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ANALYSIS OF COMPLIANCE OF BIOMEDICAL SHEETING BASED ON POROUS MATERIALS. PART 2

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Abstract. Compliance of damping biotissue and its hypothetical artificial analogues (sheetings, implants) made of adaptive poromaterial is analysed. The adaptive reaction provides for minimisation of traumatic contact stress initiated by static or quasi-static loading of human body. An individual pore of high-density poromaterial is assumed as a mezoscopic structural unit with *a priori* unknown moving internal boundary. For numerical solution of contact problem the variational theory and boundary elements technique are implemented. The optimal pore localisation is determined by iteration procedure. Aimed at rehabilitation effect, the activated implants from metastable poromaterials are proposed.

Key words: adaptation, biojoints, damping layer, poromaterial, compliance, contact pressure, boundary elements

Introduction

When developing promising designs, it is often useful to turn to natural objects because biostructures impress by their exclusive rational perfection. Remarkable advantages of biostructures in many instances are connected with the fact that these systems become adaptable, i.e. they acquire an ability to modify their properties in response to changes in outer conditions.

So, the high durability of locomotor system may be explained by adaptive damping reaction, which is possible, probably, due to controlling compliance of biotissues, in particular, due to resorption process. This reaction helps to effectively reduce peak contact stresses in vertebral column, hip and knee joints.

But in prosthetic designs, adaptive damping properties have been realised inadequately because the compliance of artificial materials and structures is constant for any traumatic action, as a rule. At present, only numerical modelling of adaptive damping reaction provides for the possibility to study the mechanism of adaptation carefully and to create more effective functional materials mentioned above.

This paper is devoted to the mezoscopic analysis of biotissue with porous structure that plays the role of an adaptive damping material. An individual pore is assumed as a structural unit of material and optimal pore localisation under static or quasi-static contact load is determined by iteration procedure. Using the space discretisation by boundary element method it is possible also to overcome limitations of rod model of low-density foams proposed in the first part [1] of the present study.

The numerical model of adaptive damping sheeting

Mechanical properties of existing and perspective types of prostheses are determined by material structure (Table 1). Ideal prostheses should imitate non-linear deformation behaviour of vital prototypes. Stress state of the simplest variant of damping implants made of homogeneous material under contact loading is heterogeneous and causes the irrational utilisation of material strength. Even fabrication of gradient materials ensures only static optimal structure. Most effective (perspective at present) metastable type of implant structure demonstrates adaptive damping reaction, i.e. reversible changing of local compliance based, for example, on multimodulus deformation behaviour [2]. Utilisation of metastable poromaterials also makes grounds for reversible changing of local compliance and, therefore, for creating dynamically optimal implants.

Table 1. Stress states of some generations of implants.

Type of structure	Stress state characteristics
Homogeneous	Heterogeneous
Layered	Interface stress concentration
Gradient	Statically optimal
Metastable	Dynamically optimal

For example, two biostructures related to locomotor system with porous damping sheeting (coating) are shown in Fig. 1. These are vertebras and intermediate elastic fibrous rings filled with viscous core, hip or knee head and cavity having quasi-elastic cartilage filled with synovial fluid. It must be clear that a compliance of intermediate damper element mentioned above is a dominant in comparison with practically underformed counterbody made of high-modulus bone material.

It is important to note the dependence of contact parameters versus pores localisation. Thus we have a possibility to control local compliance of sheeting.

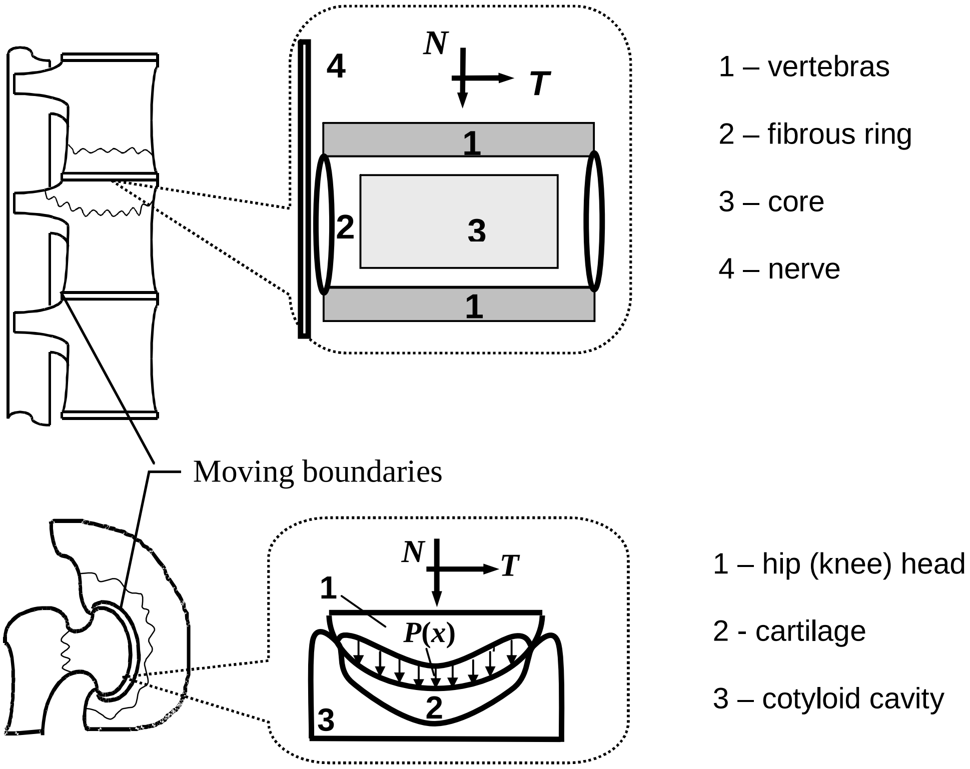

Fig. 1. The examples of biostructures with porous damper sheeting (vertebral disk, cartilage).

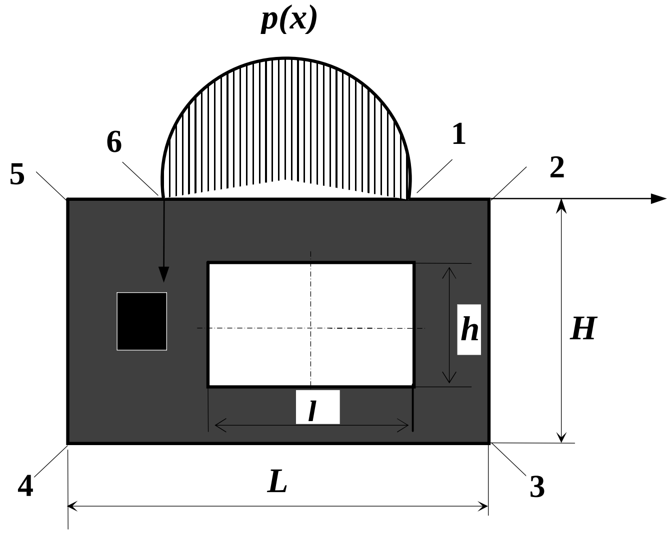

Fig. 2. Mezoelement of porous sheeting.

For simplification of numerical model we assume that a thick elastic porous layer between non-Hertzian (conformed) contacting surfaces may approximately reproduce deformation behaviour of both damper structures under normal loading. Contact interactions by shear loading in these biojoints are essentially different: strong adhesive bonds in case of vertebral column and low friction resistance to relative motion in joints. But we may easily consider these ultimate situations of tangential stress factor in terms of one model if describe the tangential resistance by very high and very low friction coefficient, respectively.

According to mezomechanical approach let us introduce a minimal fragment of poromaterial, namely, an individual pore as a finite length elastic layer (unit) with a cavity (Fig. 2). The scheme of loading illustrates compression of this mezoelement adhered to rigid foundation by another rigid body under normal (*N*) and tangential (*T*) loads. Pressure distribution *p(x)* may be calculated by solving contact problem for known values of elastic moduli, loads and prescribed geometry of interacting bodies.

It follows that expected redistribution of contact pressures on the surface and complience of poromaterial due to physical resorption process could be described in terms of localisation principle, i.e. optimal stabilisation of moving boundaries in time. Similar to other biostructures with moving boundaries [2], localisation result is not given *a priori*, but is determined in the process of numerical studies with the help of a certain criterion similar to well-known condition of attained full-strength of bone tissues.

Let us consider the numerical simulation of adaptive damping reaction as time-dependent process of transition from the initial non-uniform contact stress distribution to uniform one.

The theoretical possibility of adaptive damping reaction is based on the existence of relation between pressure distribution *p(x)* and geometry of poromaterial structure. This implicit relation is stated by the criterion: “pores localisation should minimise deviation between pressure distribution and its mean value”

$$min \int_{S_c} (p_0 - p(x))^2 \, dx, \qquad (1)$$

where p_0 is the mean value of pressure; S_c is the contact area.

It is important that mean value of pressure p_0 is *a priori* unknown in general case (excluding the joint with prescribed size of contact interface). But using the criteria (1), the dynamic configuration of porous structure may be calculated by iteration procedure.

Two commonly used boundary conditions are implemented to characterise the contact problem with friction [3]: characteristics of unilaterial contact when there is the constant sign of pressure

$$\gamma\ (x, u) = \psi(x) + v(x) - \delta_N(t) = 0, \quad p(x) > 0;$$
$$\gamma\ (x, u) > 0, \quad p(x) = 0\ , \tag{2}$$

(where $\psi(x)$ is the function of counterbody profile; $v(x)$ is the normal displacement of surface; δ_N is the gap function), and relation between tangential stress $\tau(x)$ and distributed frictional force *fp(x)* in accordance with Amonton's law, for example, in static form

$$|\tau| < f\,|p|, \quad u = 0;$$
$$|\tau| = f\,|p|, \quad u = -\lambda\tau,\ \lambda > 0 \tag{3}$$

(where f is the friction coefficient; u is the tangential surface displacement (microslip)).

For each step of minimisation (1), the contact problem may be formulated in terms of variational inequality theory developed by Duvaut and Lions [3]. The numerical solutions of similar problems are given in [4, 5].

It was shown in [3, 4] that the classic quasi-static formulation of contact problem with friction is equivalent to variational inequality solution or optimisation problem with limitations in the form of inequalities

$$J(u) = min\,max\left[0.5\left(a(w, w) - L(w) + j(w)\right)\right];$$
$$w \in K,\ p \le 0, \tau \le f\,|p| \tag{4}$$

where $a(w,w) = \int_\Omega a_{ijkl}\,\varepsilon_{kl}(w)\varepsilon_{ij}(w)\mathrm{dw}$ is a quadratic component of the functional; $L(w) = \int_{S_F} F\, w\,\mathrm{ds}$ is the linear one and $j(w) = \int_{S_C} [\,p\gamma\ (x,w)(w_T - u_T)]\mathrm{ds}$ is the non-linear one; w is a kinematically possible displacement; a_{ijkl} is the tensor of elasticity; ε_{ij} is the tensor of strain; F is the vector of external forces; $\gamma\ (x,w)$ is the function describing contact geometry; u_T is the exact solution for tangential displacement.

Thus, adaptive damping reaction modelling consists of sequence of solutions (4) for some cavity coordinates varying to satisfy condition (1).

The boundary element method implementation

There are two peculiarities of numerical modelling adaptive reaction under discussion: high gradient of contact stresses and time-dependent boundary conditions. For simulation of corresponding transient phenomenon, i.e. for compliance optimisation, we have developed the effective numerical procedure based on relaxation algorithm using space discretisation by the boundary element method techniques since this approximation gives an opportunity to solve similar problems with high accuracy and more economically than by the finite element method code [6].

Let us turn to the solution of Kelvin's problem for concentrated force in infinite region.

In case of plane strain, this solution is expressed by [6]

$$g(x, y) = -\frac{1}{4\pi(1-\nu)}\ln(x^2 + y^2)^{-0.5}, \tag{5}$$

where x, y are coordinates of the point under consideration; ν is Poisson's ratio.

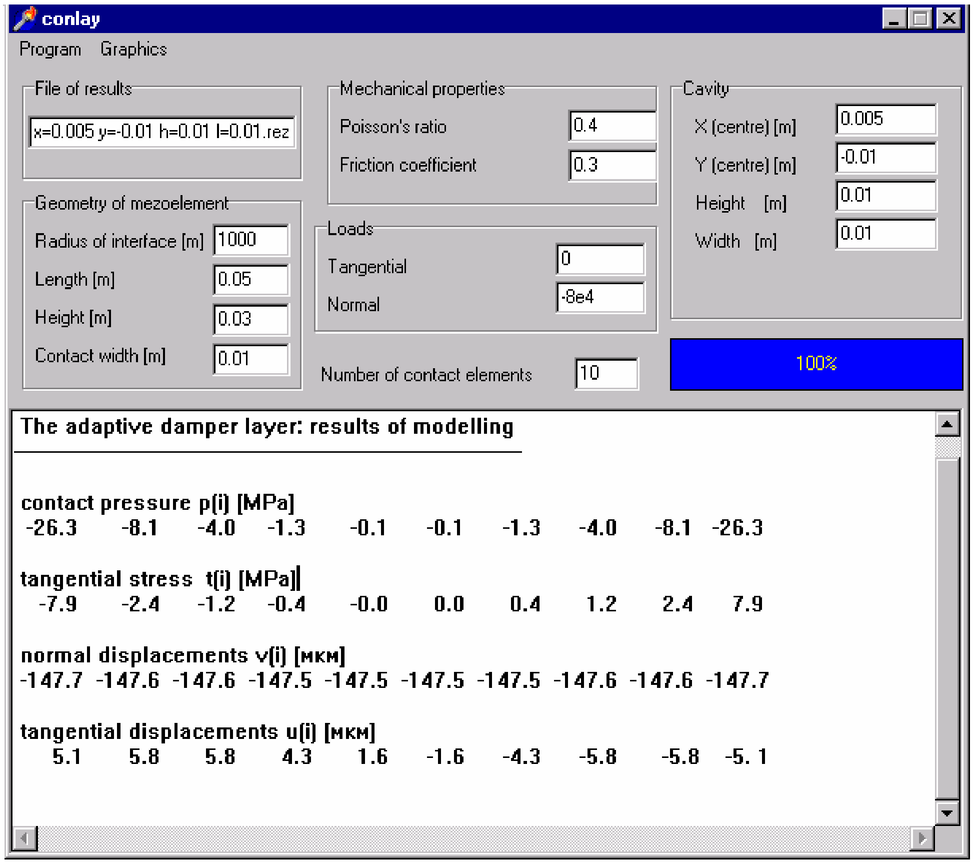

Fig. 3. Menu of computer program.

The displacements may be determined by the formulae

$$u_x = \frac{F_x}{2G}[(3-4\nu)g - xg_{,x}] + \frac{F_y}{2G}(-y\,g_{,x}), \tag{6}$$

$$u_y = \frac{F_x}{2G}(-xg_{,y}) + \frac{F_y}{2G}[(3-4\nu)g - y\,g_{,y}], \tag{7}$$

where F_x and F_y are force components; G is shear modulus.

For the boundary element method calculations of displacements a simple approximation has been implemented with partitioning of the surface into linear segments. Nodal displacements were determined by the superposition

$$\begin{Bmatrix} u_{T\,j} \\ u_{N\,j} \end{Bmatrix} = \begin{Bmatrix} K_{ij}^{\mathrm{TT}} & K_{ij}^{\mathrm{TN}} \\ K_{ij}^{\mathrm{NT}} & K_{ij}^{\mathrm{NN}} \end{Bmatrix} \begin{Bmatrix} \tau_i \\ p_j \end{Bmatrix}. \tag{8}$$

The coefficients in matrix equation (8) in case of elastic unit with finite sizes have been determined in terms of the boundary element method fictitious loads version using the expressions for stress and displacement of the boundary element with number i [6]

$$u_s^i = \sum_{j=1}^{M} B_{ss}^{ij} P_s^j + \sum_{j=1}^{M} B_{sn}^{ij} P_n^j, \quad u_n^i = \sum_{j=1}^{M} B_{ns}^{ij} P_s^j + \sum_{j=1}^{M} B_{nn}^{ij} P_n^j; \tag{9}$$

$$\sigma_s^i = \sum_{j=1}^{M} A_{ss}^{ij} P_s^j + \sum_{j=1}^{M} A_{sn}^{ij} P_n^j, \quad \sigma_n^i = \sum_{j=1}^{M} A_{ns}^{ij} P_s^j + \sum_{j=1}^{M} A_{nn}^{ij} P_n^j, \tag{10}$$

$$i = 1, \ldots, M;$$

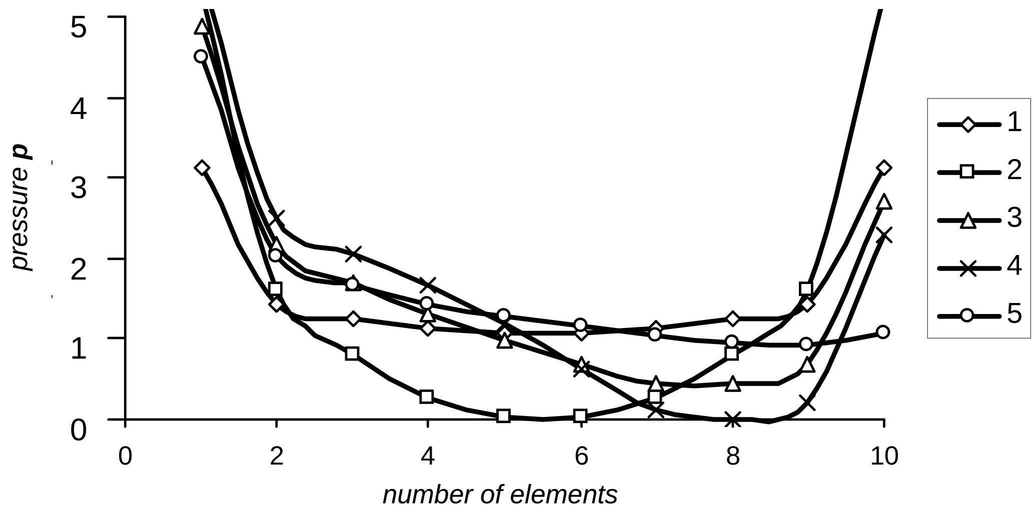

Fig. 4. Dimensionless contact pressure *p(x)* vs. cavity localisations.

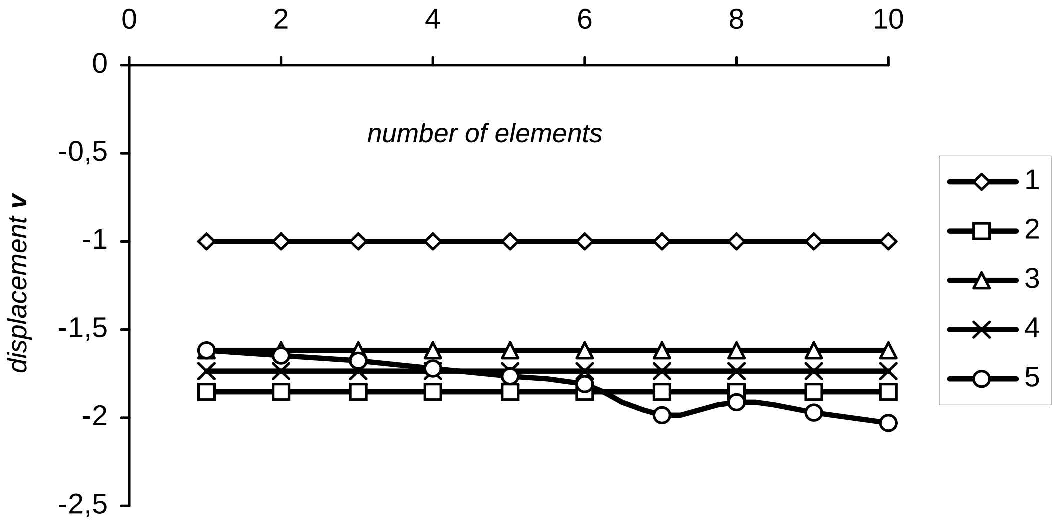

Fig. 5. Compliance of sheeting *v(x)* vs. cavity localisations.

where M is the number of boundary elements; $B_{ss}^{ij}, \ldots, A_{ss}^{ij}$ are influence coefficients obtained from singular solution of Kelvin's problem; P_s^j, P_n^j are tangential and normal components of fictitious (imaginary) loads.

Numerical results

Menu of the original computer program "BEL/CONLAY", which is given in Fig. 3, contains data and calculated values of normal and tangential components of stresses *p(i)*, $\tau(i)$ and displacements *v(i)*, *u(i)* at the interface for initial state of mezoelement when cavity is placed on symmetrical axis.

Test results were obtained for rectangular mezoelement (see Fig. 2) made of elastic material with Young's modulus $E = 1000$ MPa, Poisson's ratio $\nu = 0.4$ and friction coefficient f = 0.3 pressed by rigid plate with a large radius of curvature (approximately flat surface) in plane strain conditions. Length L and height H of mezoelement were 5 cm and 3 cm, respectively (Fig. 2). The data includes constant normal load N = 80 kN/m and static friction coefficient f_0 = 0.4. The range of discretisation of expected interface includes 10-100 boundary elements. Good convergence rate and accuracy have been achieved for 10 contact elements.

Variations of most important parameters – normal pressure and displacement distributions, which characterise local sheeting compliance for some moments of adaptive damping reaction are illustrated in Figs. 4, 5.

The initial state corresponds to contact deforming of mezoelement without cavity. For testing of procedure the optimisation has been performed only on the right side of mezoelement.

It was shown that at the finite stage of adaptation process (curves 5 of distributions mentioned above) effective decrease of stress concentration occurred near the right sharp edge of the counterbody in comparison with the similar zone near its left edge, where material adaptation was absent.

Conclusions

Estimation of compliance of high-density porous material under contact load may be realised by mezoscopic analysis. It was shown that numerical solution based on variational inequality theory and boundary element method demonstrated high efficiency.

Development of metastable poromaterials with moving boundaries of pores allows to improve damping characteristics of biomedical sheeting and coating, including the orthopaedic implants.

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АНАЛИЗ ПОДАТЛИВОСТИ ЗАЩИТНЫХ ПОКРЫТИЙ БИОМЕДИЦИНСКОГО НАЗНАЧЕНИЯ НА ОСНОВЕ ПОРОМАТЕРИАЛОВ. ЧАСТЬ 2

С.В. Шилько (Гомель, Беларусь)

Анализируется податливость демпфирующих биотканей и их гипотетических искусственных аналогов (прокладок, имплантатов) из адаптивного пороматериала. Адаптивная реакция обеспечивает минимизацию травмирующего контактного напряжения, вызванного статическим или квазистатическим нагружением человеческого тела. Отдельная ячейка пороматериала высокой плотности рассматривается как мезоскопическая структурная единица, имеющая *a priori* неизвестную подвижную внутреннюю границу. Для численного решения контактной задачи использованы вариационная теория и метод граничных элементов. Оптимальная локализация ячейки определяется при помощи итерационной процедуры. С целью эффективного протезирования предлагаются имплантаты из метастабильных пороматериалов. Библ. 6.

Ключевые слова: адаптация, демпфирующий слой, пороматериал, податливость, контактное давление, граничные элементы

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