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EVALUATION OF THE EFFECT OF SKI-JUMPER'S AERODYNAMIC QUALITY ON SAFETY OF LANDING

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Abstract: The problem of safety in ski-jumping is actual because any fall is connected with risk for health and life of a sportsman. This paper concentrates on one of the factors contributing to safety of skier's landing, namely the velocity of approach to the hill surface and the influence of ski-jumper's aerodynamic quality on it. The main difficulty lies in finding the aerodynamic forces of drag and lift in the flight phase. The authors chose the simplest model of investigated system: plane plate in the stream of ideal gas. We understand the weaknesses of this model and so we gave attention to its identification with experimental data. Analytic expressions were produced for drag and lift. Our investigation showed that there exists some optimal value of plate thickness (and thus graph of aerodynamic quality) which minimizes landing velocity component normal to the hill surface. We hope that this conclusion will be valid for more precise models of investigated system. This work also contributes to creating a technique of exploring of one of the main factors affecting safety of landing.

Key words: ski-jumping safety, aerodynamic quality, landing velocity

Introduction

The main index of a ski-jump is the distance. Studies [1-6] concentrate on flinging and optimization of flight distance. Safety of landing also is an important factor. Take-off technique, forming of flight posture and its stability, technique of landing contribute to this complex factor [7-8].

Landing velocity component normal to the jumping hill slope was insufficiently studied in literature, but this parameter is not the least important for safe landing. It was regarded in the paper [4] for the first time, and flight trajectories were calculated with constraint on landing velocity.

This study shows the influence of aerodynamic quality of the model of skier-on-skis landing velocity normal to the surface and thus, to the certain degree, on landing safety of a ski-jumper.

Evaluation of the maximal safe velocity of landing

Let us consider a simple model of a ski-jumper pictured in Fig.1. It is a material particle of mass m in the place of sportsman's hip joint. Reference system A_0xy is bound to skis. This reference system is not inertial but this fact has no effect on the movement of ski-jumper normal to the hill surface. After touch-down the ski-jumper dampens his velocity component v_y normal to the slope. The point A_0 is the position of the point A at the moment of touch-down (t=0), and the point A_1 is the final position of the point A, where the velocity component v_y normal to the hill surface is reduced to 0. The y-coordinate of the point A_0 is 0, and the y-coordinate of the point A_1 is h.

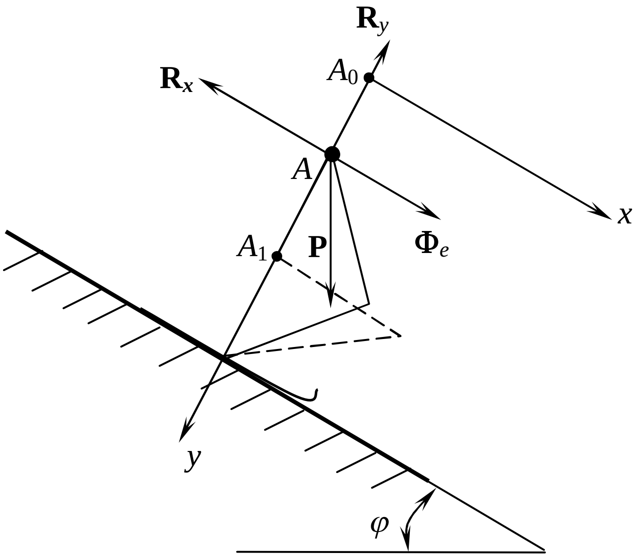

Fig.1. The model of a ski-jumper is one material particle of mass *m* in the place of sportsman's hip joint.

The ski-jumper is affected by the force of gravity $\mathbf{P}$, two components of the ground reaction force $\mathbf{R}_x$ and $\mathbf{R}_y$ and Eulerian force of inertia

$$\mathbf{\Phi}_e = -m\mathbf{a}_e, \tag{1}$$

where $\mathbf{a}_e$ is transfer acceleration, which is directed opposite to the *x*-axis.

We assume that ground reaction $|\mathbf{R}_y|$ is proportional to the length of the way of the point *A* in direction normal to the plane

$$R_y = ky. \tag{2}$$

The second Newton's law gives us differential equation of skier's movement along the *y*-axis

$$mv_y \frac{dv_y}{dy} = mg\cos\varphi - ky. \tag{3}$$

The initial value of velocity is $v_y = v_1$ (initial landing velocity); the final value of velocity, when the point *A* covers the distance *h* between the points A_0 and A_1, is 0. Integration of (3) gives us

$$v_1 = \sqrt{\frac{kh^2}{m} - 2gh\cos\varphi}. \tag{4}$$

According to (2), the maximal deceleration force is

$$R_y = kh. \tag{5}$$

We suppose that it is n times higher than weight of ski-jumper, so

$$v_1 = \sqrt{(n - 2\cos\varphi)gh}. \tag{6}$$

If n=5 [7], φ=36°, g=9.8 m/s^2, h=0.4 m (hip length) then v_1=3.6 m/s.

Now we know the maximal safe landing velocity and we must mind it in any further flight analysis.

A simple model of skier-on-skis system for evaluation of aerodynamic quality

The main purpose of this study is determination of influence of aerodynamic quality K of skier-on-skis system on safety of landing. Aerodynamic quality is equal to the force of lift divided by force of drag. The desire to find an analytic expression for K has led us to the following simple model of investigated system (see Fig.2).

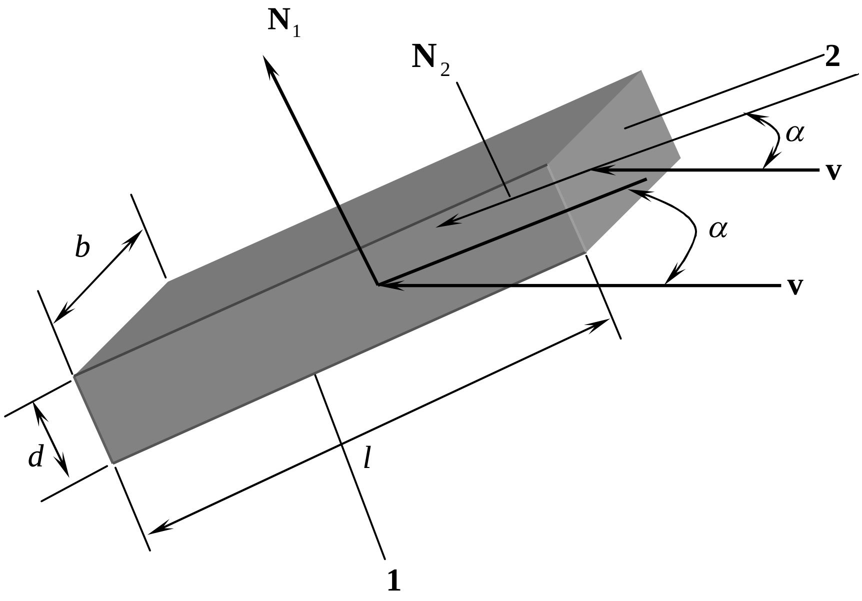

Fig.2. The model of skier-on-skis is the plate of finite thickness.

Let us consider the airflow of speed v around the plate of finite thickness d, length l (skis length) and width b.

The angle α between the plate and the airflow is the attack angle. Let us find drag and lift forces as if finding the pressure of moving stream of ideal liquid on an immovable surface. This method does not take into account the finiteness of the object but it gives qualitative dependence of forces on the attack angle. The numerical method of studying the airflow around the plate, which was used in the paper [4] is not enough accurate in considering plate thickness, when thickness is little, due to the error of the method.

Let us now consider the influence of the airflow on the side 1 of the plate (see Fig.2).

According to the momentum theorem, normal force of air stream pressure on the side 1 (Fig.2) is [9]

$$N_1 = \rho v^2 S \sin\alpha\,, \tag{7}$$

where ρ is air density (ρ=1.23 kg/m^3 when t=0° and air pressure p=760 Hg mm), S is lateral area of the stream.

$$S = lb\sin\alpha\,, \tag{8}$$

and (2) give us

$$N_1 = lb\rho v^2 \sin^2\alpha\,. \tag{9}$$

The force of pressure on the side 2 is determined in the same way

$$N_2 = ld\rho v^2 \cos^2\alpha\,. \tag{10}$$

Lift force modulus Q is equal to the algebraic sum of the components of vectors $\mathbf{N}_1$ and $\mathbf{N}_2$

$$Q = N_1\cos\alpha - N_2\sin\alpha\,; \tag{11}$$

applying of (9) and (10) brings us to the final equation for aerodynamic force of lift

$$Q = b\rho v^2 \sin\alpha\cos\alpha(l\sin\alpha - d\cos\alpha)\,. \tag{12}$$

The resulting force of drag is determined by components of $\mathbf{N}_1$ and $\mathbf{N}_2$ parallel to the velocity $\mathbf{v}$

$$R = b\rho v^2(l\sin^3\alpha + d\cos^3\alpha)\,. \tag{13}$$

Considering (12) and (13), the aerodynamic quality $K = \dfrac{Q}{R}$ takes the following form

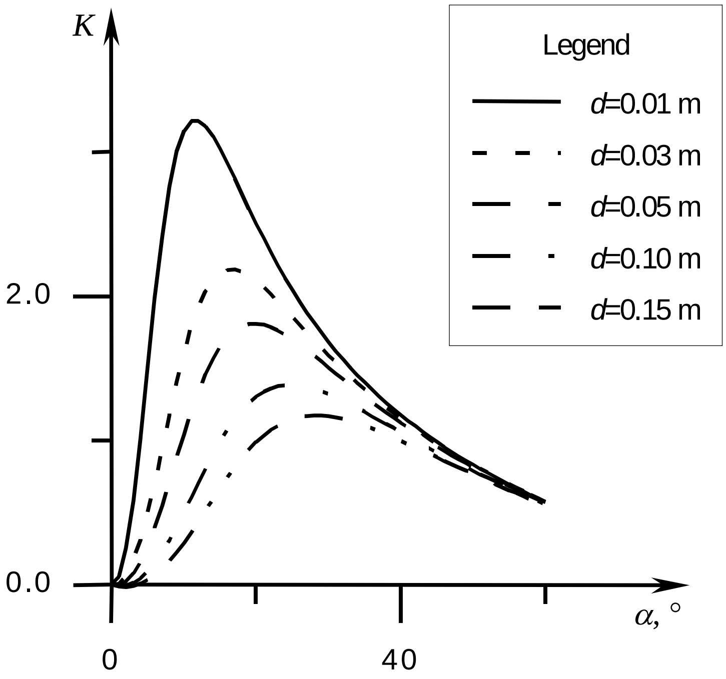

Fig.3. Aerodynamic quality of the moodel of skier-on-skis system for different thicknesses of the system.

$$K = \frac{\sin\alpha\cos\alpha(l\sin\alpha - d\cos\alpha)}{l\sin^3\alpha + d\cos^3\alpha}. \tag{14}$$

Fig.3 shows the graph of K versus attack angle α for l =2.4 m. Five lines are drawn for d =0.01 m, d =0.03 m, d =0.05 m, d =0.10 m and d =0.15 m, respectively. It is seen that increase of d leads to decrease of K for all attack angles and movement of maximum K to the area of higher attack angles. Note that thickness of the plate represents the relative position of skier and his skis. Higher aerodynamic quality is achieved in the modern V-style, when they are in one plane.

Investigation of ski-jumper's flight

Modern large jumping hills give sportsmen the possibility of long jumps. We investigated the live videorecord of a World Cup Competition "Four Hills" held on K110 jumping hill in Innsbruck on 4 January 1998. Parameters of K110 jumping hill in Innsbruck are the following: θ = -12°, φ = -36°; maximum flight distance L =110 m, minimum flight distance (the distance between break-off point and the beginning of landing zone) B = 90 m, height of break-off table above jumping hill surface T=4 m; parameters of hill's slope H =63.2 m, N = 87.0 m (see Fig.4).

The ski-jumper is affected by forces of gravity $\mathbf{P}$, drag $\mathbf{R}$ and lift $\mathbf{Q}$ (see Fig.5). The differential equations of movement have the following form

$$\begin{cases} m\dfrac{d^2x}{dt^2} = -R\dfrac{v_x}{v} + Q\dfrac{v_y}{v}, \\ m\dfrac{d^2y}{dt^2} = mg - R\dfrac{v_y}{v} - Q\dfrac{v_x}{v}, \\ v = \sqrt{v_x^2 + v_y^2}, \end{cases} \tag{15}$$

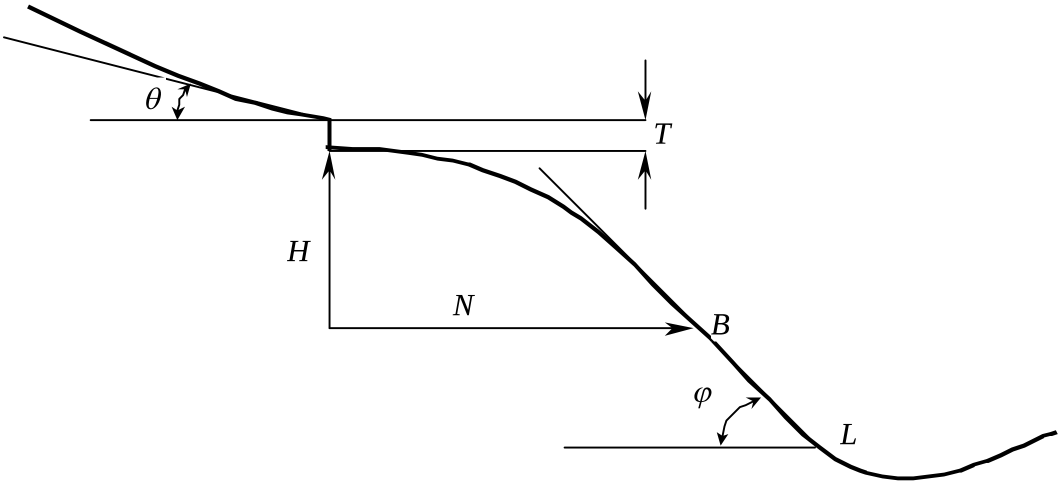

Fig.4. Basic geometrical elements of a jumping hill.

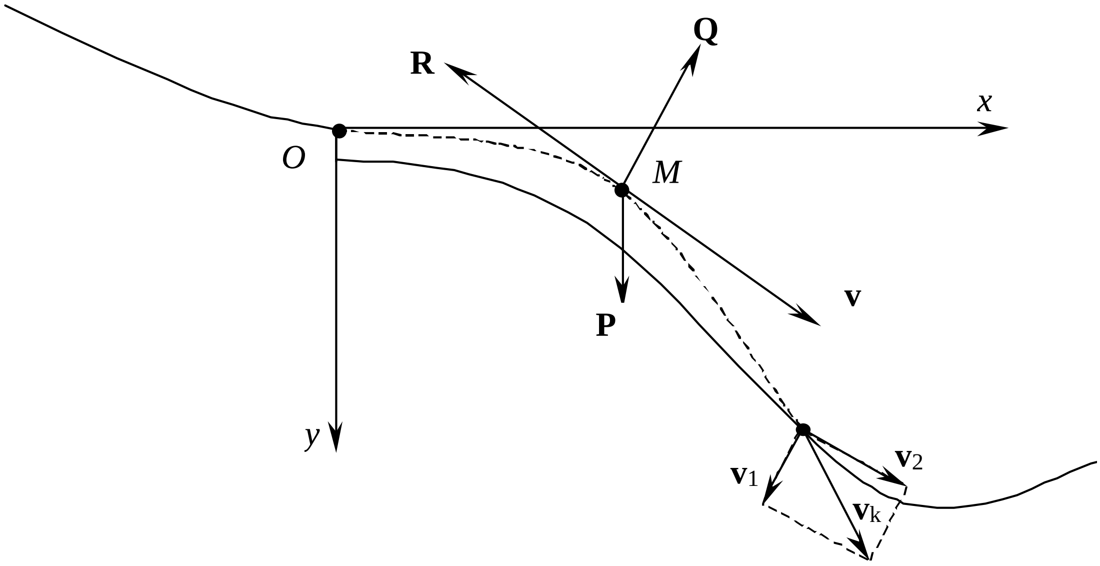

Fig.5. Forces affecting the ski-jumper in the flight.

where x, y are coordinates of ski-jumper,

v_x, v_y are the projections of his velocity on coordinate axes,

Q and R are calculated according to equations (12) and (13), respectively.

The set of equations (15) differs from the equations used in earlier works [1-6] by method of finding drag and lift (Q and R).

These equations are reduced to 4 differential equations of the first order and numerically solved by the method of Runge-Kutt with known initial conditions for coordinates and velocity.

The coordinates of ski-jumper form the flight trajectory; landing velocity component normal to the hill surface was found in the touch-down point. For identification of the mathematical model we chose the sportsman who held his skis almost horizontally and his body was almost in the plane of skis, that is who performed V-style jump perfectly. His flight distance was 113 m. The mass of fully equipped sportsman with skis was 60 kg. Initial velocity was v_0 =24.7 m/s.

Identification was held separately for each thickness of the plate d and l =2.4 m. Plate width b was found from the solution of the problem of minimization the difference between calculated distance and real distance. It occurred that this nonlinear problem had more than one solution, and physically senseless results were discarded.

Results

Fig.6 shows the graph of landing velocity normal to the hill surface versus plate thickness. The minimum was achieved for d =0.07 m (v_1 =0.04 m/s, b =1.67 m) when the ski-jumper lands almost without impact. Note that plates of lesser thickness have higher aerodynamic quality but the trajectory is not optimal from the point of view of landing safety due to high v_1 (landing velocity component normal to the hill surface).

We should point also that v_1 did not exceed the limit v_1 =3.6 m/s found previously in present paper for all investigated d. It means that this jumping hill is safe for a wide variety of ski-jump techniques. But big increase of d results in rapid fall of aerodynamic quality K, and the sportsman would not be able to achieve the real distance of 113 m for any b. This is so already for d =0.1 m.

Conclusions

The influence of aerodynamic quality of skier-on-skis system on landing velocity was investigated for a fairly simple model of the system – the plate of finite thickness. It was shown that there exists optimal thickness for which landing velocity component normal to the surface of the jumping hill has minimal value. The investigation showed the way of solving

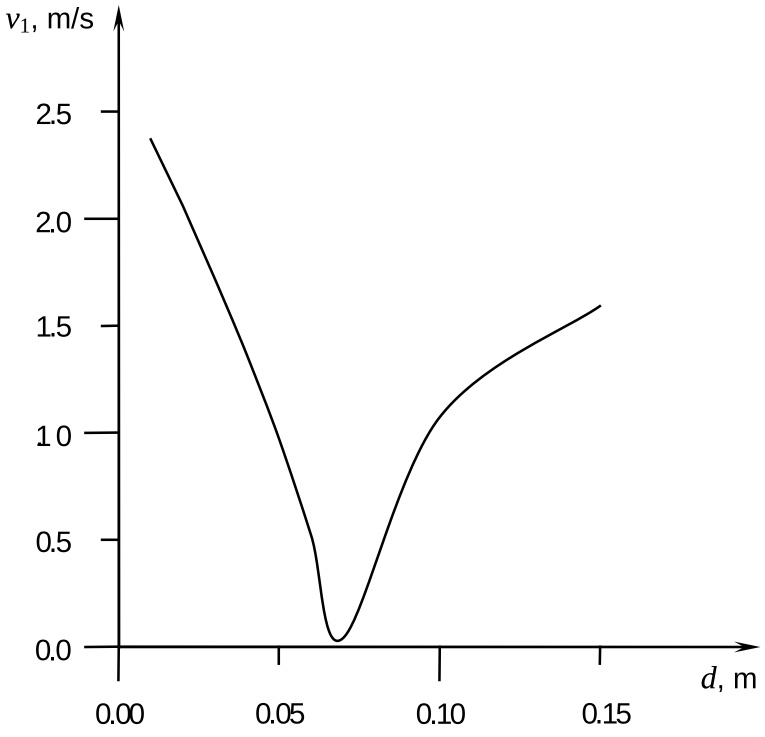

Fig.6. Dependence of landing velocity normal to the hill surface on plate thickness.

the real problems of increasing the quality of the ski jump. The main problem is in exploring the airflow around the ski-jumper and more precise finding of aerodynamic forces of drag and lift.

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ОЦЕНКА ВЛИЯНИЯ АЭРОДИНАМИЧЕСКОГО КАЧЕСТВА СИСТЕМЫ ЛЫЖНИК-ЛЫЖИ НА БЕЗОПАСНОСТЬ ПРИЗЕМЛЕНИЯ ПРЫГУНА С ТРАМПЛИНА

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Проблема безопасности прыжка на лыжах с трамплина актуальна, поскольку любое падение в этом технически сложном виде спорта сопряжено с риском для здоровья и жизни спортсмена. В статье обсуждается один из факторов, влияющих на безопасность приземления прыгуна – скорость приближения лыжника к поверхности горы приземления. Исследуется влияние аэродинамического качества системы лыжник-лыжи на эту скорость.

Основная трудность задачи состоит в определении лобового сопротивления и подъёмной силы в фазе полёта лыжника. В настоящее время мы не можем обеспечить достаточно точное решение, позволяющее выработать практические рекомендации тренерам, поскольку для этого необходимо решение трёхмерной внешней задачи обтекания потоком воздуха геометрически сложной системы лыжник-лыжи. Попытка использовать экспериментальные данные по обдуву манекена лыжника в аэродинамической трубе [10] также не привела к успеху, поскольку там приведено лишь отношение подъёмной силы к силе лобового сопротивления, а этого недостаточно для построения траектории полёта лыжника.

Авторы выбрали простейшую модель исследуемой системы в виде плоской пластины конечной толщины и соответственно этому приближению – гидравлический

подход к определению давления потока воздуха на пластину. Были получены аналитические выражения для аэродинамических сил, что позволило детально исследовать влияние угла атаки и толщины на скорость приземления прыгуна. Понимая недостатки используемой модели, мы обратили внимание на идентификацию расчётного прыжка реальным прыжком.

Полёт лыжника рассматривается как движение материальной точки. Дифференциальные уравнения движения интегрировались численно методом Рунге-Кутта. От работ [1-6] эта часть работы отличается зависимостями от параметров задачи сил, действующих на лыжника и определяющих траекторию полёта.

В приведённой модели получается, что аэродинамическое качество K при всех углах атаки α монотонно уменьшается с ростом толщины пластины. Зависимость K от угла атаки для пластин конечной толщины качественно совпадает с экспериментальными данными [10]. Однако существенное различие получается для бесконечно тонкой пластины, и такая модель системы неприемлема.

Исследование показало, что существует некоторое оптимальное значение толщины пластины (и, соответственно, коэффициента K), при котором нормальная к поверхности составляющая скорости приземления минимальна. Можно надеяться, что этот вывод сохранится и для более точных моделей исследуемой системы. Библ. 10.

Ключевые слова: безопасность прыжка на лыжах с трамплина, аэродинамическое качество, скорость приземления

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