ON INFLUENCE OF MODEL ON PREDICTING MODULUS VALUE OF TRABECULAR BONE TISSUE

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Abstract: Different structure models of trabecular bone tissue are considered in the paper. For every model, the relation between elastic modulus and apparent density is received. The Hooke’s law as classic one is chosen to describe the mechanical response on the external action. Using models estimating calculations is made to determine a validity of every model. The simple models are illustrated to use in practical problems. Different models result in quantitatively different relations between modulus and density. The hypothesis is proposed that different models can be used for theoretical prediction of mechanical properties of different locations of trabecular bone tissue.

Key words: trabecular tissue, elastic modulus, bone density

Introduction

Problem of theoretical prediction of mechanical behavior of trabecular bone tissue is one of the most important issue of biomechanics through the ages and so far. Many papers which devote to theoretical and experimental researching are published. Many measurement technologies are considered and used in practice: ultrasound [1,2], micromeasurement [2,3,4], optical measurement [3]. The values of strength [5], elastic modulus [2,6-9,10,11], fatigue characteristics [12] are measured and dependence of each on others is analysed and approximated. Their relations to apparent density of trabecular bone take especial part in research directions, e.g. [9].

The determination of such relationships and creation of mathematical models of mechanical behavior of bone tissue would allow advancing individual medication. The use of theoretical models is a necessary step in a design of medication of bone tissue in vivo, and in a diagnostic methods, and in other actual problems of clinic. For instance, internal architecture of human patella was studied by quantitative stereological analysis of microdiographs made from sagittal and horizontal sections [4]. The modeling can use the stereological analysis as one of start point of theoretical calculations for a state, parameters of medication and a prediction of dynamics of bone tissue properties change. It is one of numerous possible ways.

From the other hand, the experimental data of many researches require, as a rule, to be described by theory. But it does not take place always. The one reason of that is frequent attempts to use one characterize relation for predicting of bone tissue behavior of different location. A parameters of relation are determined to adjust the experimental data [2,10,11,13,14]. Usually, the relations are linear or power laws. For last case the power is elected to relate mechanical properties. For example, for relation modulus-apparent density the value of power equals roughly 1.5 [11]. In paper [2], with reference to [15], the relation is proposed as cubic polynomial one. The attempts to utilize universal law [10,11] lead to significant discrepancy between experimental points and approximation curves. A more accurate theory does not guarantee good agreement with experiment [14]. But there are papers which prove that the differences between properties of tissue depends on location of bone
within organism and on the location within bone too. Furthermore, a material response for mechanical loading depends on geometry of a specimen [13, 16].

That is the choice of model, which has to describe the bone tissue behavior, seems to be very essential to our mind. And the choice is not unique. The law and its parameters are varied. Cowin et al. proposed [17] a relation between elasticity modulus and architecture parameters of trabecular bone by fabric tensor. This model is preference because it allows to predict anisotropy properties of trabecular bone using average of structural characteristics of tissue in the investigated location of bone. But the drawback of this theory is a complication of its application. There is the necessity to determine many parameters. It is a diagnostic problem. But this idea can be one of bases in investigation of fundamental problem about relation between architecture (and density together) of trabecular tissue and its mechanical characteristics.

In this paper the series of simple structural models of trabecular tissue are proposed which allow to calculate elasticity modulus by structural parameters of bone tissue, and, consequently, by density. The aim of paper is to illustrate that a result depends on type of applied model. Moreover authors want to emphasize impossibility of universal model of trabecular bone tissue, but the numerous models have been used in practical calculation. A choice of model has to be determined by factors such as bone location, experimental data of considered bone, its structural and anatomical peculiarities, and, possibly, some other markers which are unknown yet.

The elaboration of models is conducted in frame of structural elasticity theory by means of definitions and theorems cited in [18]. Certainly that information can be found in any monograph on continuum mechanics.

Models

The elaboration of structural models of bone materials leads always to problem of division bone tissue into two types: cortical and trabecular bone tissue. First one is compact bone formation and serve as construction material of bone wall. Other tissue is more responsible for bone behavior in the region near joint.

In this paper the trabecular bone is considered.

Macroscopic model. A continuum analog of bone tissue needs to receive apparent characteristics. From the point of macroscopic representation spongy tissue is modeled as isotropic one with ordered apparent density.

A two-dimensional model of trabecular bone is considered. The observed specimen is rectangular plate subjected to compressive loads on top and bottom edges. The coordinate axes are directed parallel to loaded edge (axis OX), to free one (axis OY) and orthogonal to the plate, Fig. 1.

A plane state of stress is assumed, i.e. \( \sigma_{ix} = 0, \forall i = x, y, z \). Taking into consideration that volume forces and particle acceleration are neglectible, the equations of equilibrium state can be written in the form:

\[
\vec{\nabla} \cdot \vec{\sigma} = 0
\]

or in a component form
\[
\dot{\varepsilon}^i \frac{\partial}{\partial x^i} \sigma^{ij} \varepsilon_i \dot{\varepsilon}^j = \frac{\partial \sigma^{ij}}{\partial x^l} \varepsilon_l = 0 \Rightarrow \frac{\partial \sigma^{ij}}{\partial x^j} = 0. 
\] (1)

Taking into account that \( \sigma_{ic} = 0, \forall i = x, y, z \)

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0. 
\] (2)

The relation between the stress and the strain are defined by Hooke’s law

\[
\varepsilon_x = \frac{1}{E} \sigma_x - \frac{\nu}{E} \sigma_y, \quad \varepsilon_y = \frac{1}{E} \sigma_y - \frac{\nu}{E} \sigma_x. 
\] (3)

Instead model shown in the Fig.1 it is more convenient to investigate a plate supported by bottom side. Because of symmetry the stress state in new object is the same as in one in Fig. 1. Then boundary conditions are written in the form: for the boundary \( \Gamma_1: u_y = 0, \sigma_y = -p \), for the boundary \( \Gamma_2: \sigma_x = 0, \tau_{xy} = 0 \), for the boundary \( \Gamma_3: \sigma_y = -p, \tau_{xy} = 0 \), and for the boundary \( \Gamma_4: \sigma_x = 0, \tau_{xy} = 0 \). Consequently, the problem of linear elasticity results to

\[
\frac{\partial u_x}{\partial x} = \frac{\nu p}{E}, \quad \frac{\partial u_y}{\partial y} = -\frac{p}{E}. 
\] (4)

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**Fig. 2. The schematic structure of spongy tissue:**

a) whole structure, b) its representative element.
That is the strains are homogeneous over the volume of plate.

**Structural models.** 1. Model based on hexagonal structure.

For calculations a structure of investigated region is specified by using of experimental data. The model of spongy tissue is truss cell structure. Every cell is a hexagonal polygon with equal sides. The side of cell is modeled as a beam. There is no any filling compound. Every beam of cell imitates a trabecula. The structure is illustrated in Fig. 2. Of course, this model is not quite adequate for a prediction of mechanical behavior of trabecular tissue.

Geometrical parameters of this model are: length of each beam (of each trabecular) $l$, its thickness $t$, and slope angle noted as $\theta$. And M and N denote the numbers of cells series along horizontal and vertical edges, respectively. Two last characteristics relate structural geometry and the macroscopic sizes of specimen, Fig. 2. The weights of beams are neglected according to the assumption of macroscopic model. On the bottom side the system leans on the horizontal plane. The load (pressure $p$) subjected to the top side $\Gamma_3$, see Fig. 1, is modeled by node forces $F$ that are subjected to the top series of nodes, Fig. 2 (every node force $F$ is distributed homogeneously along distance in perpendicular direction to the plate).

**Trabecula as a beam.** The trabecula is modeled as a beam. In plane case the beam has six degrees of freedom. The strains are supposed to be small. Then the theory of beam flexure can be employed without consideration of tensile/compressive strains [18]. It is assumed that if value of N is rather great then vertical displacement of nodes far from edges is constant and equal $\Delta$. The deformation of separated cell is considered. The cell is far from edges of plate. It is supposed that the structure is sufficiently large to neglect the boundary effects. Therefore, because of symmetry of infinite structure with respect to axis $L$, Fig. 2b, vertically oriented beams are not bent and, consequently, its compressive strains are neglected with comparison to flexure of sloped trabeculae. Further, each vertical beam is subjected by force $F$ since the force field is homogeneous in the structure.

Since the strains are homogeneous in the structure they can be determined by strains of separated element. The beam AB is selected, i.e. the lowest beam of inner cell, Fig. 2b. Fig. 3 illustrates a process of its deformation.

The displacement $\delta$, shown in Fig. 3a, is determined accurately by means of theory of beam flexure [18]. Perpendicular force is equal to $F\cos\theta/2$, tangential $n(x)$ and normal $h(x)$ displacements in the beam, Fig. 3b, equal
\[ \tau(x) = a, \quad n(x) = \frac{F \cos \theta}{12E_s I} x^3 + bx^2 + cx + d \]  

where \( E_s \) is modulus of elasticity of beam material, \( I \) – moment of inertia of cross-section area. The coefficients \( a, b, c, d \) are determined by conditions of beam fixation

\[
\begin{align*}
\tau(0) \sin \theta + n(0) \cos \theta &= 0, \\
- \tau(l) \cos \theta + n(l) \sin \theta &= 0, \\
n'(0) &= n'(l) = 0.
\end{align*}
\]  

(6)

(7)

Really, the value \( n' \) equals to an angle of rotation of cross-section (or of axis line) of beam. For a rigid frame angles between beams connected in point A (Fig. 3b) do not change since vertical and sloped beams do not rotate. So value \( n' \) equals zero on the edges of sloped beams.

Finding coefficients for (5) by equations (6) and (7) transformations result to

\[ \delta = \frac{F l^3}{24E_s I} \cos^2 \theta. \]  

(8)

Then the strain \( \varepsilon_y \) of structure with rather large \( N \) and \( M \) is found as

\[ \varepsilon_y = \frac{\delta}{l(1 + \sin \theta)}. \]  

(9)

Consequently, according to the formula (4), apparent modulus \( E \) is equal to

\[ E = E_s \frac{1 + \sin \theta}{\cos^3 \theta} \frac{12I}{l^3 d}. \]  

(10)

The pressure \( p \) for the cell structure is separated in node forces \( F \) over a length of the cell along horizontal axis (axis OX)

\[ p = \frac{F}{2l \cos \theta}. \]  

(11)

Let the cross-section of beam be rectangular, \( I = \frac{1}{12} t^3 d \) (\( d \) is a thickness of cross-section in picture perpendicular direction). Then

\[ \frac{E}{E_s} = \frac{1 + \sin \theta}{\cos^3 \theta} \left( \frac{t}{l} \right)^3. \]  

(12)

For the cell as hexagonal one, Fig. 2b, geometric and density characteristics are related

\[ \frac{t}{l} = \frac{2}{3} \cos \theta (1 + \sin \theta) \frac{\rho}{\rho_s}. \]  

(13)
Then

\[
\frac{E}{E_s} = \frac{8}{27} (1 + \sin \theta)^4 \left( \frac{\rho}{\rho_s} \right)^3
\]  

(14)

**Trabecula as a rod.** How is the relation changed if trabecula is modeled by a rod which has four degrees of freedom? It is clear from the physical analysis for the structure constructed by rods the continuum analog needs to modify. Namely, the boundary conditions have to be changed: for the boundaries \( \Gamma_2 \) and \( \Gamma_4 \) the condition \( u_x = 0 \) is written instead \( \sigma_x = 0 \). In opposite case the structure is folded and cannot endure any loads. In real situation the constrain for boundaries \( \Gamma_2 \) and \( \Gamma_4 \) can mean the position of trabecular substance between rigid walls of tube bone. The strains of new boundary problem are

\[
\varepsilon_x = 0, \quad \varepsilon_y = -\frac{P}{E} (1 - \nu^2),
\]  

(15)

where \( E \) denotes apparent modulus of elasticity in direction along axis \( OY \). The analytical solution for one cell constructed by rods results to

\[
\varepsilon_y = -\frac{2pld \cos \theta}{SE_s (1 + 2 \sin \theta)} \left( 1 + \frac{1}{\sin^2 \theta} \right)
\]  

(16)

where \( S \) is the area of cross-section of the trabecula (rod). Since the strains in formulae (15) and (16) are the same, the result follows

\[
\frac{E}{E_s} = \frac{t}{2d} \frac{(1 - \nu^2)(1 + 2 \sin \theta) \sin^2 \theta}{(1 + \sin^2 \theta) \cos \theta}
\]  

(17)

or

\[
\frac{E}{E_s} = \frac{l}{d} \frac{(1 - \nu^2)(1 + 2 \sin \theta) \sin^2 \theta}{(1 + \sin^2 \theta) \cos \theta} \frac{\rho}{\rho_s}
\]  

(18)

2. **Model as a structure of quadratic cells.** Let model structure have view shown in Fig. 4. Any separate cell is quadratic polygon. The same assumptions as for the first structure are made. It means that deformation of alone sloped trabecula has the same scheme as it is shown in the Fig. 3a, only the angle \( \theta \) equals 45°. Proceeding in analogous way the vertical displacement of edge with respect to other one is calculated by formula (8). For this model

\[
\delta = \frac{F}{4E_s d} \left( \frac{l}{t} \right)^3
\]  

(19)

Then the strain \( \varepsilon_y \) of structure with rather large \( N \) and \( M \) is found by expression

\[
\varepsilon_y = \frac{\delta}{l \sin \theta}.
\]  

(20)
as for hexagonal structure. Consequently, according to the equation (4), apparent modulus $E$ equals

$$E = 2E_s\left(\frac{t}{l}\right)^3 \quad \text{or} \quad \frac{E}{E_s} = \frac{1}{4}\left(\frac{\rho}{\rho_s}\right)^3. \quad (21)$$

**Estimate calculation**

Sometimes in experiment a range of elasticity modulus variety with respect to apparent density is so small that the approximation of series of data can be described both by linear and power law over all range of density value change. And the model choice is complicated. But this fact simplifies the situation simultaneously since it allows to use different models for one experiment without significant error. Obviously, models have to predict the value of modulus with necessary exactness.

We try to estimate and compare predicted value of elasticity modulus and compare with existing experimental results. The typical sizes of trabecula are chosen [19] as: length $l=1\text{mm}$, thickness – about $t=30\ \mu\text{m}$, elastic modulus of cortical tissue – $E=22.7$ GPa. With these parameters for model based on hexagonal cells with $\theta=30^\circ$ a value of apparent elastic modulus is equal to (using formula (12))

$E=1,415$ MPa.

For model with trabecula as a rod according to formula (18) (with $t=d$)

$E=9,541$ MPa.

For quadratic cell using formula (22)

$E=1,226$ MPa.

These results for models based on beams have good agreement for some experimental data cited in [15]. But many results, especially recent [2,9-11,13], differ from estimation by many times and even by orders (in general, the range of experimental value of elasticity modulus is from 0.1 MPa to 1.5 MPa).
Results and discussion

The difference between predicted mechanical properties and some experimental values can be interpreted by superfluous idealization of models. By comparison of structure of model and photographs of spongy tissue [20] it can be seen that the images are rather different but periodicity takes place. For instance, in some bones investigated in [4] it is possible to use model based on rectangular cells or on parallelepiped cells for spatial analysis. This issue has to be investigated in more details. Obviously, in the same paper other scheme of trabecular issue can be found which need other structural models.

The preferences of beam models have to be noted. The values of apparent elastic modulus are very close to real ones though models are very approximate and values are estimated. It proofs that similar simple models can be used in some range of application which have to be determined by experiment. Their simplicity is indisputable preference in comparison to complicate models that consider many factors.

The disparity of experimentally measured modulus of considered region of bone from theoretically calculated value of modulus means that the model can not be applied for prediction of properties of this region. This situation can because different locations of trabecular bone have distinct properties [6,8].

Received relations, equations (14), (18), (21), are linear and cubic laws. Note that clamped trabeculae correspond to cubic law, but hinged trabeculae – to linear law. Experiments, as a rule, illustrate [10,11] that the power of relation has intermediate value (from 1.03 to 1.88 [11]). Possibly, more adequate model can be created by means of elastic supports of trabeculae. Such model is analogous to Cosserat model. There were some attempts to describe experimental data by Cosserat continuum [14]. Good quantity agreement with experiment was reached.

Although the powers of relations modulus–density are different the values predicted by models based on beams have quite reasonable magnitude of modulus. Model used on rods as trabeculae yields great overestimation of the value of modulus. The conclusion has to be drawn that model used rods for trabecular imitation can not be used for predicting of mechanical behavior of trabecular bone tissue.

From the point of view of authors, a direction of further development is to create more adequate models which use another structural elements, maybe more complicated than beam and rod. Because of anatomy of trabecular tissue it is necessary to consider filling liquid (or marrow). A purposeful experimental research is to find factors out that have more effect on the modulus, the strength, and the density. The practical appliance of models demands the methods of characteristic measurement have to be noninvasive. These models are interested practically, and their applications in clinic sets are real contribution of biomechanics in human treatment.

References

О влиянии модели на вычисленные значения модуля трабекулярной костной ткани

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Важным направлением в биомеханике является разработка методов индивидуального лечения пациентов на основе неинвазивной диагностики. При заболеваниях костных тканей выбор оптимального метода лечения во многих случаях зависит от значений механических факторов пораженной кости пациента (модуль Юнга, предел прочности и др.). Весьма перспективным способом определения этих параметров является денситометрическое исследование различных элементов кости с помощью компьютерной томографии, магнитного резонанса и других методов. В основе этих методов должна лежать установленная биомеханическая связь между значениями эффективной плотности $\rho$ и модуля Юнга $E$, а также другими параметрами.

В данной работе строятся соответствующие биомеханические модели для трабекулярной костной ткани. Для этой цели аналитически решается задача равновесия идеализированной костной структуры при различных конфигурациях и свойствах ячеек представляющих собой трабекулы (гексагональные элементы с шарнирным или жестким закреплением, квадратные элементы и т.д.). Показано, что различным структурам трабекулярной кости соответствуют различные соотношения...
связи эффективная плотность - модуль Юнга в виде линейного и кубического законов. Отмечается, что различным элементам костного скелета и различным локализациям в данной кости могут соответствовать различные законы $E=E(\rho)$. Библ. 20.

Ключевые слова: трабекулярная костная ткань, модуль упругости, плотность, биомеханическая модель

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