rates of the body’s conjugate boundaries. In addition, rigid adhesion among the boundaries of area 3 and those of areas 1, 2 is postulated:
\[ v^+ = v(x_i, \delta_0/2); \quad v^- = v(x_i, -\delta_0/2); \quad x_i \in [F, C]. \quad (6) \]

By considering the rates of the traction vectors of layer 3 as boundary conditions for areas adjacent to it [5, 8], we come to a simultaneous solution of variational equilibrium equations for body 1:
\[ \int \sigma \cdot \delta v \, ds + \int \tilde{\sigma}_{22} \delta v_2 \, dx_1 + \int \tilde{\sigma}_{11} \delta v_1 \, dx_1 + \int \tilde{\sigma}_{21} \delta v_1 \, dx_1 = \int P \cdot \delta v \, dl \quad (7) \]
and body 2:
\[ \int \sigma \cdot \delta v \, ds - \int \tilde{\sigma}_{22} \delta v_2 \, dx_1 - \int \tilde{\sigma}_{21} \delta v_1 \, dx_1 + \int \tilde{\sigma}_{21} \delta v_1 \, dx_1 = \int P \cdot \delta v \, dl. \quad (8) \]

The equations (7), (8) need to be closed with concrete defining relations that connect the stress rates with the strain ones. The behavior of the material of bodies 1 and 2 under active loading (\( \sigma \cdot \sigma > 0 \)) is determined by the following physical relations:
\[ \tilde{\sigma} = 2G^{(i)} \tilde{\varepsilon}, \quad \tilde{\rho} = 3K^{(i)} \tilde{\theta}, \quad (9) \]

where \( i = 1, 2; \tilde{\sigma} \) is the stress deviator rate tensor; \( \tilde{\varepsilon} \) is the traceless part of the strain rate tensor; \( \tilde{\rho} = \sigma \cdot E \); \( K^{(i)} \) is the volume compression modulus; \( G^{(i)} \) is the shear modulus \( G^{(i)} = G^{(i)} \) at \( T = T^{(i)} \). \( G^{(i)} = G^{(i)} \) at \( T > T^{(i)} \); \( T^{(i)} \) is the yield limit of the corresponding material; \( T \) is the intensity of tangential stresses.

In the unloading state (\( \sigma \cdot \sigma \leq 0 \)), the defining relations are written as:
\[ \tilde{\sigma} = 2G^{(i)} \tilde{\varepsilon}, \quad \tilde{\rho} = 3K^{(i)} \tilde{\theta}, \quad (11) \]

In the layer’s material, the defining relations are assumed to be reasonable for the average rate characteristics of the stress-strain state along the layer’s thickness:
\[ \tilde{\sigma} = 2G^{(3)} \tilde{\varepsilon}, \quad \sigma \cdot \sigma > 0; \quad \tilde{\rho} = 3K^{(3)} \tilde{\theta}, \quad (13) \]

where \( K^{(3)} \) is the volume compression modulus of the layer material; \( G^{(3)} \) is the shear modulus of the layer material \( G^{(3)} = G^{(3)} \) at \( T \leq T^{(3)} \). \( G^{(3)} = G^{(3)} \) at \( T > T^{(3)} \); \( T^{(3)} \) is the yield limit of the layer material.

As a result of substituting expressions in the defining relations (13)-(15) the component of the average strain rate
(2), (5) average stress rates are determined through the boundary velocities and their derivatives. Thus, the solution of the system (7)-(12) is reduced to determining the velocity field \( \mathbf{v}(x_1, x_2) \) in bodies 1 and 2 (see Fig. 1). In this case, the velocities of the NS boundary of body 2 will be present in equation (7) (see Fig. 1), and the velocities of the FC boundary of body 1 in equation (8) (see Fig. 1).

After determining the velocity field for finding the displacement and strain, we use evolutionary relations:

\[
\mathbf{u}(t_1) = \int_0^{t_1} \mathbf{v} dt,
\]

\[
\varepsilon(t_1) = \int_0^{t_1} \mathbf{w} dt,
\]

where \( \mathbf{u}(x_1, x_2) \) is the displacement field; \( \varepsilon \) is the strain tensor.

Taking into account the defining relations (9)-(12), we find the stress field in bodies 1 and 2:

\[
\sigma(t_1) = \frac{1}{\delta_0} \mathbf{\sigma} dt,
\]

and from (13)-(15), by using the values of the layer's boundary velocities and relations (2), (5), we determine the layer's average stress field:

\[
\bar{\sigma}(t_1) = \frac{1}{\delta_0} \mathbf{\bar{\sigma}} dt.
\]

Destruction conditions for the adhesive layer (AL) are formulated for \( \delta \)-elements of the layer of \( \delta_0 \times \delta_0 \) in size. This is a consequence of a main physical allowance, the destruction covers the material particle of a typical size \( \delta_0 \) [8, 11-15]. As the cohesive fracture criterion of the AL we will use the Coulomb criterion, according to which, by using the evolutionary relations (17), (18), we find stresses and strains \( \bar{\sigma}_{ij}, \bar{\varepsilon}_{ij} \). A transition to the plastic state is related to average SSS properties. The elastic-plastic problem is solved on the basis of the "elastic solution" method [32]. We will determine the location of each finite element's secant shear modulus on the basis of average components \( \bar{\sigma}_{ij}, \bar{\varepsilon}_{ij} \) and consider the secant modulus to be constant within the finite element.

Each element node is characterized by nodal velocity \( \mathbf{v}' \) and nodal force rate \( \mathbf{F}' \). By using the evolutionary relations:

\[
\mathbf{u}'(t_1) = \int_0^{t_1} \mathbf{v}' dt,
\]

\[
\mathbf{F}'(t_1) = \int_0^{t_1} \mathbf{F}' dt,
\]

nodal displacements and nodal forces are determined in the nodes.

We postulate that the layer's element is fully destructed under cohesive fracture during time interval \( \Delta t \). In this case, at the time of destruction \( t_1 \) we will mentally remove the element of size \( \delta_0 \times \delta_0 \) from the layer where the stress state determined according to (20) corresponds to criterion quantity \( \hat{\sigma}_{ij} \) and compensate its effect on the body with the external nodal load \( \mathbf{F}^{(i)} \), \( \mathbf{F}^{(j)} \) from the element according to Fig. 2.

![Fig. 2. Equivalent loads of the layer element at the time of predestruction](image)

We assume that during interval \( \Delta t \) \( \delta \)-element will be completely destructed, if the load that affects the body from the layer element becomes equal to zero. In this case, during time interval \( \Delta t \), under unchanged external load \( \mathbf{P} \), it is necessary to unload new material surfaces. We will consider the unloading process to be simple and depending upon one
parameter $\Delta t$. This corresponds to the formulation of rates of nodal external loads:

$$F^{NF} = -F^{(i)} / \Delta t, \quad F^{NN} = -F^{(j)} / \Delta t. \quad (23)$$

Thus, considering (23) as the external load presented in Fig. 3, it is possible to model the formation of new surfaces on a characteristic size for the finite element formulation of problem (7)-(15). The simple unloading process within interval $\Delta t$ can be divided into several intervals, however, we will limit ourselves to one step in this article.

![Fig. 3. Load during the destruction of the layer element](image)

The main problem is to find nodal forces $F^{(i)}$, $F^{(j)}$ at the time of predestruction, and, consequently, to form boundary conditions (23). The discrete solution (7)-(15) for the subcritical deformation along the layer conjugation boundaries contains the nodal velocity vector $\mathbf{v}^{(i)}$, $\mathbf{v}^{(j)}$, as unknown quantities, along which the displacement vector $\mathbf{u}^{(i)}$, $\mathbf{u}^{(j)}$ can be formed from (16).

In order to determine the nodal forces $F^{(i)}$, $F^{(j)}$ it is proposed to use the procedure of the repeated loading that consists in replacing the effect of the destructible element by setting the process of change with the "time" of nodal displacements of the element, when the law of external effect on the body is repeated, according to Fig. 4.

![Fig. 4. Repeated loading diagram](image)

The finite element solution of the repeated loading in the nodes with the set nodal velocity field $\mathbf{v}^{(i)}$, $\mathbf{v}^{(j)}$, as known quantities, will contain the vectors of nodal force rates $F^{(i)}$, $F^{(j)}$ along the destructible element's boundaries.

Depending on the distribution of the main external load, as a result of destruction of $\delta$-element, the separation process can continue at an unchanged value of the main external load (unstable fracture) or terminate. Herewith it is necessary to consider that as a result of $\delta$-unloading (connected with the AL destruction), the additional loading with an entry into the plastic area is possible in the surrounding layers. This circumstance does not allow to directly use the unloading theorem by A.A. Ilyushin [32].

### 3. Solution Results

Let us consider the composite material, under plane deformation, that consists of two elements with material properties close to alloy D16 as an example of calculation: $G = 2.8 \cdot 10^{10}$ Pa; $G_p = 5.2 \cdot 10^8$ Pa; $K = 6 \cdot 10^{10}$ Pa; $T_p = 3 \cdot 10^8$ Pa is the elasticity limit; $\sigma_5 = 4.2 \cdot 10^8$ Pa is the strength limit bonded by epoxy resin with the following properties: $G = 1.3 \cdot 10^5$ Pa; $K = 1.7 \cdot 10^5$ Pa; $\sigma_5 = 9 \cdot 10^7$ Pa. The geometric characteristics of the composite were taken as follows: $AD = 5 \cdot 10^{-3}$ m; $\delta_0 = FN = 10^{-3}$ m; $MQ = 2 \cdot 10^{-2}$ m; $AB = 10^{-1}$ m; $DF = 5 \cdot 10^{-2}$ m. The external load rate $P = 1$ Pa/s is directed at angle $\pi/4$ to axis $0X_1$.

As a test, let us consider the problem of destruction of the layer element in the elastic approach. Let the critical state in the $\delta$-element be determined by the external load $P = 1$ Pa. The computational convergence of the solution establishes a partition of the boundary area conjugate to the $\delta$-element into four finite elements [8]. Taking into account the quadratic law of distribution of the velocity field on a finite element, nine nodal forces along the upper and lower boundaries of the destructible $\delta$-element will determine the equilibrium of the composite at the time of predestruction (see Fig. 2). The corresponding nodal forces determined by the repeated loading method are given in Table 1. The forces presented in Table 1 are related to a minimum value of the modulus of their projections. The nodes are numbered from left to right.

### Table 1

<table>
<thead>
<tr>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$F_3$</th>
<th>$F_4$</th>
<th>$F_5$</th>
<th>$F_6$</th>
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<td>1.3</td>
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<td>2.0</td>
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<td>-3.3</td>
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<td>-1.4</td>
</tr>
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<td>8.0</td>
<td>3.8</td>
<td>7.2</td>
<td>3.6</td>
</tr>
</tbody>
</table>
Without loss of generality, the minimal module of projections of the found nodal forces can be considered as local unloading parameter \( \Delta \) for determining the boundary conditions (23).

Graph 2 in Fig. 5, a shows the distribution of dimensionless vertical displacements \( u_2^b \) along the boundary of body 1 and layer 3 towards the end of the subcritical deformation, graph 3 shows the distribution of displacements due to the process of destruction of the layer structural element, graph 1 shows the superposition of the displacement fields. The displacements are related to the value of displacement \( u_2^b(0, \delta_0/2) \) at the beginning of the process of destruction. The dimensionless coordinate in the direction of the abscissa axis is defined as \( x_1^b = x_1/\delta_0 \).

Graph 1 in Fig. 5, b determines the corresponding vertical displacements when solving a problem without the layer's first structural element at the end of interval \( \Delta t_1 \). This solution simulates the "kill element" approach used in the finite element modeling [15].

Graph 2 in Fig. 5, b repeats curve 1 in Fig. 5, a. As we see from Fig. 5, b, the coincidence of graphs 1 and 2 shows the adequacy of the results of the proposed procedure of unloading new surfaces during the destruction process to the "kill element" approach within elastic deformation. It is necessary to note that the situation with horizontal displacements is identical.

Meanwhile the main maximum tensile stress \( \sigma_{\text{max}} \) in the layer's second element increases, which suggests a catastrophic fracture in this diagram.

By comparing the solution of the elastic problem for the set external load to the layer's first structural element and without it, let us note that the stress intensity \( (I = \sigma \cdot \delta) \) decreases for many finite elements. Fig. 6 shows the finite element area of bodies 1 and 2 (see Fig. 1), where the stress intensity has decreased (the elements are highlighted), when solving the problem without the layer's first structural element, as compared to the solution with the structural element. This circumstance allows us to assume that the generation of a zone of unloading of plastic areas is possible under elastic-plastic deformation.

Fig. 6. Configuration of the finite element area where the stress intensity decreases at the elastic solution of the problem. The configuration is made on the basis of the “kill element” approach.

However, the "kill element" approach does not allow to correctly describe this process. Let us consider the solution of this problem on the basis of the proposed procedure.

Fig. 7 shows a plastic deformation area in the state of predestruction. The value of the external load is denoted as \( P_0 \). The main maximum stress in the adhesive layer on the characteristic element is equal to the strength limit, and the strength limit is not reached outside the layer. Consequently, destruction in the layer will take place faster, than in the materials conjugate to it. We assume that the adhesive bond resin-alloy is sufficiently strong, and the destruction will take place along the array of the adhesive component.

The destruction of the first element leads to the redistribution of the plasticity zone and unloading of a number of the composite's elements. In this case, the result of considering destruction as a thermomechanical process is different from the result obtained with the "kill element" procedure. In Fig. 8, the additional plastic load zone is marked with a dark filling, and the lighter parts show the elements where elastic unloading from the plastic area took place, after the destruction of the first \( \delta \)-element.
By solving the problem of loading the composite with critical load $P_1$ without the layer's first structural element, we come to the distribution of the plastic area shown in Fig. 9. By comparing Fig. 8 and Fig. 9, we see that the unloading from the plastic area is not taken into account in the "kill element" approach, and the plasticity area is slightly larger than the combination of the unloading areas from the plastic area and the additional plastic load shown in Fig. 8.

The destruction of the first element leads to an excess of the strength limit on the second element of the layer, which means its destruction at a fixed external load. Thus, this type of loading results in a catastrophic destruction.

4. Conclusion

This work suggests a description of the discrete destruction of the material volume on the basis of the simple unloading hypothesis. The repeated loading procedure with the set nodal displacement field found from the solution of the subcritical deformation problem is used to determine the internal nodal forces that affect the destructible volume. The results of the calculations according to the proposed model within the elastic behavior of the material do not contradict the known "kill element" calculation method. In the case with an elastic-plastic material, the stress state obtained by modeling the destruction process with the local unloading method may considerably differ from the state determined by the "kill element" method. The proposed approach allows us to take into account the redistribution of plastic zones and the possibility of forming new zones of destruction as a result of local unloading. The proposed method can be used to model the destruction of elements in an unspecified finite element continuum.

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