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# GAUGE MODELS OF OPERATING BY BODY ORIENTATION OF FALLING OR RUNNING ANIMAL

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**Abstract**: Mechanical model of falling or running animal is suggested. This model is used for investigation of kinematics of its turn in space by means of change of animal shape when external forces and torques are absent. The animal is presented as a system of point masses connected with each other by weightless rods. The body form is specified by angles between rods and rods lengths. Computer realization of model can take into account more than 100 degrees of freedom. The computer package has an objective-oriented structure that provides a flexibility of modeling process. Biological constraints of degrees of freedom are taken into account in the model such as restrictions on bending angles and lengthening of rods. The gauge mechanics of nonrigid bodies is applied for interpretation results and planning model investigations. The main notions of the gauge theory and comprehensible example of its application are considered for pedagogical purposes. The detailed computation was carried out for the biological model with 12 degrees of freedom. The effectiveness of different scenarios of body deformation like bending of legs and twisting of body for providing animal's turn around horizontal axis is analyzed.

**Key words**: body orientation, gauge models, falling, running

## Introduction

The main feature that differs biomechanical problems from the "usual" mechanics problems is a presence of controlling subsystem that biological system in vivo usually has. For example, if in the "usual" mechanics the flow of fluid in the tube of fixed cross-section is considered the corresponding problem of biomechanical interest is the fluid flow in the tube of varying in course of time cross-section. In contrast to the problems of rigid body mechanics in biomechanics the problems of nonrigid bodies with controlled shape are important.

Biological objects are the class of complex systems, where hierarchy of molecular and biomolecular process is presented, which in turn result in the process on physiological level, like action of the neurosystem and the muscles and on the top of the hierarchy visual controlled mechanical motions take place which are of our interest. It is obvious that ideas, concepts and terminology of object-oriented analysis and design are very useful for operating with biosystems. Biosystems (like animal or plant) may be considered as a set of objects those are connected with each other. Every object has its own "methods" or functions which can be divided into internal and external ones. In biomechanics, we restrict ourselves to some level of abstraction, taking into account only mechanical part of phenomena. External "methods" are presented in the problem as controlled parameters which effect on mechanical movements of objects.

The main goal of biomechanical investigation is to find out the scenarios of biological system behavior. The scenario has to possess some invariant features those are independent (to some extent) on the concrete values of masses, velocities and so on. For example, it is known for the problem of falling cat that if "back down" is cat's starting position it lands on the ground by its "back up". This result (scenario of cat's falling) is independent of the cat size, mass and can be considered as an "invariant". The useful tool for scenarios investigations is suggested by gauge theory of nonrigid bodies that is developed by number of authors (see published review paper [1]).

It is proposed on the one hand that a biomechanical system like usual mechanical system can be described by a set of mechanical parameters: masses, inertia moments, different type connections and so on. However, on the other hand in the addition to that, some of the listed parameters are under control or in other words they can be changed according to given law (input parameters). For example, in the swimming problem [2] input parameters are those that define the body shape of animal swimming in the fluid. The change of the parameters (body shape) is the cause of the displacement of animal in the fluid (the displacement is an output parameter in this case).

The main goal of the given paper is the design of models of falling or running animal. Note that in contrast to the walking, running is specified by stage when the runner does not touch the ground or he is separated from the surface on which he is running. So in our model it is assumed that the run process includes two stages: 1) the short-term pushes, when the animal takes momentum $\mathbf{P}$ and angular momentum $\mathbf{L}$ and 2) the long-term intervals of the "free-flight". The goal of control is to correct the orientation of animal's body during time of fall or run process to provide the safety landing. The only way for animal to control its space orientation is a change of its shape (by bending legs, twisting body, etc.). The result of these mechanical manipulations is landing on the legs or preservation of balance.

In our mechanical model an animal is presented as a system of material points those are connected with each other by weightless rods; the angles between rods and the lengths of rods are controlled parameters. By this model detailed scenario of the internal movements' sequence according to certain algorithms of parameter change may be investigated. The natural constraints and estimation of the required energy are taken into account by computer model.

The gauge theory of mechanics is important for interpretation and planning of computer experiments. The main notions of this theory: configurations space, shape space, connectivity, holonomy are explained in the paper by presenting comprehensive example of nonrigid body. In particular, it follows that the control of the body turn in space is possible only if there are no less than two shape parameters under control.

The content of the paper is as follows. In the next section the main notions of the gauge theory as applied to the problems of nonrigid body mechanics are considered and detailed example is given for pedagogical purposes. In the third section, algorithms included into computer package are described. In the last section the results of computer research obtained by the developed computer model are presented. Some future problems are discussed in conclusion. It has to be mentioned that the problem of the falling cat attracted investigators for a long time. There are many papers devoted to this problem [3, 4]. As compared with them, the developed algorithm can be used to examine models and scenarios those are more complicated.

## Gauge theory in mechanics of nonrigid bodies

### *Fundamental notions*

As known the rigid body can not change its space orientation in the absence of the external forces and torques. The angular momentum $\mathbf{L}$ linearly depends on the angular velocity $\boldsymbol{\omega}$

$$\mathbf{L} = I \cdot \boldsymbol{\omega}, \tag{1}$$

where $I$ is the inertia tensor, the dot means the contraction of the indexes of tensor $I$ and vector $\boldsymbol{\omega}$. It follows that if the initial angular momentum is equal to zero $\mathbf{L} = 0$ then angular velocity is also equal to zero $\boldsymbol{\omega} = 0$ and the rigid body can not perform a turn in the absence of an external torque. It will be seen from the following that in contrast to the rigid body the deformable one can change its orientation in space due to the controlled redistribution of masses even in the absence of the external forces.

As noted in the introduction, we will model a deformable body as the system of $n$ point masses ($\{m_a\}$, $a = 1, 2, \ldots, n$) which are connected with each other by definite way with the help of massless rods. The parameters defining the position of the $n$-bodies system can be divided into two parts: shape parameters and orientation coordinates. The last ones specify the position of the system as a whole relative to the laboratory frame (L-frame in short). The shape parameters specify positions of point masses relative to each other.

Let us introduce the frame system that is attached to the body and name this frame as B-frame. The position of any point in L-frame of our system may be described as: 1) its position in respect to B-frame and 2) position of B-frame relative to L-frame. Three Euler angles or other parameters like them can be applied for description of B-frame relative position and L-frame [4, 5]. There is some problem in choice of B-frame for body that change its shape. There are three known methods which solve the problem: 1) Eckart frame ($B_e$-frame), 2) the principal axes of inertia tensor frame ($B_i$-frame) and 3) the "principal" body frame (B-frame). For biomechanical purposes it is natural to choose the last case, where B-frame is attached to the "main body" of a biosystem. The "principal" body is vitally important organ for animal that has to be saved.

One of the important goals of a body orientation is the preservation of principal organ in safe state, as it can be destroyed in case of unsuccessful fall. One of the main notions on which gauge theory based is the notion of fiber bundle. Let $(q_1, \ldots, q_{3n-6})$ be the set of parameters of a body shape and let some $B_q$-frame is given that uniquely is defined by the body shape, i.e. by given set of parameters $(q_1, \ldots, q_{3n-6})$. As the shape parameters are fixed the body can be considered as rigid one and all its orientations are uniquely defined by $B_q$-frame. Suppose now that for any set $(q_1, \ldots, q_{3n-6})$ $B_q$-frame is uniquely defined. The whole $(3n-3)$-dimensional configuration space can be splited onto two subspaces: 1) $(3n-6)$-dimensional space of the sets of parameters $(q_1, \ldots, q_{3n-6})$ which is named the base space and 2) 3-dimensional space that define orientation of the body of given shape. These spaces can be specified e.g. by Euler angles of $B_q$-frames and they are considered as "layers" under "base space".

The construction of the fiber bundle is shown schematically in Fig. 1. The horizontal axes is $(3n-6)$-dimensional shape space of the body and the vertical axis is 3-dimensional orientation space for every given body shape. The vertical lines (fibers) represent possible orientations of a given body shape. The curved line (really it is a surface) S marks the position when B-frames' axes orientations coincide with the axes of L-frame. This line is called the section of fiber bundle. If some other choice of the B-frame will be done section S changes its position.

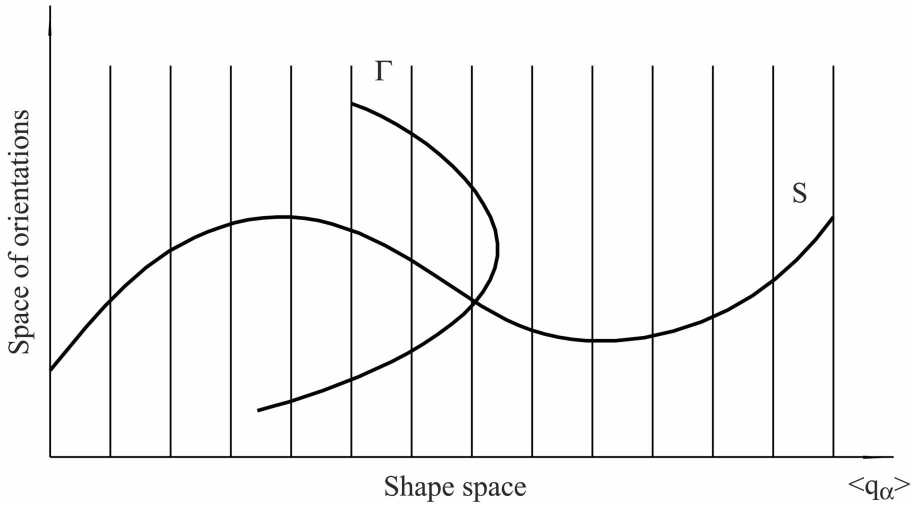



Fig.1. Fiber bundle.

It is obvious that the observed quantities can not be dependent on the choice of $B_q$ - frame or in other words on the section S of fiber bundle. It may be said that section S determines some choice of gauge. The physical consequences can not depend on the gauge choice or they are gauge invariants. Introduction of fiber bundle gives intuitive and visual geometric meaning of mechanics of nonrigid bodies. In addition important geometric notions of the gauge theory like connectivity, curvature, holonomy and others may be used for interpretation of calculation results (detail discussion of geometry of the gauge theory may be found in many books, see for example [6]). Below some of these notions are shortly discussed and the others are mentioned in the next section in the course of discussion of an example of nonrigid body mechanics.

We remind that $n$-dimensional manifold $M_n$ is a point set that is locally being in one-to-one correspondence with some region of $R^n$ space. This correspondence may not exist globally. For example 2-d sphere $S^2$ can not be mapped onto the plane $R^2$. In the cases like this the manifold $M_n$ may be mapped on the set of regions $\phi_n^{(\alpha)}$ of $R^n$. The set of regions $\phi_n^{(\alpha)}$ and plus the prescribed rule of pasting together all regions $\phi_n^{(\alpha)}$ which are in neighborhood to each other are in one-to-one correspondence with manifold $M_n$.

To explain idea of construction of fiber bundle let us consider manifold $B$ and let any point $P$ of $B$ ($P \in B$) is appropriate to some manifold $F$. The manifolds $B$ and $F$ are belong to $M$; $B$ is named base manifold or band and $F$ is named layer or fiber. The fiber $F$ can have additional structure. For example if $F$ is a vector space, the manifold $M$ is a named vector fiber bundle (or vector bundle). If $F$ is one-dimensional vector space, $M$ is line bundle. Physical examples of fiber bundles are: 1) Newtonian space-time where base $B$ is time axis and fiber $F$ is 3-d space $R^3$ and 2) space-time plus spin states of elementary particle, where base is the set of space-time coordinates $(x, y, z, t)$ and the fiber is isospin space.

It is defined projection $\Pi$ on fiber bundle $M$ that is $\Pi : M \to B$. It maps all elements of given fiber $F$ onto point $P \in B$ to which fiber $F$ is attached. Locally the manifold $M$ is equal to direct product of $B_\alpha \times F$ ($B_\alpha \in B$, $B = \sum B_\alpha$) but global topological properties of $M$ are described by the rule (or connectivity) that determines the assembling $M$ from the small parts $B_\alpha \times F$. As an example consider the circle as a base and let to every point of it the stripe is glued. In the case when all stripes are perpendicular to the plane of the circle the fiber bundle is the ring. But in other case when every stripe is turned a little bit and then return to

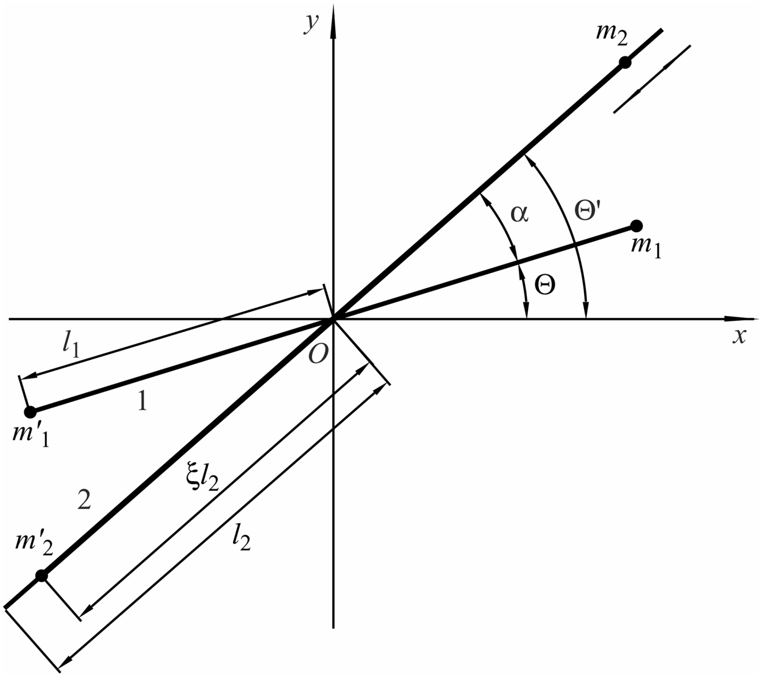



Fig. 2. Example model of a nonrigit body.

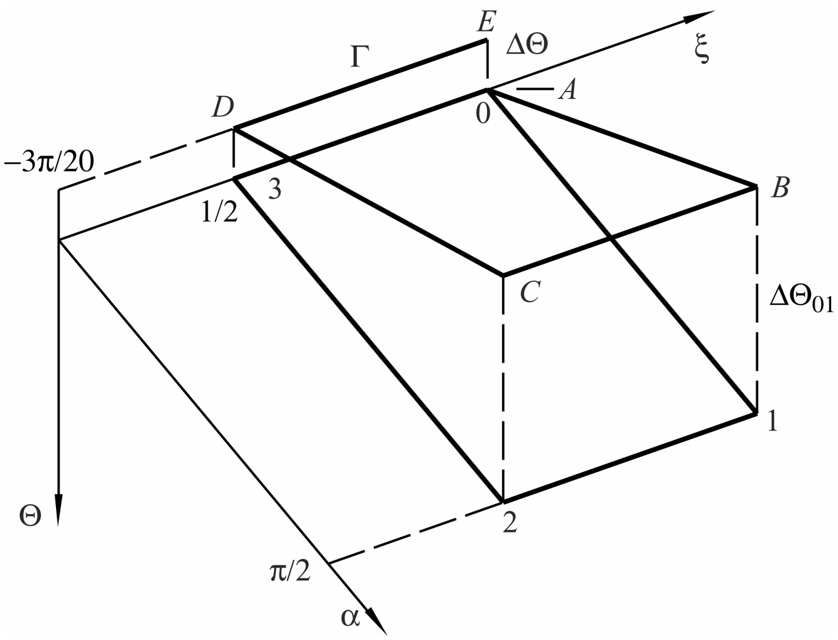



Fig. 3. Calculated path in configurational space.

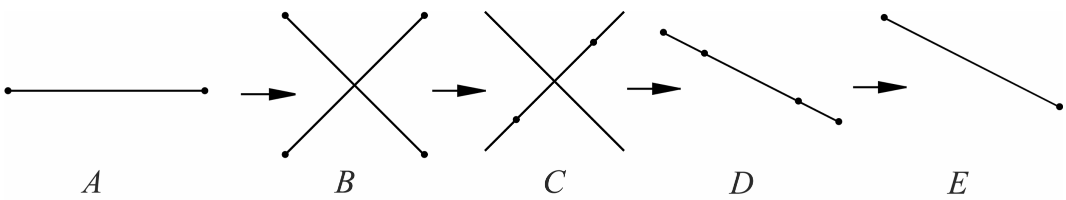



Fig. 4. Change of shape that corresponds to the path 0-1-2-3-0.

the initial point with opposite orientation the fiber bundle will be the Mobius strip. As the fibers for points of $B$ are independent of each other it is needed an additional rule that help to compare elements of different fibers. This rule, named connectivity, defines parallel transport of elements of fiber $F_1$ in point $P_1 \in B$ into element of fiber $F_2$ in point $P_2 \in B$ along the definite path in $B$. The end point of some path $\Gamma \in M$ that is projected on the closed path $\Gamma_B \in B$ belongs to the same fiber $F$ but not coincide with each other in generally. The distance between end points of $\Gamma$ is named holonomy (see below Fig. 3). For infinitesimal closed path the holonomy is characteristic of connectivity curvature. The connectivity defines linear dependence between infinitesimal displacements along the fiber and area of closed path on the base.

### *Example of nonrigid body mechanics*

In this section the simple example of nonrigid body is considered and the notions of the gauge theory are explained on the base of this concrete system. The model of nonrigid body is shown in Fig. 2. It consists of two massless, rigid rods 1, 2 and four masses $m_1, m_1', m_2$ and $m_2'$ on them.

Masses are balanced in such a way that the inertia center is fixed in the point $O$. It is supposed that our system can change its shape due to alteration of angle $\alpha$ between rods 1 and 2 and a distance $\chi\, l_2$ from point $O$ to $m_2$ or $m_2'$ position. These internal movements of masses $m_2$ and $m_2'$ can be provided by some engines which are not shown in the picture. Biological objects can change its shape due to muscular contraction.

The angle $\Theta$ defines the rod 1 orientation relative to L-frame. Thus we fix a frame to be connected with the rod 1 and consider this rod as a "principal" body. The rod 2 can be also chosen as a general body or the B-frame can be fixed by some way for every given value of the controlled parameters $\chi$ and $\alpha$. If the values $\Theta, \alpha$ and $\chi$ are fixed radius vectors $\mathbf{r}_1, \mathbf{r}'_1, \mathbf{r}_2, \mathbf{r}'_2$ of the masses $m_1, m_1', m_2$ and $m_2'$ can be written as

$$\mathbf{r}_1 = l_1\mathbf{e}_1,\ \mathbf{r}'_1 = \mathbf{r}_1,\ \mathbf{r}_2 = l_2\mathbf{e}_2,\ \mathbf{r}'_2 = \mathbf{r}_2. \tag{2}$$

Unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are directed from the center $O$ towards the corresponding points $m_1$ and $m_2$. In these equations it was proposed for simplicity that $m_1 = m_1'$ and $m_2 = m_2'$. The components of these vectors in L-frame can be written as

$$\mathbf{e}_1 = (\cos\Theta, \sin\Theta),\ \mathbf{e}_2 = (\cos(\Theta+\alpha), \sin(\Theta+\alpha)). \tag{3}$$

Suppose now that the internal parameters $\alpha$ and $\chi$ are changed in the course of time in a way. This leads to the "external" parameter $\Theta$ change. We can find how $\Theta$ depends on $\alpha$ and $\chi$ using the law of angular momentum $\mathbf{L}$ conservation. The value of $\mathbf{L}$ is given by equation

$$\mathbf{L} = \sum m_a [\mathbf{r}_a, \mathbf{V}_a], \tag{4}$$

where $\mathbf{V}_a = \dot{\mathbf{r}}_a$ - the velocities of masses and the summation is taken over the points. After a differentiation over time Eq. (2), (3) we find that

$$\mathbf{V}_1 = \dot{\mathbf{r}}_1 = l_1\dot{\mathbf{e}}_1,\ \mathbf{V}'_1 = -\mathbf{V}_1, \tag{5}$$

$$\mathbf{V}_2 = \chi l_2\dot{\mathbf{e}}_2 + \dot{\chi} l_2\mathbf{e}_2,\ \mathbf{V}'_2 = -\mathbf{V}_2. \tag{6}$$

Using (3), the velocities of unit vectors $\dot{\mathbf{e}}_1$ and $\dot{\mathbf{e}}_2$ can be found to be proportional to the vectors $\mathbf{g}_1$ and $\mathbf{g}_2$, which are orthogonal to $\mathbf{e}_1$ and $\mathbf{e}_2$ respectively

$$\mathbf{e}_1 = \dot{\Theta}\,\mathbf{g}_1,\ \mathbf{e}_2 = (\dot{\Theta}+\dot{\alpha})\,\mathbf{g}_2, \tag{7}$$

where

$$\mathbf{g}_1 = (-\sin\Theta, \cos\Theta),\ \mathbf{g}_2 = (-\sin(\Theta+\alpha), \cos(\Theta+\alpha)). \tag{8}$$

Using these equalities we can present an angular momentum (4) in the form

$$\mathbf{L} = \left[ 2m_1\, l_1{}^2 \dot{\Theta} + 2m_2\ \chi^2\ l_2{}^2 (\dot{\Theta} + \dot{\alpha}) \right] \mathbf{e}_z\,, \tag{9}$$

where the unit vector $\mathbf{e}_z = [\mathbf{e}_1, \mathbf{g}_1] = [\mathbf{e}_2, \mathbf{g}_2]$ directed along $z$-axis of L-frame is introduced. Let introduce the inertia momentum of rods 1 and 2 with masses on them:

$$I_1 = 2m_1\, l_1{}^2\,,\ I_2 = 2m_2\, \chi^2 l_2{}^2\,, \tag{10}$$

and the total inertia momentum

$$I = I_1 + I_2\,. \tag{11}$$

By this notations angular momentum (9) of nonrigid body can be rewritten in the following form

$$L_z = I(\chi)\dot{\Theta} + I_2(\chi)\dot{\alpha}\,. \tag{12}$$

Let us compare this expression with angular momentum of rigid body (1). First of all in (12) in addition to angular velocity $\boldsymbol{\omega} = \dot{\Theta}\, \mathbf{e}_z$ the derivative $\dot{\alpha}$ is presented. Note that in general case the shape of arbitrary nonrigid body described by parameters $q_1, q_2, \ldots, q_s$ where $s$ is the number of degrees of freedom the angular momentum can be expressed in the analogous form

$$\mathbf{L} = I(q_1, q_2, \ldots, q_s)\, \boldsymbol{\omega} + \sum_{a=1}^{s} \mathbf{K}_a (q_1, q_2, \ldots, q_s)\, \dot{q}_a\,, \tag{13}$$

where $I(q_1, q_2, \ldots, q_s)$ is an inertia tensor of body which depends on parameters $q_1, q_2, \ldots, q_s$, $\mathbf{K}_a (q_1, q_2, \ldots, q_s)$ are some vector coefficients also depending of $q_1, q_2, \ldots, q_s$. The kinds of these dependencies are determined by the body's internal structure.

Let us return to the analysis of our example. Suppose that the angular momentum is equal to zero $L_z = 0$. Then from Eq. (12) we can get the relation between the "external" parameter $\Theta$ and "internal" parameters $\alpha, \chi$

$$\dot{\Theta} = -\frac{I_2(\chi)}{I(\chi)}\dot{\alpha}\,. \tag{14}$$

It is the specific character of this simple problem that at the right hand side there is no term propositional to $\dot{\chi}$ defining variation of the second degree of freedom.

Eq. (14) establishes how the rod's 1 orientation changes relative to L-frame when the internal parameters are changing and the angular momentum stays invariable. Multiplying the left and right hand sides of Eq. (14) by $dt$ transforms it into the following equation

$$d\Theta = -\frac{I_2(\chi)}{I(\chi)}d\alpha\,. \tag{15}$$

As it can be seen from this relation between $\Theta$ and $\alpha, \chi$ does not depend on time $t$, it is manifestation of a geometric nature of the turn with respect to internal deformations.

Given some initial state parameters $\alpha_0, \chi_0$ we suppose that the initial orientation angle $\Theta$ is also given by value equal to zero $\Theta = \Theta_0 = 0$ (the rod 1 is aligned along the $x$ axis of L-frame). Let $\alpha$ and $\chi$ change is defined by some function $\chi = \chi(\alpha)$ or in more general case the curve $\alpha = \alpha(t)$, $\chi = \chi(t)$ is given. Then we can find with the help of Eq. (15) the change $\Delta\Theta = \Theta_1 - \Theta_0$ of the angle $\Theta$ in the form

$$\Delta\Theta = \int_{\alpha_0}^{\alpha_1} \frac{I_2(\chi)}{I(\chi)}d\alpha\,. \tag{16}$$

Table 1. Closed way in a shape space

| $i$ | $\alpha_i$ | $\chi_i$ | $\Theta_i$ | $\Theta'_i$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 |
| 1 | $\frac{\pi}{2}$ | 1 | $-\frac{\pi}{4}$ | $\frac{\pi}{4}$ |
| 2 | $\frac{\pi}{2}$ | 0.5 | $-\frac{\pi}{4}$ | $\frac{\pi}{4}$ |
| 3 | 0 | 0.5 | $-\frac{3\pi}{20}$ | $-\frac{3\pi}{20}$ |
| 0 | 0 | 1 | $-\frac{3\pi}{20}$ | $-\frac{3\pi}{20}$ |

As we have noticed above, angle $\Theta$ does not define the orientation of our body and we can not to affirm that the change of the shape leads to the change of the orientation: the angle $\Theta$ defines the rod's 1 orientation only. Suppose that the path is closed in the shape space so the body's final shape is the same as initial one, i.e. $\alpha_0, \chi_0$. Then we can compare the final and initial orientations of the system if there is a difference between them, we can conclude that the body is turned, because now two compared bodies have the same shape. Generally speaking the phase path in the space $(\alpha, \chi, \Theta)$ may be nonclosed. This case is our main interest: it follows that internal forces can turn nonrigid body.

We can calculate the turn of the body in our example for some closed path in $(\alpha, \chi)$-space in the explicit form. Let $\lambda$ designates a ratio of lengths $l_1$ and $l_2$, $\mu$ designates a ratio of masses $m_1$ and $m_2$

$$\lambda = \frac{l_1}{l_2}, \ \mu = \frac{m_1}{m_2}, \tag{17}$$

then the relation (14) with respect to (10) can be formulated as

$$d\Theta = -\frac{\chi^2}{\chi^2 + \mu^2\lambda^2} d\alpha \ . \tag{18}$$

Let us choose a closed path 0-1-2-3-0 in the shape space $(\alpha, \chi)$ that is given by Table 1 and shown in Fig. 3.

For simplicity we assume below that $\mu^2\lambda^2 = 1$. With Eq. (18) we can find that on the part of path between points $O$ and 1

$$\Delta\Theta_{01} = -\frac{1}{2}\frac{\pi}{2} = -\frac{\pi}{4}, \tag{19}$$

on the parts 1-2 and 3-4

$$\Delta\Theta_{12} = \Delta\Theta_{30} = 0, \tag{20}$$

and finally on the part 2-3

$$\Delta\Theta_{23} = \frac{1}{5}\frac{\pi}{2} = \frac{1}{10}\pi \ . \tag{21}$$

The resulting turn after a closed path passage is equal to

$$\Delta\Theta = \Delta\Theta_{01} + \Delta\Theta_{12} + \Delta\Theta_{23} + \Delta\Theta_{30} = -\frac{3}{20}\pi \ . \tag{22}$$

We have determined that the body will turn relative to L-frame (after a loop 0-1-2-3-0 in shape space) by the angle $\Delta\Theta = -\frac{3}{20}\pi$ radians.

Simultaneously phase point of this system moved along the path $\Gamma = ABCDE$ in the complete phase space $(\alpha, \chi, \Theta)$. As it can be seen from Fig. 3, the initial and final positions $A$ and $E$ of the path $\Gamma$ do not coincide with each other. It is usual in the gauge mechanics to name the distance $AE$ as a holonomy. In our case the holonomy is a turn angle $\Delta\Theta$ calculated above along a closed loop in shape space (line 0-1-2-3-0 in a plane $(\alpha, \chi)$ Fig. 3). It must be underlined that the change of angle $\Delta\Theta$ does not depend on the velocity of movement along path. This fact has the "geometrical" sense (like a "Berry phase"). Certainly this does not mean that the orientation change takes place for the geometrical reasons. In fact, the change can be produced because of work generated by the internal energy sources. It must be marked again that the found change of orientation does not contradict the law of angular momentum conservation, but it follows from this law. It is evident that any change of isolated system's shape (which we are discussing here) can not lead to a translational movement (center of mass translation). The change can produce a redistribution of masses but the total mass of a system does not change. From the law of momentum conservation of an isolated system it follows that

$$\mathbf{P} = \sum m_a \mathbf{V}_a = const(t) \tag{23}$$

and the inertia center

$$\mathbf{R} = \frac{\sum_a m_a \mathbf{r}_a}{\sum_a m_a} \tag{24}$$

remains at rest or moves with fixed velocity. In particular if it rests at the initial time, it will hold still later.

Let us discuss now our example in respect of the invariance of a frame choice, linked to a nonrigid body. The body's orientation as a whole will be defined by the new angle $\Theta'$ with the B-frame's axis related to the rod 2 (Fig. 2). $\Theta'$ and $\Theta$ depend on each other by the equation

$$\Theta' = \Theta + \alpha . \tag{25}$$

Instead of Eq. (3) and (18) we have now

$$\begin{aligned} &\mathbf{e}_1 = (\cos(\Theta' - \alpha), \sin(\Theta' - \alpha)), \dot{\mathbf{e}}_1 = \chi(\dot{\Theta}' - \dot{\alpha})\, \mathbf{g}_1, \\ &\mathbf{e}_2 = (\cos\Theta', \sin\Theta'), \dot{\mathbf{e}}_2 = \dot{\Theta}' \mathbf{g}_2, \end{aligned} \tag{26}$$

and for an angular momentum $L_z$ the following equation can be received

$$L_z = 2m_1\, l_1(\dot{\Theta}' - \dot{\alpha}) + 2m_2 \chi^2 l_2{}^2 \dot{\Theta}' . \tag{27}$$

Setting $L_z$ equal to zero we can find the relation between $\Theta'$ and $\alpha$ in the form

$$\dot{\Theta}' = \frac{I_1}{I}\dot{\alpha} = \frac{\lambda^2\mu^2}{\chi^2 + \lambda^2\mu^2}\dot{\alpha} . \tag{28}$$

It can be calculated using the same closed path 0-1-2-3-0 (Table 1) and assuming $\mu^2\lambda^2 = 1$ that

$$\Delta\Theta'_{01} = \frac{\pi}{4},\ \Delta\Theta'_{12} = \Theta'_{30} = 0,\ \Delta\Theta'_{23} = -\frac{2}{5}\pi . \tag{29}$$

It follows that the resulting rotation is equal to the same angle after a passage along the closed path

$$\Delta\Theta' = -\frac{3}{20}\pi \,. \tag{30}$$

Thus, we have shown that on the closed path the resulting angle increment does not depend on coordinate frame choice, i.e.

$$\Delta\Theta' = \Delta\Theta \,. \tag{31}$$

It is usual to name the choice of coordinate frame linked to body as a gauge convention. Physical parameters can not depend on gauge convention, thus they must be gauge invariant. In these terms we can claim that the resulting turn is the gauge invariant value.

We can rewrite the relation between $\Theta$ (or $\Theta'$) and shape parameters (18) (or (28)) in the next form:

$$d\Theta = A(\chi)\, d\alpha \,,\ d\Theta' = A'(\chi)\, d\alpha \,, \tag{32}$$

where

$$A = \frac{\chi^2}{\chi^2 + 1} \,,\ A' = \frac{1}{\chi^2 + 1} \,. \tag{33}$$

$A(A')$ is defined as a gauge potential. As we can see from these equations the potential is not an invariant quantity $A \neq A'$. Let us examine the increment of turn angle $\Delta\Theta$ when the system shape makes the infinitesimal cycle in the neighborhood of the arbitrary point $(\alpha, \chi)$ in the shape space $(\alpha, \chi)$, $(\alpha + d\alpha, \chi)$, $(\alpha + d\alpha, \chi + d\chi)$, $(\alpha, \chi + d\chi)$, $(\alpha, \chi)$. The changes of $\Theta$ and $\Theta'$ in this cycle are equal to each other and have the value

$$\Delta\Theta = \Delta\Theta' = B(\chi)\, d\chi\, d\alpha \,, \tag{34}$$

where the gauge field $B(\chi)$ is introduced

$$B(\chi) = \frac{dA(\chi)}{d\chi} = \frac{dA'(\chi)}{d\chi} = \frac{2\chi}{(1+\chi^2)^2} \,. \tag{35}$$

We can state that the field $B(\chi)$ is invariant under coordinate frame change. It is important that we can also perform gauge invariant evaluation. The turn angle on arbitrary closed path $\Gamma$ using the double integral on the area element $\Sigma$ rested on $\Gamma$ is equal to:

$$\Delta\Theta = \iint_{\Sigma} B(\chi)\, d\chi\, d\alpha \,. \tag{36}$$

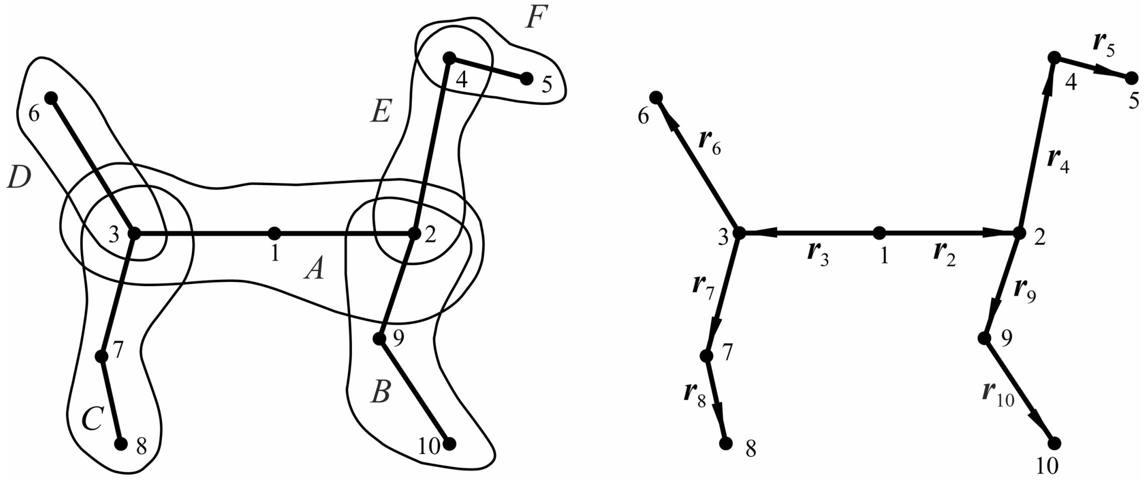
**Animal's model**

In our model an animal is represented as a system of point masses, connected to each other by the massless rods (Fig. 5). The rods form a space graph having no loops (tree graph). It is supposed that the body of an animal can change its shape because of muscular contractions. It is evident that these internal forces can not produce a total torque so the total angular momentum also can not be changed. The lengths of rods and some chosen angles between rods $e_{\mathrm{ab}}$ are the variable parameters (degrees of freedom of animal shape). The angles have constraints caused by an animal's anatomy.

## *Algorithms and program package*

The programming technology is based on a modular structure of algorithms and computer model. This structure helps easily rebuild the program package by modifying modules or including new ones into it.

The mechanics of biological objects like shown in Fig. 5 (a) is modelled by the developed package. The principle body is placed in the «root» of the tree graph structure (point 1 in the Fig. 5 and Fig. 6). The system of orthogonal unit vectors $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ forms




a b

Fig. 5. Mass-rods model of animal (a) and its oriented graph (b).

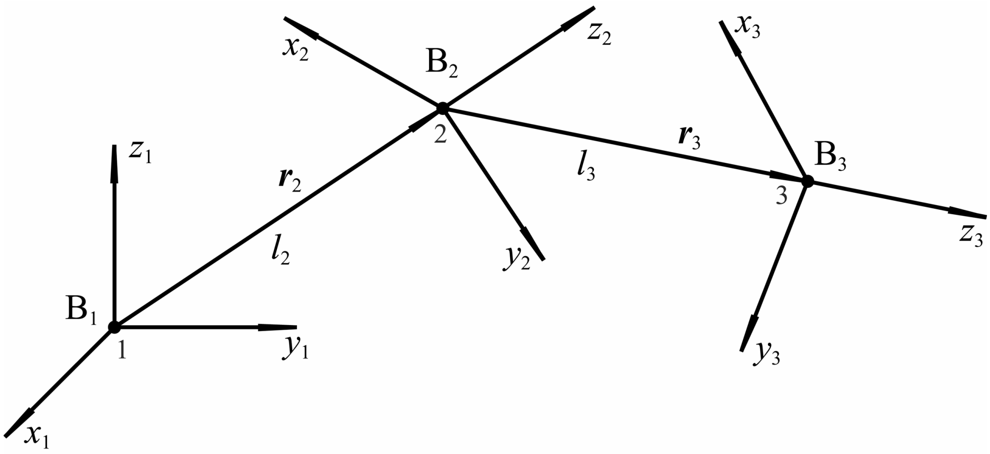



Fig. 6. $B_a$-frame definition.

$B_1$-frame that is attached to this body. The body itself is not shown in Fig. 6 and it is symbolized by bases vectors of $B_1$ reference frame.

A recurrence procedure was developed to define the positions of the other nodes of the tree with respect to the laboratory reference frame (L-frame) when shape of the body is changed. To be exact, the procedure calculates coordinates of nodes $x_a, y_a, z_a$, a=1,..., $n$ in L-frame when we change the angles between tree branches and lengths of rods $l_a, \vartheta_a, \varphi_a$. The idea is to generate reference frames $B_a$ attached to every node a by the successive transformations of $B_1$-frame. For this purpose we first transform the tree graph of animal model into oriented graph. The orientation that attributed to every edge is chosen in direction away from the principal body that is placed in the root node 1 (Fig. 5 (b)). It is obvious that every node a of a tree graph may have only one incoming edge (parent branch) and many outgoing edges (offspring branches). We denote the oriented incoming to node a edge by vector $\mathbf{r}_a$. The length of $\mathbf{r}_a$ is equal to the length of the edge $l_a$ and the unit vector along $\boldsymbol{r}_a$ is denoted by symbol $\mathbf{e}_a$ so that the equality $\mathbf{r}_a = l_a \, \mathbf{e}_a$ takes place. For the following let us name the node b which is a starting point for vector $\mathbf{r}_a$ as a parent node for node a.

To transform $B_1$-frame into $B_2$-frame attached to the node a=2 in Fig. 6 that is in the neighborhood to the «principal» body 1 the following steps must be done:

1. The origin of $B_1$-frame (point 1) is translated by the vector $\boldsymbol{r}_2$ that is incoming to node 2.

2. The rotation $\hat{R}_{12}$ around the axis $n_{12}$ perpendicular to the plane that is formed by vectors $\langle \mathbf{e}_{z1}, \mathbf{e}_2 \rangle$ by the angle $\vartheta_2$ equal to the angle between $\mathbf{e}_{z1}, \mathbf{e}_2$ is performed.
3. The axes of $B_2$-frame are denoted by $\mathbf{e}_{x1}, \mathbf{e}_{y1}, \mathbf{e}_{z1}$ where $\mathbf{e}_{z2} = \mathbf{e}_2$ and Euler angles $\vartheta_2, \varphi_2, \psi_2$ which describe position of $B_2$-frame with respect to $B_1$-frame are calculated and saved.

To produce $B_a$- frame in arbitrary node a by transformation of $B_b$-frame attached to the parent node b the analogous steps must be done:

1. The origin of $B_b$-frame (placed at point b) is translated by the vector $\mathbf{r}_a$ outgoing of node b and incoming to the node a.
2. The rotation $\hat{R}_{\mathrm{ba}}$ around the axis $n_{ba}$ perpendicular to the plane that is formed by vectors $\langle \boldsymbol{e}_{\mathrm{b}}, \boldsymbol{e}_{\mathrm{a}} \rangle$ by the angle $\vartheta_{\mathrm{a}}$ equal to angle between $\mathbf{e}_b, \mathbf{e}_a$.
3. The axes of $B_a$-frame are denoted by $\mathbf{e}_{xa}, \mathbf{e}_{ya}, \mathbf{e}_{za}$ where $\mathbf{e}_{za} = \mathbf{e}_a$ and Euler angles $\vartheta_{\mathrm{a}}, \varphi_{\mathrm{a}}, \psi_{\mathrm{a}}$ which describe position of $B_a$-frame with respect to $B_b$-frame are calculated and saved.

Now we can return to discussed problem of algorithm for counting body coordinates in L-frame for fixed body shape. The body shape is uniquely defined by the set of spherical coordinates of vectors $\{\boldsymbol{r}_{\mathrm{a}}\}$ attached to every node a. These spherical coordinates $l_{\mathrm{a}}, \vartheta_{\mathrm{a}}, \varphi_{\mathrm{a}}$ are the control parameters in our problem. They have to be given with respect parent reference frame ($B_b$-frame) for considered node a. To calculate coordinates of every node a with respect to L-frame some sequence of inverse transformations has to be performed. First we find the way on the tree graph from the node a to the «principal» body 1. It follows through sequence of parents nodes. For example if node b is parent for a, node c is parent for b, node 2 is parent for c and node 1 is parent for node 2 the following way from node a to 1 can be found: a-b-c-2-1. To transform coordinates of vector $\mathbf{r}_a$ given in $B_b$-frame which we denote by symbol b, as $\mathbf{r}_a^{(\mathrm{b})}$ into coordinates of this vector with respect to $B_1$-frame, which we denote by symbol 1, as $\mathbf{r}_a^{(1)}$ the following transformation has to be done:

$$\mathbf{r}_a^{(1)} = \hat{\mathrm{R}}_{12}^{-1} \hat{\mathrm{R}}_{2c}^{-1} \hat{\mathrm{R}}_{cb}^{-1} \mathbf{r}_a^{(\mathrm{b})}.$$

As every rotation operator $\hat{R}$ depends on angles between branches of tree graph, this formula reflects the complicated dependence of body coordinates on its form.

Bringing about the transformations to $B_1$-frame over all vectors $\mathbf{r}_a$ (a=2,3,..., $n$) it is possible to find the coordinates of every node a with respect to origin 1 of «principal» body. In particular for considered example it can be calculated that

$$[\mathbf{r}_a^{(1)}]_1 = \mathbf{r}_2^{(1)} + \mathbf{r}_c^{(1)} + \mathbf{r}_b^{(1)} + \mathbf{r}_a^{(1)}.$$

Conversion of $\mathbf{r}_a^{(1)}$ to $\mathbf{r}_a^{(\mathrm{L})}$ - coordinates of $\mathbf{r}_a$ with respect to L-frame is realized by operator $\hat{R}$ that is calculated by the main algorithm of the package discussed below.

To take into account that the center of mass has to hold still during the changes of shape (because we have assumed that the external forces do not exist) in our computer model the following procedure was included:

1. Calculate the initial position $\mathbf{R}_0 = (X_0, Y_0, Z_0)$ of the center of mass.
2. Specify an increment of shape parameters.
3. Calculate position of the center of mass of the deformed system $\mathbf{R}_1 = (X_1, Y_1, Z_1)$.
4. Calculate the displacement vector $\mathbf{R} = \mathbf{R}_1 - \mathbf{R}_0$.

5. Translate the system as a whole by the vector $\mathbf{R}$.

The main algorithm of the program package is the calculation of the change of orientation of a deformed system.

1. It is supposed that initial positions of all point masses $m_a$ in L-frame i.e. vectors $\boldsymbol{r}_a^{(L)}$ are known.
2. Specify increments of the angles $\Theta_a, \varphi_a, \psi_a$ which define orientation of vector $\boldsymbol{r}_{a-1a}$ with respect to $B_{a-1}$-frame. Calculate the new set of $B'_a$-frames at every node using recurrent procedure described above.
3. Find the displacement of every mass in L-frame $d\boldsymbol{r}_a^{(L)}$.
4. Calculate the change of angular momentum

$$d\mathbf{L} = \sum_a m_a \left[\mathbf{r}_a^{(L)}, d\mathbf{r}_a^{(L)}\right]. \qquad (41)$$

5. Calculate the inertia tensor

$$I_{ik} = \sum_a m_a \left[x_{ai} x_{ak} - r_a^2 \delta_{ik}\right]. \qquad (42)$$

6. Calculate the compensation turn

$$\boldsymbol{\omega} dt = I^{-1} \cdot d\mathbf{L}, \text{ or } \mathbf{n}\, d\varphi = I^{-1} \cdot d\mathbf{L}, \qquad (43)$$

where $\mathbf{n}$ and $d\varphi$ are the axis vector and the angle of the turn respectively.

The program package contains three base modules. The first module is the library of classes of objects which define computer model of animal (see below). The second module is the library of methods by which the objects can interact with each other. This library helps to present the processes which take place in the system. The third module contains the library of methods of design and visual representation of 3D-objects.

The external libraries providing matrix and vector calculus, user interface and other subsidiary functions are also used in the package.

To realize the computation process additional code was created that operates by the libraries as external ones. This helps to remove underlying details of implementation from the user so he can operate only by the biological and physical objects.

The package was created using the object-oriented language Java (JDK 1.3.1) and OpenGL library. HTML language and XML technology were used to present results of calculations and to save working projects.

## *Simulation of the movement of the running or falling animal*

As noted above it is most rational to apply an object-oriented analysis and design [7] for modeling of complex biological objects. Object-oriented programming has multiple levels. It provides faster and cheaper development and maintenance, the modeling process becomes simpler and produces a clear, manageable design, the elegance and clarity of the object model and the power of object-oriented tools and libraries makes programming a much more pleasant task, and programmers experience an increase in productivity. To this end it must introduce some new types and classes. It is simpler to understand package classes which correspond to the application conception. The developed package uses a collection of classes defining the biological structure of an animal. Classes are introduced in the following manner. The developed package uses a collection of classes defining the biological structure of an animal. Classes are introduced in the following manner. Deformation of biological object means the change of some parameters of physical object (like angles $\Theta_a$, $\varphi_a$, etc.).

Table 2. Model initial parameters

| Biological object | Description | Physical object | $l$ | $m$ | $\Theta$ | $\varphi$ | $\psi$ |
|---|---|---|---|---|---|---|---|
| A | Body | 1 | 0 | 1 | 0 | 0 | 0 |
| | | 2 | 1 | 1 | $\frac{\pi}{2}$ | $\frac{\pi}{6}$ | 0 |
| | | 3 | 1 | 1 | $-\frac{\pi}{2}$ | $-\frac{\pi}{6}$ | 0 |
| B | Right front leg | 2 | 1 | 1 | $\frac{\pi}{2}$ | $\frac{\pi}{6}$ | 0 |
| | | 9 | 0.75 | 0.75 | 0 | 0 | 0 |
| | | 10 | 0.75 | 0.75 | 0 | 0 | 0 |
| C | Right hind leg | 3 | 1 | 1 | $-\frac{\pi}{2}$ | $-\frac{\pi}{6}$ | 0 |
| | | 7 | 0.75 | 0.75 | 0 | 0 | 0 |
| | | 8 | 0.75 | 0.75 | 0 | 0 | 0 |
| D | Tail | 3 | 1 | 1 | $-\frac{\pi}{2}$ | $-\frac{\pi}{6}$ | 0 |
| | | 6 | 2 | 0.01 | $-\frac{\pi}{3}$ | 0 | 0 |
| E | Neck | 2 | 1 | 1 | $\frac{\pi}{2}$ | $\frac{\pi}{6}$ | 0 |
| | | 4 | 0.5 | 0.5 | $-\frac{\pi}{3}$ | 0 | 0 |
| F | Head | 4 | 0.5 | 0.5 | $-\frac{\pi}{3}$ | 0 | 0 |
| | | 5 | 0.5 | 0.5 | $\frac{\pi}{2}$ | 0 | 0 |

Table 3. Orientation angles of the animal's body after one cycle manipulations by its shape according to listed algorithms.

| Algorithm | $\Theta$ , rad | $\varphi$ , rad | $\psi$ , rad |
|---|---|---|---|
| Algorithm 1 | 0.0200 | 0.0043 | 0.0000 |
| Algorithm 2 | -0.3083 | -0.0246 | 0.0589 |
| Algorithm 3 | -0.6193 | 0.0443 | 0.1277 |
| Algorithm 4 | -0.1229 | 0.0067 | 0.0435 |

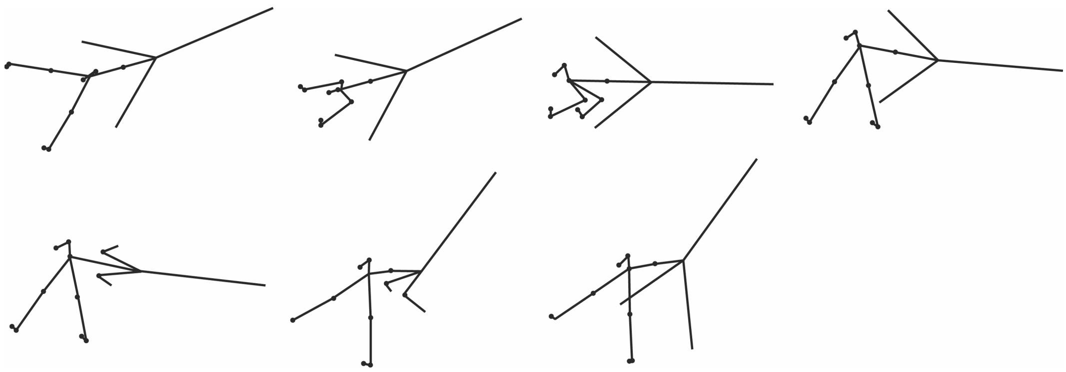


Fig. 7. Animal modelling (Algorithm 3).

Collection of biological objects forms a biological system. Biosystem can have a big number of objects (>100), degrees of freedom and parameters. It is important to emphasize that the degrees of freedom are not equivalent for goals of moving animal. The most important is the space orientation of the "principal" body which can be considered as the orientation of an animal as a whole.

Let us consider examples of biological objects included in our package. We define a biological object "animal's leg" consisting of three physical objects: - joint, knee and foot. We can let this biological object to be deformed (to bend) changing the parameters of physical object "knee". It is evident that we must concentrate on deformations having the "biological" meaning (named biological methods), e.g. "to turn head", "to bend leg", etc. These deformations must have specific constraints - an animal can bend its leg only in plane formed by three physical object joint, knee and foot and the angle's change must be limited according to an animal anatomy.

The proposed model (Fig. 5) is constructed in accordance with these principles but it can not be a precise animal model. To make an exact model a big number of objects has to be introduced which take into account the movement of every muscle, bones and passive masses of the body. In our research we had a more limited goal to investigate: the scenarios of shape and body parameters change of falling or running animal. The model shown in Fig. 5 consists of physical objects listed in Table 2.

Biological methods introduced for this model are as follows:

1. "Bend body" means a change of angles $\Theta_2$ and $\Theta_3$ which are properties of the physical objects 2 and 3 respectively.
2. "Turn body" means a change of angles $\varphi_2$ and $\varphi_3$ which are properties of physical objects 2 and 3.
3. "Bend leg in the elbow" (e.g. for the right front leg (biological object "B" in Table 2) means a change of angle $\Theta$, physical object 9).
4. "Bend leg in the knee" (e.g. for the right front leg (biological object "B") means a change of angle $\Theta$, physical object 10).
5. "Bend neck" means a change of angle $\Theta$, physical object 4).
6. "Turn head" means a change of angle $\varphi$, physical object 5).

Modeling of a complete set of classes (biological objects) which possess relevant numbers of methods makes the procedure of planning scenarios much simpler.

Concrete examples of animal movement's scenarios are given below. Our goal is to find the most effective scenario of an animal turn in space. Four algorithms were investigated for that purpose

Algorithm 1

1. Turn the front part of the body relative to its hind part.
2. Bend front and hind legs in the knees.

3. Turn the front part of the body to return to initial position relative to the hind part.
4. Bend front and hind legs in the knees to return to their initial position.

Algorithm 2
1. Bend front legs in the knees and in the elbows.
2. Turn the front part of the body relative to its hind part.
3. Bend front legs in the knees and in the elbows to return to their initial position.
4. Turn the front part of the body to return to initial position relative to the hind part.

Algorithm 3 (Fig. 7)
1. Bend front legs in the knees and in the elbows.
2. Turn the front part of the body relative to its hind part.
3. Bend front legs in the knees and in the elbows to return to their initial position.
4. Bend hind legs in the knees and in the elbows.
5. Turn the front part of the body to return to initial position relative to the hind part.
6. Bend hind legs in the knees and in the elbows to return to their initial position.

Algorithm 4
1. Simultaneously bend front legs in the knees and in the elbows and turn the front part of the body relative to its hind part.
2. Simultaneously bend front legs in the knees and in the elbows to return to their initial position and bend hind legs in the knees and in the elbows.
3. Simultaneously turn the front part of the body to return to initial position relative to the hind part and bend hind legs in the knees and in the elbows to return to their initial position.

Physical object 1 was considered as the "principal" body, so the change of Euler angles of physical object 1 can determine a change of orientation of the biological object as a whole. The results of calculations are shown in Table 3.

It can be seen from Table 3 the third algorithm is the most effective among others, after five cycles Euler angles of "principal" body are equal to $\Delta\Theta = -0.1858$, $\Delta\varphi = 2.8466$, $\Delta\psi = -3.1677$. Comparing our results with the movie of falling cat (see e.g. http://www.ffden2.phys.uaf.edu/211.fall2000.web.projects/) it seems that real cat makes only one cycle of manipulation to return to proper orientation of it's body while in our model animal takes five cycles to come to the same position. The reason for that discrepancy may be as follows. First our model is taking into account not the right lengths, masses and other parameters. Second it may be so that restrictions for bending angle, etc. we take into account are narrower then in the reality.

## Conclusions

In the course of our investigation the computer model of animal was designed, that gives us a possibility to analyze the shape changes' scenarios and to compute the change of orientation in the space. The computer package can visualize these processes having a descriptive-geometric sense.

The interpretation of the results is performed with the help of the gauge theory - in terms of space of shapes, holonomy, etc. where the scenario is the path in the shape space. The developed model solves the problem of computation of animal orientation by operation of its shape. The model can initiate the solving the following problems:

1. "Control problem": how to change the shape to obtain a given orientation?

2. "Optimization problem": how to optimize this change of shape with respect to expenses of energy, time, etc.

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# КАЛИБРОВОЧНЫЕ МОДЕЛИ УПРАВЛЕНИЯ ОРИЕНТАЦИЕЙ ТЕЛА ВО ВРЕМЯ ПАДЕНИЯ ИЛИ БЕГА ЖИВОТНОГО

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Построена механическая модель животного, позволяющая исследовать кинематику его поворотов в пространстве во время свободного падения или в процессе бега в отсутствие внешних воздействий за счет изменения формы тела. Животное представлено как система точечных масс, соединенных между собой невесомыми стержнями. Варьируемыми параметрами служат углы между стержнями и длины стержней, определяющие форму тела, а также величины масс. Модель реализована в виде компьютерной программы, которая позволяет учитывать до 100 степеней свободы. Программа имеет объектно-ориентированную структуру, что обеспечивает гибкость моделирования, достигаемую за счет введения небольшого числа основных классов, свойства и методы которых наследуются биологическими объектами. В модели учитываются биологические ограничения степеней свободы: допустимые углы поворота конечностей, пределы изменения длин связей и т.п. Для интерпретации результатов моделирования используется калибровочная теория механики нежестких тел. Для полноты изложения ее основы кратко рассмотрены во втором разделе статьи. Рассмотренние иллюстрируется простым примером нежесткой системы, который позволяет детально проследить механизм поворота тела за счет деформации формы и сопоставить его с понятиями калибровочной теории. Конкретные расчеты с использованием построенной программы проводились для системы с 12 степенями свободы, для которой проведен анализ эффективности различных сценариев движения конечностями и скручивания туловища, для обеспечения поворота тела вокруг горизонтальной оси. В заключении обсуждается круг биомеханических задач, которые могут быть рассмотрены с помощью построенной модели. Библ. 12.

Ключевые слова: ориентация тела, калибровочные модели, падение, бег

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