Russian Journal of Biomechanics
www.biomech.ac.ru

COMPUTER SYNTHESIS OF COMPLEXLY COORDINATED HUMAN MOTIONS BASED ON ANTHROPOMORPHIC MODEL WITH NON-STATIONARY CONSTRAINTS

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Abstract. Some aspects of formulation and solution of anthropomorphic mechanism motion synthesis problem are considered. Necessity of differential-algebraic equations system employment in order to achieve desired quality of motion synthesis is provided. It is also shown that approach to motion synthesis based upon solution of mixed problem of dynamics of solid bodies constrained by parametric links system, allows to formulate goal-oriented motions optimization problem in the observation space. Besides, employment of balance relationships for kinematic and/or dynamic motion characteristics as constraints equations and parametric adjustment of mathematical model allows ensuring coincidence of measured and calculated characteristics of motion. In this approach, anthropomorphic mechanism motion synthesis problem is actually a basis for solution of a more general problem that is identification of mathematical model. In the framework of the proposed approach we show how computer synthesis of acrobatic motion in both support and supportless phases can be realized. Modeled kinetic relationships data are given.

Keywords: human motion modeling, dynamics of solid bodies system, goal-aimed motion, sports motions

General Points

Mathematical modeling of human and animal musculoskeletal apparatus is a universal basis for analysis, synthesis, development and design of new biomorphic mechanisms. Classic field of mathematical modeling employment is analysis of experimental kinematic and dynamic data, obtained through video registration of object's motion as well as from other independent additional sources (acceleration and force measuring devices, etc.). Analysis of experimental data gives kinematic and energy-force picture of motion. This one reflects real picture with certain accuracy. Adequacy criterion value usually depends on difference between values of motion characteristics obtained through modeling and through experiment. Presently capabilities of measuring devices are essentially growing due to computer controlled measuring systems. Therefore there arises a problem concerning value of experimental data sufficient for adequate modeling. This question retains its importance despite intense development of measuring devices systems proposed by leading manufacturers (e.g. Ariel Dynamics Inc. (http://www.arielnet.com/), Oxford Metrix Ltd. (http://www.vicon.com)).

In papers [1-3] there have been considered difficulties in employment experiment data analysis results for motion computer synthesis and suggested ways to cope with them. It should be noted that regular approach to the adequacy criterion problem formulation and choice of mathematical model with minimal number of parameters, satisfying this criterion, could be realized only through a reliable procedure of synthesis of complexly coordinated motion. At the same time possibility to synthesize anthropomorphic mechanism motion with required kinematic and dynamic characteristics yields all data necessary for testing of analysis problem or to be more correct (taking into account that structure of model is varying) for the problem of mathematical model identification. And finally, only if one possesses a reliable algorithm of motion synthesis he can consider the problem of minimal volume of additional measurements (alongside with video registration) required for development of such a mathematical model that would adequately describe real motion of musculoskeletal apparatus.

In this paper, the main attention is paid to the technique of human musculoskeletal apparatus motion modeling which allows to structure control action so that motion goal would be formalized in the space of geometric and kinematic characteristics and used directly as a control factor. In the simplest case programmed motions are assigned. In general case of mathematical model the system of differential-algebraic equations is needed [2, 4, 5].

The problems of control actions structuring in biomechanics directly parallel with principles of control actions formation in problems of solid bodies systems dynamics [2]. The necessity of consideration of these questions is quite acute because musculoskeletal apparatus motion synthesis requires that constraints upon control be not only satisfied but also mutually coordinated and realizable via only internal (muscular, for example) action [6, 7].

Main Equations and Model Features

System of differential-algebraic equations of a system of solid bodies constrained by geometric and kinematic non-stationary links can be put down as follows:

$$\begin{aligned} &A(q)\ddot{q} = B(q,\dot{q},t) + U(q,\dot{q},t), \\ &F(q,t) = 0, \\ &G(q,\dot{q},t) = 0, \\ &H(q,\dot{q},\ddot{q},t) = 0, \end{aligned} \tag{1}$$

where $q = (q_1,\ldots,q_n)^T$ is a column of generalized coordinates; $\dot{q}, \ddot{q}$ are its derivatives with respect to time; $A(q)$ is $(n\times n)$ square matrix of quadratic form of kinetic energy; $B(q,\dot{q},t)$ is $(n\times 1)$ column of inertial elements; $F(q,t), G(q,\dot{q},t), H(q,\dot{q},\ddot{q},t)$ ($H''_{\ddot{q}\ddot{q}^T} \equiv 0$) are $(m\times 1), (l\times 1), (k\times 1)$ columns, comprised from constraints equations; $U(q,\dot{q},t)$ is $(n\times 1)$ column of generalized forces, which formation method defines the main possibilities of motion synthesis.

Let $U(q,\dot{q},t)$ be a sum of three terms:

$$U(q,\dot{q},t) = U_1(t) + U_2(q,\dot{q}) - P'\lambda(q,\dot{q},t)\,. \tag{2}$$

Here $U_1(t)$ is generalized control, defined a priori. The second term $U_2(q,\dot{q})$ describes passive visco-elastic properties of joints muscles and tendons as well as deformation of bodies-superelements incorporated in the model. As for superelement presentation of the musculoskeletal apparatus, we consider it to be a system of solid bodies [4], but employ principle of two-level modeling. Each superelement is the system of solid bodies forming a kinematic link of certain form [6]. Superelement modeling simplifies process of initial model data determination, allows employment of parametric control if deformations in internal superelement "joints" are relatively small. Besides, in the framework of the same model of solid bodies system one can model elastic behavior of superelements due to visco-elastic description of internal joints properties.

Non-linear function $U_2(q,\dot{q})$ is introduced in order to describe visco-elastic properties of generalized joints:

$$U_{2k}(q,\dot{q}) = -\beta_k \dot{q}_k - A_k \tan\,(d_{1k} q_k + d_{2k}) - d_{3k}\,, \quad k = \overline{1,k}\,. \tag{3}$$

Correct adjustment of values of coefficients $\beta_k, A_k, d_{1k}, d_{2k}, d_{3k}$ [2, 6] allows to imitate restriction on joints mobility or to assign desired position of anthropomorphic mechanism. Besides, relationship (3) allows us to describe model visco-elastic contact with external objects and obstacles, assigned by a system of inequalities. In particular, human musculoskeletal apparatus contact with non-linear visco-elastic support is presented by external forces, acting upon anthropomorphic mechanism if certain set of anthropomorphic mechanism points "makes attempts" to cross support level border. If one uses superelement joints coordinates as such a set, then character of contact changes with respect to changes in structure, shape and internal visco-elastic properties of superelement components. One of the simplest implementations of superelement modeling is, for example, description of contact of elastic rod of given shape with visco-elastic support.

The term $P'\lambda(q,\dot{q},t)$ is generally speaking an algorithmic function of $q, \dot{q}$ and t which can be calculated through solution of the following system of linear algebraic equations:

$$(P'^T A^{-1} P')\lambda = P'^T A^{-1}(B + U_1 + U_2) + D(q,\dot{q},t)\,, \tag{4}$$

where $P' = (F'_q, G'_{\dot{q}}, H'_{\ddot{q}})^T$ is $n \times (m + l + k)$ Jacobi's matrix of constraints equations, $\lambda = (\lambda_1, \ldots, \lambda_{m+l+k})^T$ is a column of Lagrange coefficients. The column $D(q,\dot{q},t)$ has sizes $(m + l + k) \times 1$ and is obtained through differentiation of constraints equations with respect to time i.e. $D(q,\dot{q},t) = (F'_q \ddot{q} - \ddot{F}, G'_{\dot{q}} \ddot{q} - \dot{G}, H'_{\ddot{q}} \ddot{q} - H)$.

Solution of the system (4) via the modernized Gauss method allows to take into account linearly dependent constraints, refraining by determination of Lagrange coefficients $\lambda = (\lambda_1, \ldots, \lambda_r)^T$, where $r \le (l + m + k)$, if the rank of expanded matrix of the system (4) is equal to r. Additive structure of the term pertaining to constraints in relationship (2) for $U(q,\dot{q},t)$ makes possible to put the rest of elements of column $\lambda_i, i = \overline{r+1, m+l+k}$ to be equal to zero.

When the non-stationary constraints equations are considered then D from (4) can be divided in two terms:

$$D(q,\dot{q},t) = D_1(q,\dot{q},t) + D_2(t)\,, \tag{5}$$

where $D_2(t)$ corresponds to program motion with respect to separate freedom degrees and their combinations aimed at achieving certain program of motion.

Modeling of prescribed motion with help of non-stationary constraints equations actually allows to formulate synthesis problem in geometric and kinematic space, which is natural for human being. From analytical mechanics point of view there is need of a mixed problem of dynamics solution. Most universal approach to solution of this problem is to put down motion program as a system of constraints equations and then to solve differential-algebraic system of equations. Human musculoskeletal apparatus motion feature is that its translation in environment is carried out due to only "internal" controlled motions. Therefore synthesis of programmed motion requires to take into account not only kinematic constraints but also requirement that the relations of general theorems of dynamics of the system be satisfied.

Constraints which are reflected by relationship $H(q,\dot{q},\ddot{q},t) = 0$ can appear as result of traditional differentiation of geometric and kinematic constraints equations. Other sources of constraints equations linearly depending on $\ddot{q}$, are theorems of center of mass motion and of momentum variation of the whole bodies system as well as of any subset of its elements. Imposition of such constraints allows to obtain synthesized motion with desired characteristics without changing in the system of generalized coordinates.

Let a compatible system of constraints equations which assign programmed motion be

$$Q'\ddot{q} + E = 0 \tag{6}$$

and conditions imposed upon control actions R by relationships of general theorems of dynamics of the whole system and its subsystems be

$$L(A(q)\ddot{q} - (B(q,\dot{q},t) + U(q,\dot{q},t)) = R, \tag{7}$$

where L is square matrix, R is column of linear combinations of generalized forces.

Then the system of motion equations and constraints equations is as follows:

$$\begin{aligned} A(q)\ddot{q} = B(q,\dot{q},t) + U_1(t) + U_2(q,\dot{q},t) - P'\lambda(q,\dot{q},t) + \\ + AL^T(LAL^T)^{-1}R - (I - AL^T(LAL^T)^{-1}L)Q'^T\mu, \end{aligned} \tag{8}$$

$$P'\ddot{q} + D = 0, \tag{9}$$

$$Q'\ddot{q} + E = 0. \tag{10}$$

This approach to synthesis of supportless phases of motion which adds to constraints equations system theorems of center mass motion and momentum variation allows to find among possible realizations of desired kinematics one which can be carried out by kinematic chain due to only "internal" control.

The non-stationary term $D_2(t)$ in typical constraints equations allows to model desired behavior of support reaction in single support phase or/and desired relative motion of certain musculoskeletal apparatus points with its possible dynamic correction, i.e. motion goal correction at current moment of time. It is convenient to use typical relationships in order to synthesize motion satisfying conditions on position and velocity at the beginning and at the end of certain span $[0,\tau]$. We used as typical relationships the following polynomial of the forth order:

$$\begin{aligned} x(t) = x(\xi\tau) = x_0(1 - \xi^2 + 2\xi^2(1-\xi) + 3\xi^2(1-\xi)^2) + \\ + x_\tau\xi^2(1 + 2\xi^2(1-\xi) - 3(1-\xi)^2) + \\ + \tau\dot{x}_0((1-\xi)\xi - \xi^2(1-\xi) + 2\xi\ (1-\xi)^2) + \\ + \tau\dot{x}_\tau\xi^3(1-\xi) + \\ + \frac{\tau^2}{2}\ddot{x}_0\xi^2(1-\xi)^2, \qquad 0 \le \xi \le 1. \end{aligned} \tag{11}$$

System of parameters $(x_0, x_\tau, \dot{x}_0, \dot{x}_\tau, \ddot{x}_0)$ adjustment allows to model start-stop trajectories at consequent time spans, satisfying conditions of continuity of coordinates, velocities and accelerations as well as complex coordination motions (with desired coordinates and velocities at the ends of spans and continuous at points of division). As generalization of considered typical relationships one can take polynomial spline-approximations. However, it should be taken into account that growth of constraints equations number leads to essential increase in the volume of data necessary for problem solution [10].

Another type of non-stationarity which can be used in description of non-linear behavior of support-reaction or/and control torques at certain time span $[0,\tau]$ can be presented as follows:

$$\begin{aligned} R_x(\xi\tau) = A_x\ A_1 \sin(\alpha_x 2\pi\xi), \\ R_y(\xi\tau) = A_1((4\xi(1-\xi)^2)^{\alpha_1} - A_2(4\xi(1-\xi)^2)^{\alpha_2}), \end{aligned} \tag{12}$$

where different combinations of $A_1, A_x, \alpha_1, \alpha_2, \alpha_x$ allow to vary support reaction behavior. In particular, two-peak character of vertical component of support reaction R, for walking, jumping and running can be modeled. If measurements of modeled variable are taken at certain set of trajectory points, then spline-approximation or non-linear regression (e.g. according to the least squares method) should be employed. Suggested approach to motion synthesis does not require presetting of generalized forces values, for example, interelement torques directly, which makes possible to increase number of uncontrollable freedom degrees and easily vary structure of musculoskeletal apparatus model. This gives researcher a tool for formation of testing examples in order for identification algorithm adequate choice in analysis problem.

Let us consider more closely one essential aspect of constraints equations realization. Apparently that even relatively simple systems of constraints equations can be either incompatible or degenerate. Degeneracy of the system of constraints equations can, for example, result from linear dependence of imposed constraints equations. One of possibilities of solving such a problem has been considered above. Much more difficulties arise when we have to deal with incompatibility of constraints equations both one with another and with kinematic capabilities of mathematical model. For example, such situation appears when the constraint equation prescribes the position of a joint out of the region of workspace and it cannot be precalculated by any way. This brings us to the necessity of including into the program software of additional blocks of analysis of constraints equations with automatic "switching them off" in case of they cannot be realized.

As a partial sign of incompatibility of certain constraints equation, there can serve a phenomenon of decrease of integration step to its minimum during procedure of numerical integration of motion equations along with the constraints equation. Let us note that geometrical incompatibility of constraints equations leads to impossibility of numerical integration procedure ("dead cycle"). Dynamic incompatibility shows itself in abrupt changes of velocities and accelerations values and practically can only be evaluated by periodical verification of their marginal values, especially during considerable slowing down of the numerical integration procedure work process.

Specific cases of geometric and dynamic incompatibilities are due to presentation of constraints equations directly in differentiated form, for example, as motion equations, corresponding to general theorems of dynamics with respect to the whole system of bodies and/or part of them. In this case, as incompatibility criterion there can be taken anthropomorphic mechanism energy losses, needed for realization of external forces and their torques changes, taking into account current configuration and distribution of velocities of anthropomorphic mechanism elements.

Motions with preset kinematics can be to a great extent comprised from prescribed motions for every freedom degree by varying of parameters of visco-elastic properties of joints.

Possibility of such varying depends essentially on the number of extra freedom degrees in comparison with the number of simultaneously acting constraints equations. Along with parametric correction there can be used another one, based on the local variation method, when in the right parts of motion equations there are introduced special time functions. However, for a big number of freedom degrees such method of motion control is very cumbersome.

Synthesis of a 4-Cycle Somersault on the Acrobatic Ban

The suggested above approach to the synthesis of complicated motions of anthropomorphic mechanism can be illustrated on the example of synthesis of a 4-cycle backward somersault in a grouped position after pushing off from an acrobatic ban (platform). Data for model formation and evaluation of initial conditions and rhythm of the motion being synthesized had been received by results of processing of a two-plane video record of the pushing off and free-fall phases of a triple backward somersault. Transferring the video-record into "avi" file allowed to get the frequency of 50 frames per second with discretion of the monitor screen of 72 dpi. The part of the video-row, corresponding to the pushing-off phase before the finishing combination of somersaults, is presented in Fig. 1.

This data allowed to calculate distribution of initial conditions for coordinates and generalized velocities at the moment of pushing off the platform, evaluate periods of support and free-fall phases and the value of kinetic moment. There had also been received evaluations of initial configuration and amplitudes of joints motion.

For evaluation of the interaction with the support, namely coefficients of non-linear visco-elastic interaction, except for the video-data there had been used data of acrobatic platforms certification given in [11].

In Fig. 2 there is presented kinematic scheme of anthropomorphic mechanism with numeration of its elements. First five elements were used for relatively detailed modeling of visco-elastic foot arch, contacting at certain points with visco-elastic surface.

Modeled motion of the foot fragment during support phase is pictured in Fig. 3. Time dependence of forces in control points are given in Fig. 4, vertical displacement of points - in Fig. 5. In Fig. 3 one can see relative deformation of the foot arch and trajectories of motion of contact points (heal, toe, two almost coinciding points on the foot surface). We understand that this model is not fully adequate, but it does model (although in a simple way) distributed force of interaction with the surface. Kinematics of the foot arch of the model is close to that of the real one and quite essentially influences distribution of rotation and translation energy at the moment of pushing off the platform.

Motions synthesized for the support phase and by means of non-stationary constraints equations for the free-fall phase comprised the full picture of 4-cycle backward somersault. Synthesized motions kinematics is presented in Fig. 6.

It is essential that with respect to realizing synthesized motion that during the support phase a central momentum decreases (Fig. 7), but it still proves to be enough for performance of a 4-cycle backwards somersault for a relatively not very tightly grouped position, Fig. 6.

Time period of the free-fall phase is determined by value of the vertical component of the center of mass velocity at the pushing off moment (Fig. 8).

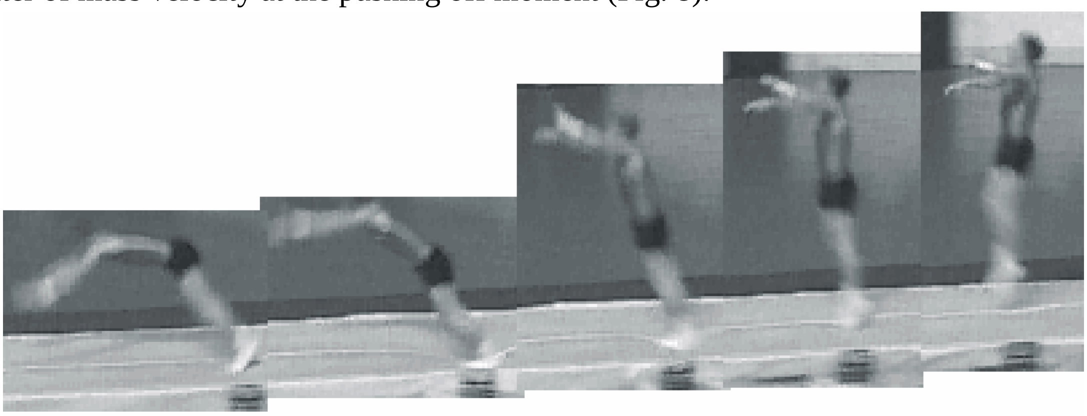

Fig. 1. Fragment of the video-film of exercises on the acrobatic platform.

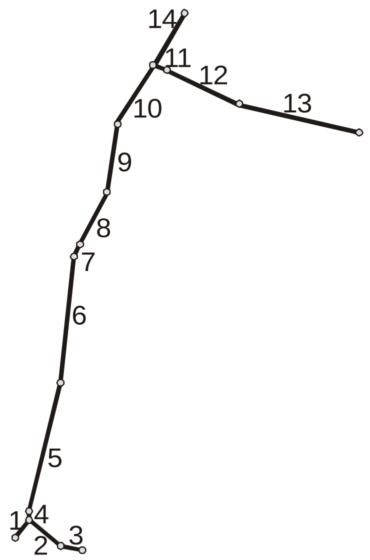

Fig. 2. Kinematic scheme of 14-element anthropomorphic mechanism for modeling of acrobatic exercise.

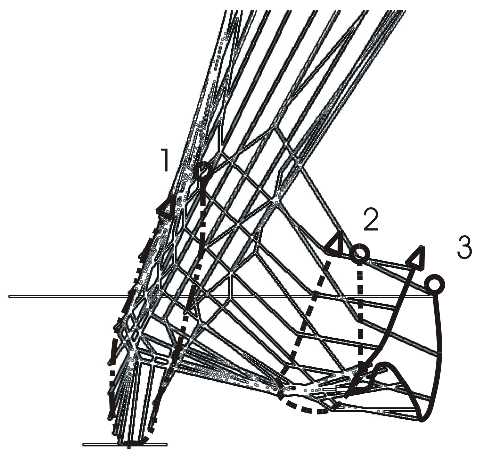

Fig. 3. Trajectory of the foots points motion in the interaction with the acrobatic platform model (1 – heel; 2 – metetersus; 3 – toe).

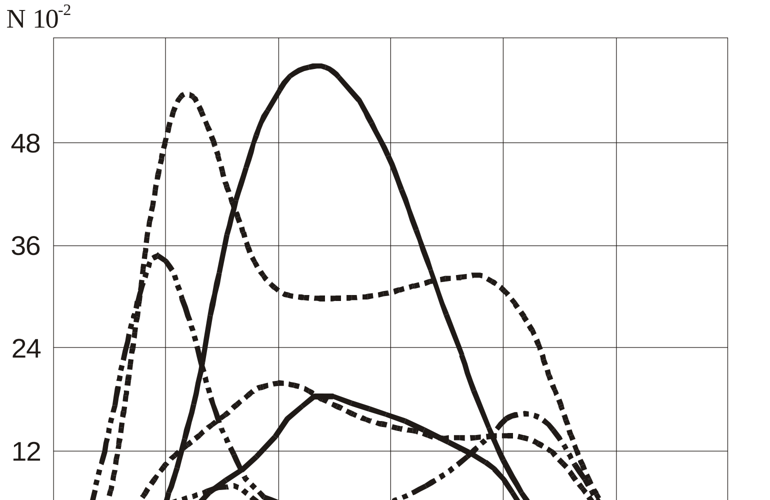

Fig. 4. Time dependence of reaction forces of the foot contact points with the elastic acrobatic platform (1 – heel; 2 – metatarsus; 3 – toe).

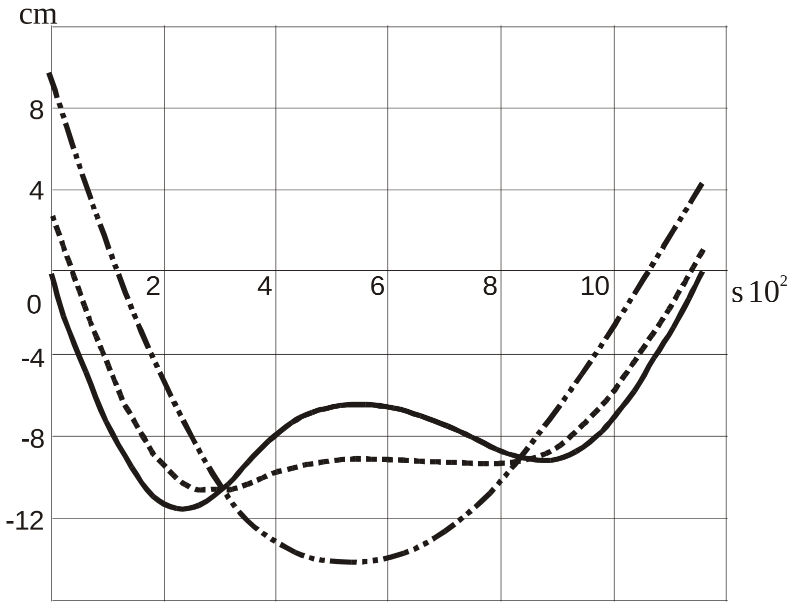

Fig. 5. Time dependence of vertical displacements of foot contact points as result of interaction with the model acrobatic platform (1 – heal; 2 – metatarsus; 3 – toe).

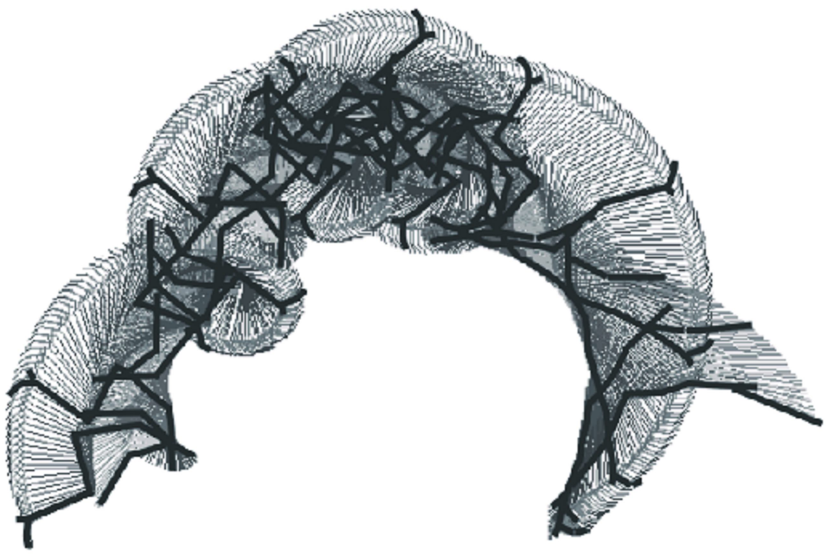

Fig. 6. 4-cycle backward grouped somersault (synthesized kinematics).

Except for contact forces of interaction of the foot and the surface our model allows to give realistic evaluation of interelement torques, corresponding to the integral value of the reaction force, which can be easily compared with available experimental data [12]. Given in Fig. 9 graphics of interelement torques as functions of time are quite close, with respect to amplitude-phasic behavior, to the principle of evenly distributed power, which presents a natural condition for real motions fulfilled under conditions of finite values of power of muscle forces. Positive values of torques at the ankle (curve 6, Fig. 9) and the hip (curve 8, Fig. 9) joints correspond to outstretching. At the knee joint we observe grouping. Calculation results show, for example, importance of essential stress of flexing muscles of the knee joint for providing of necessary amplitude of motion at the hip joint. Behavior of the interelement torque at the ankle joint allows to evaluate needed stress in the calf muscle.

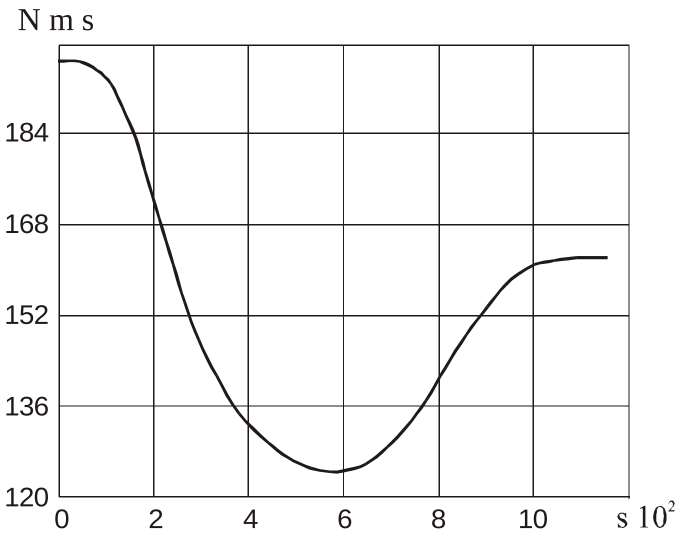

Fig. 7. Central momentum as function of time (support phase).

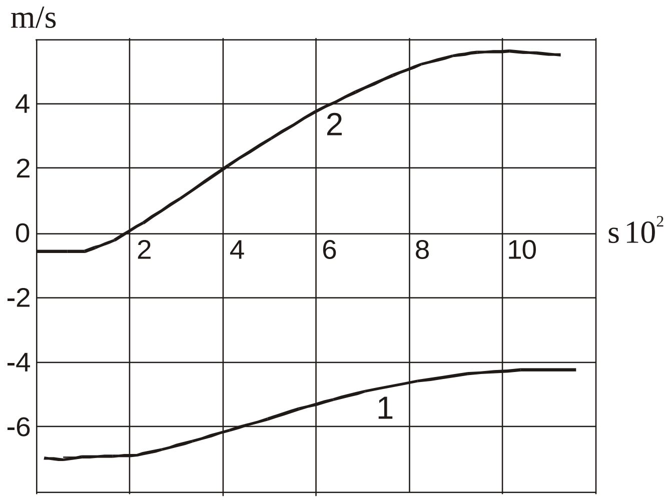

Fig. 8. Components of the center of mass velocity as functions of time (support phase) 1 – horizontal component, 2 – vertical component.

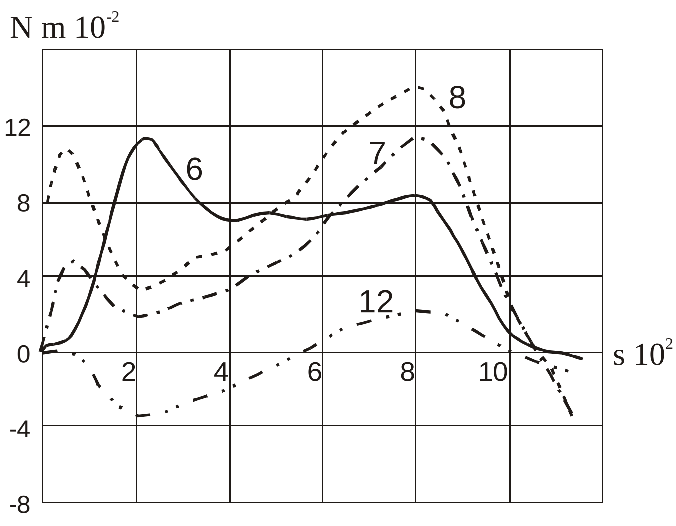

Fig. 9. Torques at the joints (6 – angle, 7 – knee, 8 – hip, 12 – shoulder).

Conclusions

Synthesis of anthropomorphic motion of human musculoskeletal apparatus model can be effectively carried out by implementation of approaches described in this article. Realization of typical constraints equations in the mathematical model (as considered above) provides for the most possible applications of the mathematical model to adequate modeling of human musculoskeletal apparatus motion [8]. Technology of such modeling implies multilevel superelement structure of the model with the following options

- parametric description of distributed visco-elastic properties for the joints;
- limitation of the region of displacements of certain points (with realization through feedback with respect to displacements and velocities under condition when such point gets on the border of the region);
- parametric equations of geometric, kinematic and force constraints.

It should also be noted that among different possibilities of mathematical model parameters variation the constraints equations allow to get the most effective results with respect to synthesis of new motions as well as for adequate modeling of real motions, since adequacy criterion implies additional dimensions which can, in fact, be used for constraints equations. Presented above approach to synthesis of motions on the basis of non-stationary constraints equations, which pre-determine the anthropomorphic mechanism motion, and stationary items (3), added to generalized forces, which provide for kinematic constraints with respect to every generalized coordinate for corresponding choice of parameters, allow to synthesize a wide range of anthropomorphic mechanism motions. Application of superelement structuring of certain elements makes easier the process of anthropomorphic mechanism structure variation. This approach to the problem of motion synthesis, based on solution of a mixed problem of dynamics of a system of solid bodies, restricted by parametric constraints, allows to formulate problems of optimization of goal-aimed motions.

The considered problem of synthesis of complex motion of a sportsman gives a clear illustration of capabilities of suggested algorithms and their effectiveness for modeling of real motions and evaluation of their energy-force characteristics.

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КОМПЬЮТЕРНЫЙ СИНТЕЗ СЛОЖНО-КООРДИНАЦИОННЫХ ДВИЖЕНИЙ ЧЕЛОВЕКА НА ОСНОВЕ АНТРОПОМОРФНОЙ МОДЕЛИ С НЕСТАЦИОНАРНЫМИ СВЯЗЯМИ

А.В. Зинковский, А.А. Иванов, В.А. Шолуха (Санкт-Петербург, Россия)

Рассмотрены особенности постановки и решения задачи синтеза движений антропоморфных механизмов. Обоснована необходимость использования системы дифференциально-алгебраических уравнений при синтезе движения с желаемыми качеством. Показано, что основанный на решении смешанной задачи динамики системы твердых тел, стесненных параметризованными связями, подход к синтезу движений позволяет формулировать задачи оптимизации целенаправленных движений в пространстве наблюдений. Кроме того, использование в качестве уравнений связей балансовых соотношений для кинематических и/или динамических характеристик движения и параметрической настройки математической модели позволяет обеспечить совпадение расчетных и измеренных зависимостей. Решаемая в такой постановке задача синтеза движений антропоморфного механизма фактически становится базисным элементом более общей задачи идентификации математической модели. В рамках обсуждаемого подхода проведен компьютерный синтез акробатического движения как в опорной, так и в полетной фазах, приведены расчитанные кинетические зависимости. Библ. 12.

Ключевые слова: моделирование движений человека, динамика системы твердых тел, целенаправленное движение, спортивные движения

Received 7 October 2000