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MATHEMATICAL MODELING IN LAMINAR DENTURE OPTIMIZATION

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Abstract: Dentofacial system with a laminar denture cannot function in the same way as with natural teeth. The use of the denture has harmful effects on the condition of denture-supporting tissues. By means of mathematical modeling an optimal construction of the removable laminar maxillary denture was found for some denture materials. Also the possibility of denture detachment while eating was investigated and it was found to be impossible. The investigations were performed for mastication and biting. In addition the stochastic optimization problem taking into account the statistical distribution of some mucosa characteristics was formulated and solved.

Key words: removable laminar denture, mucosa, optimal design, stochastic optimization

Introduction

One of prevailing ways to restore masticatory ability of an edentulous jaw is the use of a complete laminar denture. The appearance of a removable laminar maxillary denture is shown in Figure 1. However the use of such a denture results in many problems. Pathologic changes of the prosthetic bed mucosa and bone resorption under the denture are the most essential ones [3, 4, 7]. These adverse effects take place owing to irregular distribution of the masticatory load. During mastication the denture basis distributes load applied to artificial teeth over the prosthetic bed mucosa. But the supporting function is not natural for the mucosa. Mucosa compression causes ischemia of the mucosa tissues and then bone resorption. Moreover a low pain threshold of the mucosa restricts the masticatory load magnitude. As a result patients are forced to exclude hard food from their ration.

There is another serious problem. The denture basis can come off the prosthetic bed. Three conditions are needed to provide denture fixation in the patient’s mouth. Namely, a thin saliva layer between the denture basis and the mucosa, high correspondence between the prosthetic bed relief and the denture shape, and dynamic suction. An appropriate denture construction provides first two conditions. The third condition must be tested.

In our previous work [2] we created a mathematical model of the removable laminar maxillary denture together with the prosthetic bed mucosa. By means of the model the optimal values of the denture basis thickness for some basis materials were determined. Also dependence of the optimal basis thickness on elastic properties of the basis material was investigated.

In writing this paper we had three goals in mind. First we are going to find an optimal construction of the removable laminar maxillary denture. This optimal construction has to magnify the masticatory load without pain and prevent bone resorption. Second we examine a possibility of denture detachment during eating. Finally we present an attempt to take into account stochastic nature of some mucosa characteristics.

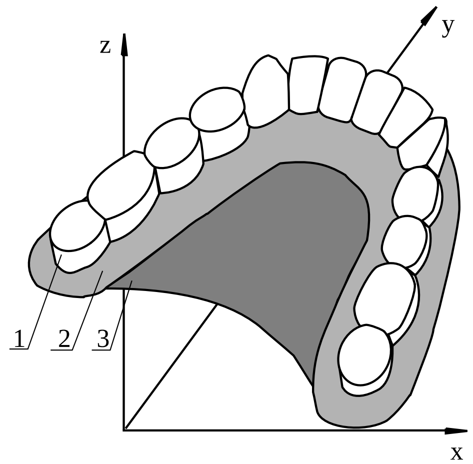


Fig. 1. The appearance of a removable laminar maxillary denture: 1 – artificial teeth (made from plastic), 2 – the external part of the denture (made from plastic), 3 – the internal part of the denture (made from metal or plastic).

Materials and methods

Before we start out we need to define some relative values.

The relative pressure in a point $\bar{x}$ (when $p(\bar{x}) \le 0$) and the specific detachment force (when $p(\bar{x}) > 0$):

$$p_o(\bar{x}) = \begin{cases} \dfrac{p(\bar{x})}{p^{th}(\bar{x})} \cdot 100\%, & p(\bar{x}) \le 0, \\ \dfrac{p(\bar{x})}{p^{ad}(\bar{x})} \cdot 100\%, & p(\bar{x}) > 0, \end{cases} \tag{1}$$

where p^{th} is the pain threshold pressure, $p^{ad} = 0.1\,MPa$ is the adhesion force [5].

The relative masticatory force:

$$F_o = \frac{F}{F_{av}} \cdot 100\%, \tag{2}$$

and the relative threshold masticatory force:

$$F_o^{th} = \frac{F^{th}}{F_{av}} \cdot 100\%, \tag{3}$$

where F is the masticatory force; F^{th} is the threshold force (when a patient feels pain); F_{av} is the average force value: $F_{av} = 150\ N$ for mastication, $F_{av} = 20\ N$ for biting [1, 5].

At this point it will be useful to introduce some terminology. The inner layer of the denture basis is said to be the internal part of the denture; the outer layer of the denture basis with artificial teeth is spoken of as the external part of the denture (Fig. 1).

Relative equivalent stresses in the internal and external parts of the denture:

$$\sigma_o^{int}(\bar{x}) = \frac{\sigma_e^{int}(\bar{x})}{\sigma_{-1}^{int}} \cdot 100\%, \tag{4}$$

$$\sigma_o^{ext}(\bar{x}) = \frac{\sigma_e^{ext}(\bar{x})}{\sigma_{-1}^{ext}} \cdot 100\%, \tag{5}$$

where $\sigma_e^{int}(\bar{x})$ and $\sigma_e^{ext}(\bar{x})$ are equivalent stresses in a point $\bar{x}$ of the internal and external parts, respectively; σ_{-1}^{int} and σ_{-1}^{ext} are the fatigue limits for the internal and external part materials, respectively.

After these preliminary remarks, we can formulate an optimization problem. As the objective function we choose the relative threshold force F_o^{th}. It defines restoration of masticatory ability. The threshold force can be magnified only by more uniform distribution of the relative pressure. Hence F_o^{th} also describes a degree of bone resorption. The internal part thickness t was selected as a variable parameter. Here we assume this thickness to be constant.

The optimization problem is to maximize the relative threshold force within restrictions on fatigue strength (8)-(9) and on the internal part thickness (10):

$$F_o \to max, \tag{6}$$

$$\max_{\bar{x}\in S_p} p_o(\bar{x}) = 100\% , \tag{7}$$

$$\max_{\bar{x}\in V_{int}} \sigma_o^{int}(\bar{x}) < 100\% , \tag{8}$$

$$\max_{\bar{x}\in V_{ext}} \sigma_o^{ext}(\bar{x}) < 100\% , \tag{9}$$

$$0 < t \le 2\ mm , \tag{10}$$

where S_p is the prosthetic bed area; V_{int} and V_{ext} are domains occupied by the internal and external parts, respectively. In the above system maximizing the relative force with restriction (7) is equivalent to maximizing the relative threshold force. We use restriction (10) because the denture with too thick basis makes speech unclear.

In order to determine pressure on the mucosa and stresses in the denture we must solve the boundary-value problem of elasticity. This problem has the following boundary conditions. The adhesion condition is given on the area where the mucosa contacts with bone tissue. A distributed load is preassigned on the contact area of the artificial teeth with the antagonistic teeth. The remaining area considered as a free surface.

To solve the problem we developed a mathematical model of the laminar maxillary denture with the prosthetic bed mucosa [2]. In accord with the model we consider the denture basis with the prosthetic bed mucosa as a two-ply elastic shell on an elastic layer, covering a rigid foundation (bone). Artificial teeth are considered as an elastic beam with a variable cross-section: triangle for incisors and canines and rectangle for premolars and molars. The appearance of the model is shown in Figure 2.

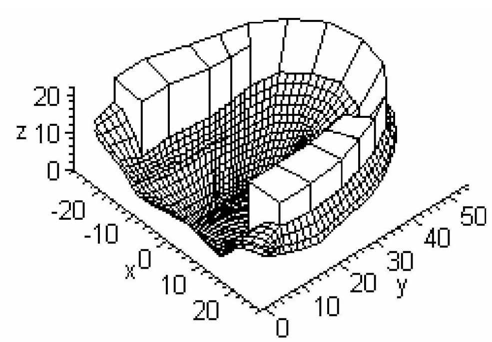


Fig. 2. The appearance of the denture model.

To determine pressure on the mucosa we use the Winckler's law [6]. Computations were provided by the finite element method. To solve the optimization problem we use an algorithm based on linearity of the elasticity problem [2].

Above we have described the determinate optimization problem. But the elastic properties of denture materials, the thickness and the pain threshold pressure of the prosthetic bed mucosa, the denture configuration and the applied masticatory force are stochastic values. So if we use mean values to solve the optimization problem, we might obtain practically useless results (since deviation of model characteristics from their mean values can essentially change the optimal solution). That is why it is important to solve a stochastic problem of denture optimization.

Before we go into this problem, we need to find values producing the greatest effect on the optimal denture thickness. According to our study, the mucosa thickness t_m and the pain threshold pressure P_{cr} are these values. We consider these values having the Gaussian distribution with the statistical expectation and dispersion, specified from an experiment [9].

For our present purposes it is essential to introduce some definitions. Let ω be a simple event, namely – the denture construction with specified values for all initial parameters. A variable parameter of the optimization problem is the internal part thickness $t(\omega)$. A stochastic function $q(t(\omega),\omega)$ is the relative force F_o : $q(t(\omega),\omega) = F_o$.

Now we can formulate the stochastic optimization problem as follows: to define the internal part thickness $t(\omega)$, maximizing the objective function $F(t(\omega),\omega)$:

$$F(t(\omega),\omega) \to max \tag{11}$$

within determinate restrictions (7)-(10).

We choose the objective function according to our task. First of all it is interesting to explore the maximal average value of the relative force. Here the objective function is the statistical expectation of the relative force

$$F(t(\omega),\omega) = \int_{\mathbf{\Omega}} q(t(\omega),\omega)\mathbf{P}(d\omega) = M\, q(t(\omega),\omega), \tag{12}$$

where $M\,q(t(\omega),\omega)$ is the statistical expectation of the stochastic function $q(t(\omega),\omega)$, $\mathbf{\Omega}$ is an event set with a probability $\mathbf{P}$. This model is named M-model [8]. M-model determines the optimal thickness corresponding to the maximum of the average relative force. It resembles the determinate optimization problem more than any other.

Also it is appropriate to investigate the maximal relative force with the minimal spread in values. Here the objective function is a combination of the statistical expectation and the dispersion

$$F(t(\omega),\omega) = \lambda\left|M\, q(t(\omega),\omega)\right| - (1-\lambda)\sqrt{D\,q(t(\omega),\omega)}, \tag{13}$$

where $\lambda > 0$ is a weighting coefficient and $D\,q(t(\omega),\omega)$ is the dispersion of the stochastic function $q(t(\omega),\omega)$. In our research we use $\lambda = 0.6$. This model is named M-D-model [8].

To solve this problem we reduce the stochastic optimization problem to a nonlinear programming problem [8, 10]. The algorithm for M-model is described below.

1. We define noncrossing subsets of $\mathbf{\Omega}$ and denote they as

$$A_i : t_m \in \left[t_m^i, t_m^{i+1}\right], P_{cr} \in \left[P_{cr}^i, P_{cr}^{i+1}\right], \quad i = \overline{1,n}.$$

2. Suppose the coming of t_m and P_{cr} to be independent events, we calculate the probability

$$\mathbf{P}(A_i) = \mathbf{P}_j^{t_m}\mathbf{P}_k^{P_{cr}},$$

where

$$\mathbf{P}_j^{t_m} = \mathbf{P}(t_m \in \Delta t_m^j),\ j = \overline{1,J},\ \sum_{j=1}^{J} \mathbf{P}_j^{t_m} = 1, \quad \mathbf{P}_k^{P_{cr}} = \mathbf{P}(P_{cr} \in \Delta P_{cr}^k),\ k = \overline{1,K},\ \sum_{k=1}^{K} \mathbf{P}_k^{P_{cr}} = 1 .$$

3. The definition of the statistical expectation is $F(t(\omega),\omega) = \lim_{n\to\infty} \sum_{i=1}^{n} q(t,A_i)\mathbf{P}(A_i)$. Using item 2 we determine an approximate value of the objective function

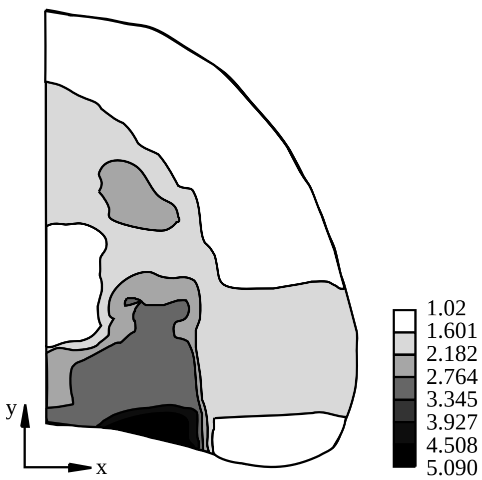
$$F_{J,K}(t,A_i) = \sum_{j=1}^{J} \sum_{k=1}^{K} q(t,A_i)\mathbf{P}_j^{t_m}\mathbf{P}_k^{P_{cr}} .$$

4. The stochastic function $q(t,A_i)$ is calculated as a polynomial function for fixed t_m and P_{cr} from the subset A_i. In the present paper we use the second-degree Lagrange polynomial: $q(t,A_i) = \alpha_0 + \alpha_1 t + \alpha_2 t^2$, where $\alpha_0, \alpha_1, \alpha_2$ are undetermined coefficients.

Results and discussion

Before presenting results we need some background. We provide our research for three internal part materials: cobalt-chromic alloy KHS, titanic alloy VT1-00 and plastic AKR-15 (elastic and fatigue properties of these materials and elastic properties of the mucosa are described in [2]). The mucosa thickness and the pain threshold pressure were obtained experimentally and reported in [9]. The distributions of these data are shown in Fig. 3.

We carry out all calculations for an occlusion moment, when the masticatory force is maximal. For mastication it is the central occlusion and for biting it is the sagittal one. The part of the masticatory force acting on an artificial tooth is determined according to the masticatory efficiency of the tooth [1, 5]. Table 1 presents values of the masticatory efficiency and force distribution over artificial teeth. Force direction is represented in Fig. 4. For the sagittal occlusion lower frontal teeth make contact with corresponded upper teeth at a line on the palatine surface of the late ones. The location of this line depends on an individual angle of the sagittal canine path (Fig. 4b). The average value of this angle is 45^{o} [5]. The force is normal to the palatine surface of upper incisors (Fig. 4b).



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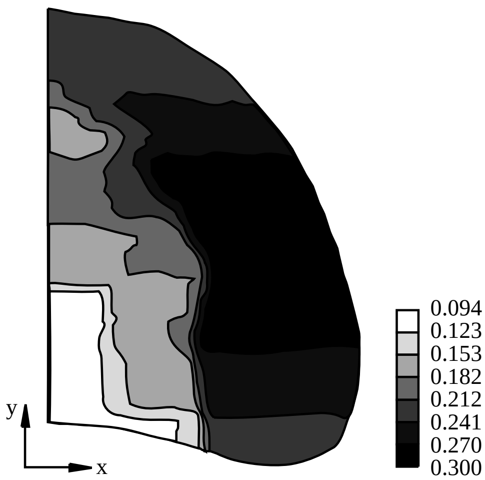

b

Fig. 3. The thickness (mm) (a) and the pain threshold pressure (MPa/mm^2) (b) of the prosthetic bed mucosa.

Table 1. The distribution of the masticatory force over artificial teeth.

Tooth	Masticatory coefficient	Force, %	
		Mastication	Biting
Central incisor	8	0	66.67
Side incisor	4	0	33.33
Canine	12	0	0
First premolar	16	21.05	0
Second premolar	16	21.05	0
First molar	24	31.58	0
Second molar	20	26.32	0

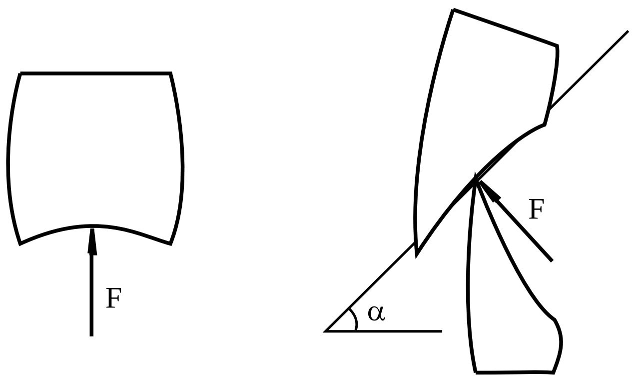

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Fig. 4. Force direction for mastication (a) and biting (b): *F* is the masticatory force, α is the angle of the sagittal canine path.

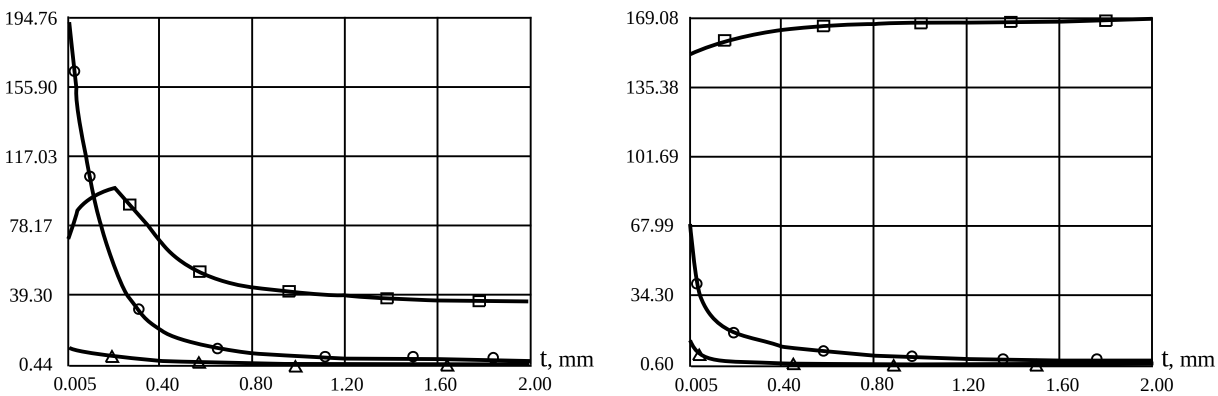

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Fig. 5. Dependencies $F_o^{th}(t)$ (□), $\max\limits_{V_{int}} \sigma_o^{int}\Big|_{F=F_o^{th}(t)}(t)$ (○) and $\max\limits_{V_{ext}} \sigma_o^{ext}\Big|_{F=F_o^{th}(t)}(t)$ (△) in the titanic denture for mastication (a) and biting (b).

Table 2. Optimal values of the internal part thickness and the maximal relative threshold force for mastication and biting.

Basis material	Mastication		Biting	
	Optimal thickness, mm	$F^{th}_{o_{max}}$,%	Optimal thickness, mm	$F^{th}_{o_{max}}$,%
KHS	0.17	92.4	2	172
VT1-00	0.20	99.4	2	169
AKR-15	1.73	92.7	1.4	158

Now let us discuss the obtained results. The dependence of the relative threshold force and the relative stresses on the internal part thickness is shown in Figure 5. The results of the optimization problem (6)-(10) are listed in Table 2. Results presented above can be generalized as follows.

1. For mastication all dependencies $F_o^{th}(t)$ have the only maximum in the investigated interval. However for biting these dependencies for the metallic dentures are increasing monotonically.
2. For mastication the relative threshold force strongly depends on the internal part thickness. But for biting the dependence takes place only for a small thickness area.
3. The optimal thickness values for mastication and biting are quite different (Table 2). But the threshold force for biting weakly depends on the internal part thickness. Also it is well known that a person executes much more masticatory acts than biting ones. So we recommend using the optimal thickness values determined for mastication. That is why in the rest of this paper we will discuss results for mastication.

As Fig. 6a indicates the denture basis leans on the alveolar process and the palatine torus during mastication. We can manage basis rigidity by changing the internal part thickness and the internal part material. So we can redistribute the masticatory load between two supports. That is why the optimal solution corresponds to an equal force distribution between two support areas. Unlike mastication, the denture basis has only one support during biting. It is the frontal part of the alveolar process (Fig. 6b). So changing the internal part thickness cannot essentially effect on neither pressure distribution nor the relative threshold force value.

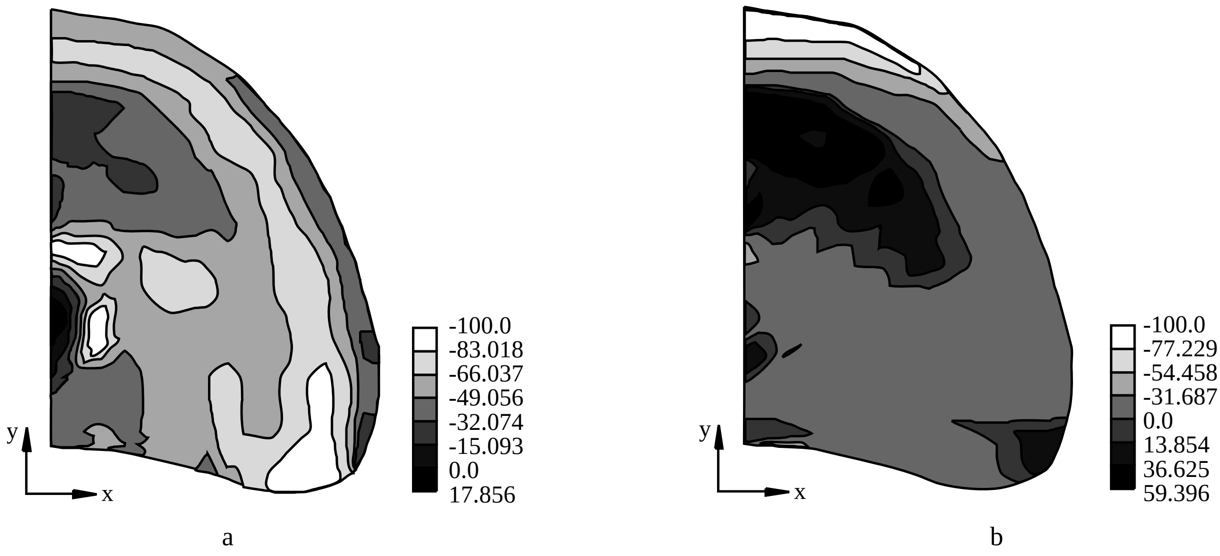
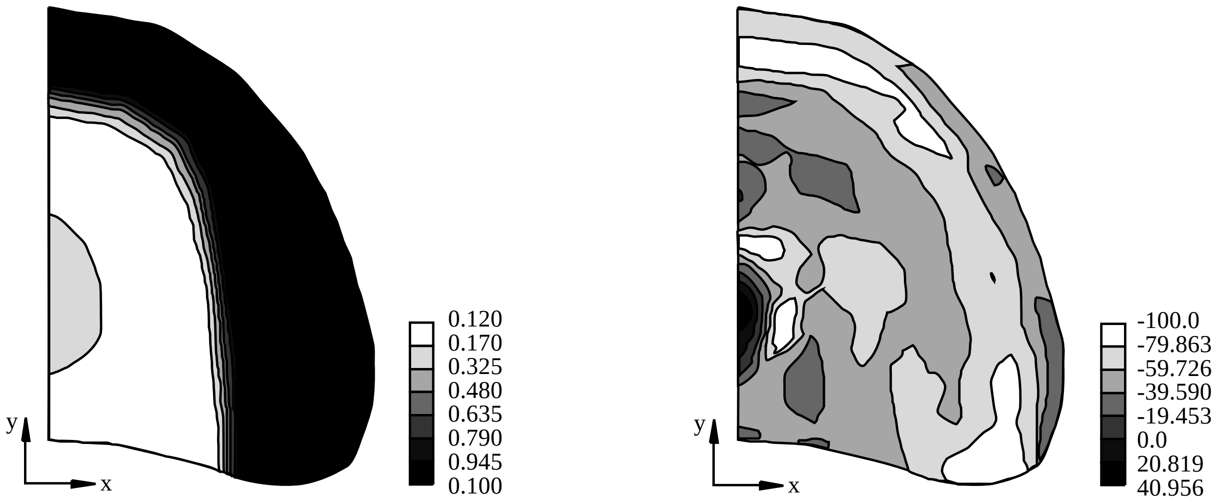


Fig. 6. The distribution of the relative pressure on the prosthetic bed mucosa (%) in the titanic denture with the optimal thickness basis for mastication (a) and biting (b).



a b

Fig. 7. The distribution of the internal part thickness (mm) (a) and the relative pressure on the prosthetic bed mucosa (%) (b) for the titanic denture with variable thickness.

Table 3. The optimal denture thickness and objective function values for different models in the stochastic optimization problem.

Model	Optimal thickness, mm	Objective function, %
Determinate model	0.20	99.4
M-model	0.18	97.3
M-D-model	0.17	56.3

For mastication the area between two supports remains weakly loaded. The use of a denture with variable internal basis thickness can help this situation. Therefore the optimization problem was solved for the denture with variable thickness. The optimal denture construction is characterized by three thickness values: $t_1 = 0.17\ mm$ in the palatine torus area; $t_3 = 1.10\ mm$ in the alveolar process area; $t_2 = 0.12\ mm$ in the remaining area (Fig. 7a). In Fig. 7b one can see the area between two supports now contains zones of the maximal pressure. The maximal threshold force runs up to 102 %.

As expression (1) indicates, positive values of the relative pressure p_o tell about the possibility of denture detachment. Two dangerous regions: the vast area at the frontal part of the denture, between the alveolar process and the palatine torus, and the area beyond the dentition are shown in Figure 6b. Although the second area is small, localization near the denture edge makes it more dangerous since air can penetrate between the denture and the mucosa and lead to denture detachment. But our investigation proves the positive relative pressures for the optimal thickness values to be under 100 %, so the denture does not detach.

We want to complete this paper by presenting results of the stochastic optimization problem. We solved the optimization problem for the titanic denture in mastication process. The results are displayed in Fig. 8 and Table 3. Table 3 documents the stochastic optimal solution to be different from the determinate one. Also Table 3 shows the M-model solution to be closer to the determinate one than the M-D-model solution. But the difference between the determinate and stochastic solutions is not so large. According to the analysis given here, determinate results can be used instead of stochastic ones, if only mucosa characteristics are assumed to be stochastic.

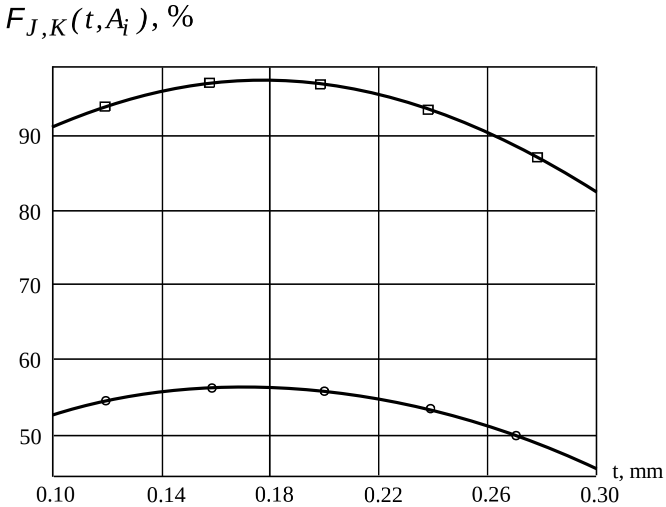


Fig. 8. Objective functions for M-model (□) and M-D-model (○) in the stochastic optimization problem.

Conclusions

In this paper we have explored the behavior of the removable laminar maxillary denture during mastication and biting. The main findings of the study may be summarized in what follows.

- With the help of the mathematical model the optimal denture structure was determined for some basis materials. This optimal construction enables to enlarge the threshold masticatory force applied to the denture and reduce bone resorption.
- The optimization problem was solved for mastication and biting. The threshold force for biting was determined to be weakly dependent on the denture thickness. So it is recommended to use the optimal thickness values determined for mastication.
- For mastication the optimal denture thickness was found to provide an equal pressure distribution between the alveolar process and the palatine torus areas.
- The optimal construction of the denture with variable thickness was determined. It was shown that this denture construction enlarges the maximal threshold force and leads to better pressure distribution over the prosthetic bed mucosa.
- The possibility of denture detachment was investigated and it was found to be impossible for the dentures with the optimal thickness.
- The influence of changing some model characteristics on the optimal solution was analyzed. The stochastic optimization problem was formulated and solved. Small difference between stochastic and determinate results proves using determinate results instead of stochastic ones, if only mucosa characteristics are assumed to be stochastic.

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ИСПОЛЬЗОВАНИЕ МАТЕМАТИЧЕСКОГО МОДЕЛИРОВАНИЯ ДЛЯ ОПТИМИЗАЦИИ КОНСТРУКЦИИ ПЛАСТИНОЧНОГО ПРОТЕЗА

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Использование съемных пластиночных протезов не может полностью восстановить жевательную способность больного и приводит к патологическим изменениям слизистой оболочки протезного ложа и атрофии кости под протезом. В работе с помощью математической модели осуществляется поиск оптимальной конструкции съемного пластиночного протеза на верхнюю челюсть, позволяющей повысить величину жевательной нагрузки, не приводящей к боли, и избежать патологических изменений тканей протезного ложа. Задача решается для процессов жевания и откусывания.

В результате получены оптимальные конструкции протеза для некоторых материалов в случае постоянной и переменной толщины базиса. Исследована возможность нарушения фиксации протеза в ротовой полости. Решена задача стохастической оптимизации протеза, позволяющая избежать погрешностей, связанных с использованием в расчетах осредненных данных. По результатам решения сделан вывод, что при учете стохастического характера только свойств слизистой оболочки, без особого ущерба можно пользоваться решением детерминированной задачи. Библ. 10.

Ключевые слова: съемный пластиночный протез, слизистая оболочка, оптимальное проектирование, стохастическая оптимизация

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