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MATHEMATICAL MODELLING OF THE MEMBRANE (HERNIA) STRESS-STRAIN STATE

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Abstract: Stress-strain state of a membrane has been modelled. The critical radius for a spherical membrane is shown to be half as much again the radius of a non-deformed one. It has been found that upon reaching the critical pressure the membrane may further deform under reduced pressure. Reinforcement of the areas of weakness in the abdominal wall with a tendon of the broadest muscle of the back during the surgical procedure allows to reduce the sag significantly and enlarge the critical intracavitary pressure up to acceptable values.

Keywords: modelling, stress-strain state, membrane, hernia

Introduction

Nowadays problems of etiology of external peritoneal hernias are still of great importance. The causes leading to herniation may be classified as either local or general. The latter in turn are subdivided into predisposing and producing. Among the predisposing factors are heredity, constitution, age, sex and muscular status. The producing factors include an increase in intra-abdominal pressure and weakness of the abdominal wall. The increase in intra-abdominal pressure is due both to pathological and physiological processes in the human organism.

The local factors imply peculiar anatomical structure of the anterior abdominal wall that has weaknesses uncovered with muscles – the inguinal area, the umbilical ring, the white line and the femoral ring. Under unfavourable conditions the peritoneum may protrude in such spots and cause further herniation.

Thus, the present research is directed at a elaboration of a mathematical model of herniation and gaining recommendations on this basis for hernia preventive and surgical treatment.

Information on the problem found in the literature [1-5] can be classified into two groups. The first group contains the knowledge of physical and mechanical properties of human (and animal) soft tissues in vivo and in vitro and the investigative techniques to determine such values as density, strength, Young's modulus, and Poisson's ratio of soft tissues and tendons. The data on mathematical modelling of like biomechanical systems should fall in the second group.

In simulating the mechanism of herniation we assume the anterior lateral abdominal wall to be a four-layer membrane. These layers are skin 2-3 mm thick, subcutaneous fat 2-20 mm thick, muscles and tendons 2-5 mm thick, and peritoneum 0.2-0.5 mm thick. The carrying capability of the anterior lateral abdominal wall is made up of carrying capabilities of its individual layers in the following proportions: the first layer – 20%; the second layer – 3%; the third layer – 70%; and the fourth layer – 7%.

1. Equilibrium in case of a spherical deformed membrane

Let us consider the forms of external hernia equilibrium and the tensions occurring in the abdominal wall.

Let us assume that the hernia in the deformed state is a spherical segment and can be simulated by a spherical membrane of radius R, thickness h and base diameter D.

To study the membrane form it is suggested that the membrane's material is isotropic and incomressible. Young's modulus is E, and Poisson's ratio is μ.

Stresses σ in the spherical membrane subject to internal excess pressure P are determined by the equation:

$$\sigma = \frac{PR}{2h}. \tag{1}$$

In an elastic material the stress increase is accompanied with tissue elongation in the direction of current efforts (in the plane tangent to the sphere) as well as with membrane thinning.

For a spherical segment under homogeneous deformation the following relationship holds true:

$$2\pi RHh = 2\pi R_0 H_0 h_0, \tag{2}$$

if the hernia has been formed from a spherical segment of radius R_0. Here H is the height of the spherical segment, index "0" refers to the parameters of the non-deformed state.

From expression (2) it follows that

$$h = \frac{R_0 H_0 h_0}{RH}. \tag{3}$$

Substitution of expression (3) in expression (1) gives

$$\sigma = \frac{PR^2 H}{2R_0 H_0 h_0}. \tag{4}$$

In this case $H = kR$, $H_0 = kR_0$, where k is a constant.

For biaxial tension ($\varepsilon_1 = \varepsilon_2 = \varepsilon$), which the membrane's material is subjected to, (normal stress is neglected because of its negligible small quantity) it is known that

$$\sigma = \frac{E\varepsilon}{1-\mu}, \tag{5}$$

where ε is the relative extension of the membrane's material, ε_1, ε_2 are the relative strains in the directions of the principal curvafuves.

The relative strain of the membrane surface is equal to

$$\varepsilon = \frac{R}{R_0} - 1. \tag{6}$$

That is why equation (5) may be written in the form

$$\sigma = \frac{E(R - R_0)}{(1-\mu)R_0}. \tag{7}$$

By equating expressions (4) and (7) let us find the equation of nonlinear relation between P and R, whence membrane pressure P can be determined

$$P = \frac{2ER_0 h_0}{1-\mu}\left(\frac{1}{R^2} - \frac{1}{R^3}\right). \tag{8}$$

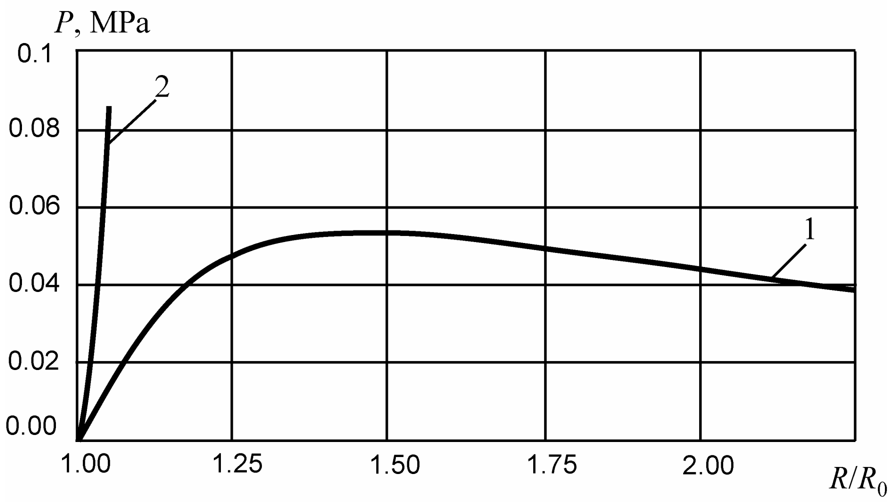

Fig.1. Curve 1 is the radius-dependent pressure of the spherical membrane, curve 2 is the stress in the membrane. E =0.932 MPa, μ =0.48, R_0 =100 mm, h_0 =10 mm.

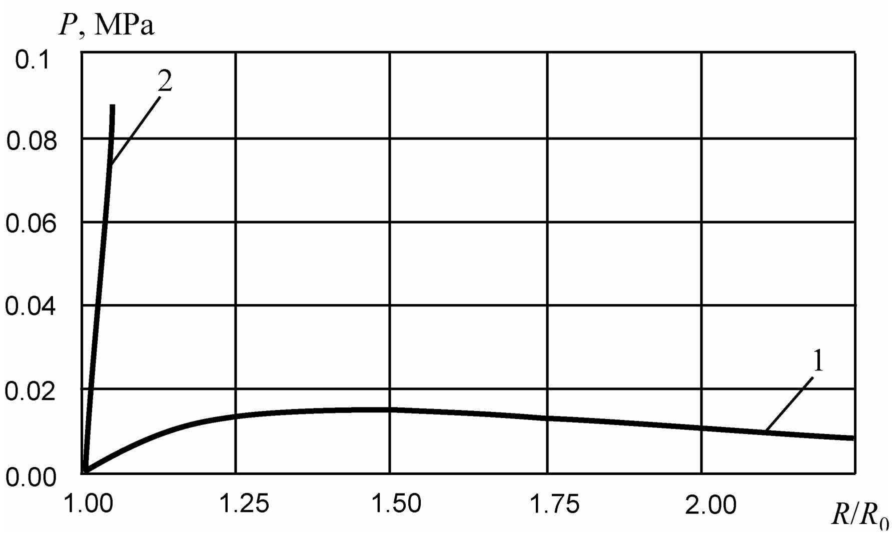

Fig.2. Curve 1 is the radius-dependent pressure of the spherical membrane, curve 2 is the stress in the membrane. E =0.932 MPa, μ =0.48, R_0 =100 mm, h_0 =3 mm.

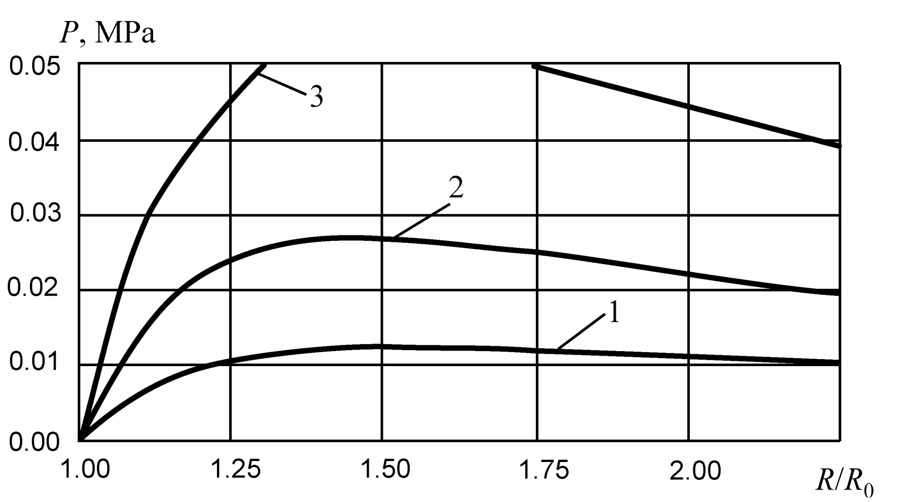

Fig.3. Radius-dependent pressure for the initial membrane thickness h_0 =10 mm (curve 1), h_0 =5 mm (curve 2), h_0 =2.5 mm (curve 3) with R_0 =100 mm, E =0.932 MPa, μ =0.48.

Analysis of the function $P(R)$ for its maximum shows that the critical values of radius R_{cr} and pressure P_{cr} are equal

$$R_{cr} = \frac{3R_0}{2}, \; P_{cr} = \frac{8Eh_0}{27(1-\mu)R_0}. \qquad (9)$$

Fig.1 gives the R - radius dependence of pressure P and stress σ in the membrane. As seen in the plots, at reaching the critical pressure the subsequent increase in the membrane radius may occur under reduced pressure. Results (9) are in complete correspondence with the equations obtained for rubber balls in [1].

Fig.2 shows that the thinner is the membrane, the lower is the critical pressure to 0.015 MPa.

Fig.3 presents the results of the radius-dependent pressure calculations for different initial membrane thicknesses.

The present analysis shows that the membrane critical pressure is proportional to the membrane initial thickness. The qualitative result obtained in the given point is as follows: hernias in their form can be subdivided into subcritical and supercritical. Under a casual increase in pressure a hernia may become supercritical and further growth in its size will occur.

2. Stress-strain state of a multilayer membrane

In simulating the mechanism of herniation we assume the anterior lateral abdominal wall to be a four-layer membrane. Layer 1 (skin) has the thickness h_1= 2 mm and Young's modulus E_1= 0.932 MPa; layer 2 (subcutaneous fat) - thickness h_2= 5 mm and Young's modulus E_2= 0.1 MPa; the third layer (muscles and tendons) - thickness h_2= 5 mm and Young's modulus E_2= 0.1 MPa; and layer 4 (peritoneum) - thickness h_4= 0.4 mm and Young's modulus E_4= 0.932 MPa. Poisson's ratio is thought to be one and the same for all the layers and equals μ = 0.48. The carrying capability of the anterior lateral abdominal wall is made up of carrying capabilities of its individual layers in the following proportions: layer 1 – 20%; layer 2 – 3%; layer 3 – 70%; and layer 4 – 7%.

The apparent stiffness of the abdominal wall, Eh, can be derived from the equation:

$$Eh = \sum_{i=1}^{4} E_i h_i ,$$

where E is the apparent Young's modulus of the abdominal wall assumed to be equal to muscles Young's modulus ($E = E_3$). So then the apparent thickness h of a single-layer membrane can be found from the expression

$$h = \frac{\sum_{i=1}^{4} E_i h_i}{E}.$$

3. Modelling of stress-strain state of a tendon allograft

In modelling the abdominal wall repair with a flap of a tendon allograft made of the broadest muscle of the back let us take Poisson's ratio equal to 0.48. The flap is up to 120 cm^2 in its area and 2 mm thick, with Young's modulus E =60 MPa [5].

Let us compare the hernial stress-strain state before surgery and after the defect's repair with tendon allograft biomaterial [4].

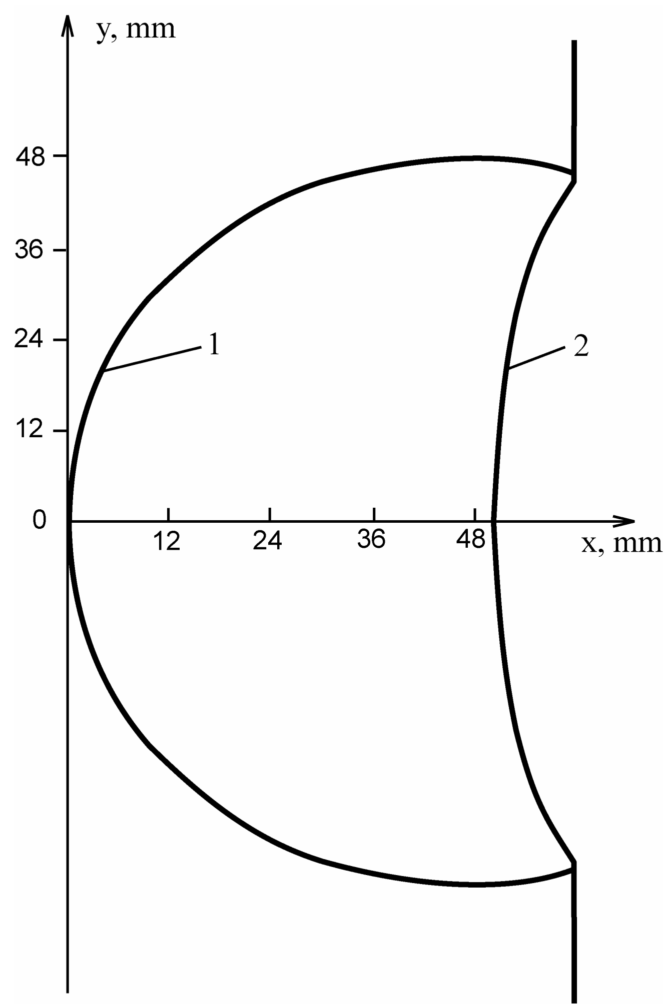

Fig. 4. Shape of the anterior abdominal wall under pressure P =48 kPa.
1 – before surgery, 2 – after surgery.

Fig.4 (curve 1) shows the shape of an axisymmetric membrane (hernia) [4] that consists of the following layers: (1) skin with h_1=2 mm and Young's modulus E_1=0.932 MPa; (2) subcutaneous fat with h_2=5 mm and Young's modulus E_2=0.1 MPa; (3) muscles and tendons are absent; and (4) peritoneum with h_4= 0.4 mm and Young's modulus E_4=0.932 MPa. The apparent Young's modulus is taken to be E_1, and the apparent thickness of the abdominal wall can be determined by the equation

$$h = \frac{\sum_{i=1}^{4} E_i h_i}{E} = \frac{0.932 \cdot 2 + 0.1 \cdot 5 + 0.932 \cdot 0 + 0.932 \cdot 0.4}{0.932} = 2.93 \text{ mm}.$$

The critical pressure in the hernia is 0.048 MPa. Fig.4 (curve 2) shows the shape of the deformed membrane that consists of the following layers (after surgery): (1) skin with h_1=2 mm and Young's modulus E_1=0.932 MPa; (2) subcutaneous fat with h_2=5 mm and Young's modulus E_2=0.1 MPa; (3) tendon with h_3=1 mm and Young's modulus E_3=65 MPa; and (4) peritoneum with h_4=0,4 mm and Young's modulus E_4=0.932 MPa. The apparent Young's modulus is taken to be E_3, and the apparent thickness of the abdominal wall h=1.042 mm. The membrane shape was determined under pressure 48 kPa.

As shown by means of modelling, reinforcement of the weak abdominal wall with a flap of tendon allograft biomaterial results in an essential reduction of the maximum sag. In this case the abdominal wall being in its subcritical position is able to withstand the interior pressure appeared.

4. Practical recommendations in line with the results of the present research

From the results of the work we can make the following conclusions:

a) The critical radius of a deformed spherical membrane is half as much again the radius of a non-deformed one.

b) When a hernia reaches the critical size, its further growth may occur even under less intracavitary pressure.

c) The techniques elaborated in the research makes it possible to calculate a critical intracavitary pressure according to the known physical and mechanical properties of the wall or its layers.

The obtained results can be applied in the analysis and research of deformation of membrane-type biological systems.

Practical recommendations are as follows:

a) The critical intracavitary pressure is reducing with an increase in the size of the anterior abdominal wall defect that is the probability of herniation becomes greater. Thus, if the defect is equal to 100 mm^2, the critical intracavitary pressure may fall within 19 to 27 kPa. Our observations testify that the use of tendon allograft is desirable even with a small-sized hernial portal. The greater is the wall thickness, the higher is the critical intracavitary pressure.

b) In accordance with the above conclusions the following ways of hernial preventive treatment can be recommended: wearing a belly bandage, training the abdominal muscles with the help of special exercises, and the combined approach.

c) Reinforcement of weak areas of the abdominal wall by the tendon of the broadest muscle of the back during the surgical procedure allows reducing the sag significantly and enlarging the critical intracavitary pressure up to acceptable values.

References

1. БЕРЕЗОВСКИЙ В.А., КОЛОТИЛОВ Н.Н. **Биофизические характеристики тканей человека. Справочник.** Киев, Наукова думка, 1990 (in Russian).
2. БРАНКОВ Г. **Основы биомеханики.** Москва, Мир, 1981 (in Russian).
3. ЕРМОЛОВ В.В. Парадокс пары резиновых шаров. **Сообщения лаборатории мягких оболочек.** ДВВИМУ. ЦБНТИ ММФ. Владивосток, вып. 14: 81-86, 1971 (in Russian).
4. КУЗИН А.А., ХАКИМОВ А.Г., ЮХИН Г.П. **Моделирование напряженно-деформированного состояния мягкой оболочки (грыжи).** Институт механики УНЦ РАН, Уфа, 32 с., 1998 (Препринт Института механики УНЦ РАН) (in Russian).
5. КУЗИН А.А. **Лечение паховых грыж аллосухожильным биоматериалом.** Диссертация на соискание ученой степени кандидата медицинских наук. Уфа, 1996 (in Russian).

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ НАПРЯЖЕННО – ДЕФОРМИРОВАННОГО СОСТОЯНИЯ МЯГКОЙ ОБОЛОЧКИ (ГРЫЖИ)

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Моделируется напряженно – деформированное состояние мягкой оболочки. Показано, что критический радиус для сферической оболочки в 1,5 раза больше радиуса недеформированной оболочки. Показано, что при достижении критического давления дальнейший рост радиуса оболочки может происходить при пониженном давлении. Упрочнение ослабленных участков стенки живота сухожилием широчайшей мышцы спины во время операции позволяет резко уменьшить прогиб и увеличить критическое внутриполостное давление до приемлемых значений. Библ. 5.

Ключевые слова: моделирование, напряженно-деформированное состояние, мягкая оболочка, грыжа

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