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THE PHENOMENOLOGICAL MODEL OF ADAPTABLE ADULT SPONGY BONE TISSUE

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Abstract: Two models of bone tissue internal remodeling are basically in use now, both of them include the balance equation of the bone matrix density, but the difference is in mechanical nature of the remodeling stimulus (strain tensor or strain energy density). Analysis of these models by means of their phenomenological analogues is carried out in present paper. The idea of taking the phenomenological approach to an adaptable elastic media consists in an obvious assumption that elastic modulus, stresses and strains are continuous and monotonous time functions during the adaptation process and have continuous derivatives. Phenomenological analogues of both models are nonlinear first degree differential operators of similar structure but differ by power of non-linearity in strain. The response of each model to the step change in axial load was investigated in the test problem of axial contraction of an elastic thick hollow cylinder made of an adaptable material. As a result of numerical solution and known experimental data that confirm the bone remodeling is just strain sensitive, preference is given to strain stimulus model.

Key words: internal remodeling, bone tissue, remodeling stimuli, phenomenological models

Introduction

The problem of mathematical description of internal remodeling of the bone tissue considered as a result of bone adaptability to changeable physiological loads is one of main problems of the bone tissue biomechanics. Its solution has important practical applications in the bone tissue orthopaedy and surgery, and makes possible to predict the evolution of different pathological processes (e.g. the femoral head necrosis, the bone resorption around implants), recovering processes of bone mechanical properties after a surgical intervention, etc.

Kinetic equations of adaptive remodeling known in present time have the same form but are based on two different assumptions about bone cell response to mechanical disturbance. S.C. Cowin and D.H. Hegedus [1] assumed that bone cell remodeling activity depends on the difference between the actual strain and the strain under normal physiological conditions in a normal bone at the same locations. The corresponding equation may be written in the following form:

$$\dot{\rho}(\mathbf{x},t) = C\left(\left\|\tilde{\varepsilon}(\mathbf{x},t)\right\| - \left\|\tilde{\varepsilon}_h(\mathbf{x})\right\| \right), \quad \mathbf{x} \in \Omega, \tag{1}$$

where ρ is a mean density of spongy bone tissue; $\tilde{\varepsilon}$, $\tilde{\varepsilon}_h$ are Cauchy strain tensors during the remodeling and at homeostatic equilibrium respectively; $\|.\|$ is an appropriate norm of strain tensor (strain intensity, for instance); C is a remodeling rate parameter, $\mathbf{x}$ is a bone particle site vector in a considered area Ω.

This assumption was verified by tests with a functionally isolated turkey ulna preparation [5], that showed high sensitivity of the adaptation process to strain distribution in the bone tissue.

The equation (1) describes such a process of bone tissue adaptation in which bone cell deformation (or some observable related to it) tries to keep its constant equilibrium value $\tilde{\varepsilon}_0$. However, it turns out that application of this equation for simulation of femur morphogenesis is impossible inasmuch as the value $\tilde{\varepsilon}_h$ in the beginning of life is not a constant because of predominant influence of growth deformations and genetic factors. This fact induced D.P. Fyhrie and D.R. Carter [2] to suppose that bone cells react on the deviation of actual strain energy density $u(\mathbf{x},t)$ away from its mean value over the whole area $\bar{u}(t)$.

Equation suggested by these authors

$$\dot{\rho} = C_1(u(\mathbf{x},t) - \bar{u}(t)) \tag{2}$$

allowed them to construct a model of morphogenesis in the proximal part of human femur that agrees with the Wolf law. However, we are not acquainted with any confirmation of the fact that bone cells react on such a mechanical stimulus as the density of strain energy.

Good results received by equation (2) do not mean that model (1) is biased as declared in [7], but show the difference of adaptive mechanisms during the morphogenesis and after it. Adaptation in an adult bone is a process of maintenance (or stabilization) of mechanical conditions of individual cells vital activity and may be described by the equation (1) or by the equation (2) in site dependent form

$$\dot{\rho}(\mathbf{x},t) = C_2(u(\mathbf{x},t) - u_h(\mathbf{x})), \tag{3}$$

where $u_h(\mathbf{x})$ is homeostatic distribution of strain energy density. The equation (3) was used for the first time with reference to femoral implant [6].

When the balance equations (1) or (3) are used in the problem of the stressed and strained conditions of bone, it is necessary to transform calculated density ρ to the elastic modulus of the bone E. The following relation is usually applied for this aim:

$$E = A\rho^3, \tag{4}$$

where the coefficient A depends on the structure parameters of bone tissue (shape and dimension of pores and bone beams), and hence it is a site dependent factor. The range of factor A values over the femoral head runs into 20% [3]. Consequently, the hypothesis about factor A constancy used in existing models of internal remodeling results in distortion of stress and strain fields. That’s why in present paper, likewise to [4], the equation (4) is excluded from the model of internal adaptation, and bone density in kinetic equations (1) and (3) is replaced with the elastic modulus:

$$\dot{E}(\mathbf{x},t) = C_3\left(\|\tilde{\varepsilon}(\mathbf{x},t)\| - \|\tilde{\varepsilon}_h(\mathbf{x})\|\right), \tag{5}$$

$$\dot{E}(\mathbf{x},t) = C_4(u(\mathbf{x},t) - u_h(\mathbf{x})). \tag{6}$$

It is interesting to compare models (5) and (6). Such a comparison is made from two points in present paper: by building up and analysing of their phenomenological analogues, and by numerical solution of test adaptability problem of dense bone tissue cylinder under step increasing of axial compressive load on it.

Methods

The idea of use the phenomenological approach to simulate the bone adaptation turns out to be successful because the structure and power of non-linearity of operator equations permits to make qualitative and quantitative comparative analysis of different models.

During the adaptive process, stresses and strains in the bone tissue and its mechanical parameters are continuously and monotonously varied. Hence, the equation of Hook low contains continuous time functions, and we can differentiate it with respect to time. In one-dimensional problem it may be written as

$$\dot{\sigma} = \dot{E}\varepsilon + E\dot{\varepsilon}, \tag{7}$$

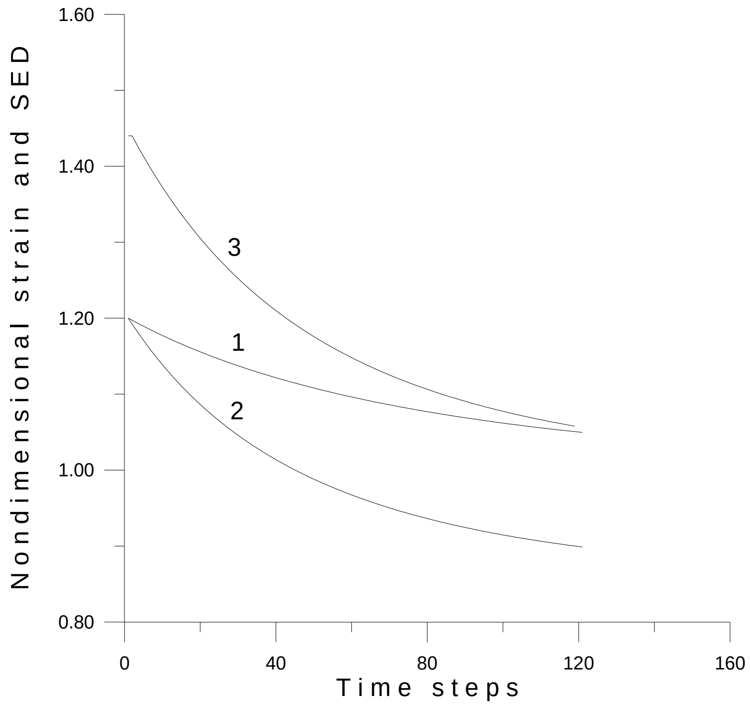

Fig. 1. Recovering of axial strain and strain energy density (SED) in thick hollow cylinder to their initial values: 1 - axial strain calculated by the phenomenological model (8);
2 - axial strain calculated by the phenomenological model (9);
3 - strain energy density calculated by the phenomenological model (9).

where σ, E, ε are stress, Young's modulus and strain respectively.

Substitution of relation (5) in the equation (7) gives us desired phenomenological analogue of this kinetic equation

$$\dot{\sigma} = C_3(\varepsilon - \varepsilon_h)\varepsilon + E\dot{\varepsilon}\,. \tag{8}$$

The phenomenological analogue of kinetic equation (6) is derived in the same way:

$$\dot{\sigma} = C_4(E\varepsilon^2 - u_g)\varepsilon + E\dot{\varepsilon}\,. \tag{9}$$

In order to compare models (5) and (6) quantitatively, the test problem about the adaptability of dense bone tissue cylinder is solved. The axial compressive load s on this hollow cylinder changes instantaneously from σ_0=2.22 MPa to σ_1=3.0 MPa. The Young modulus of the cylinder material E_0 = 15,000 MPa when $\sigma = \sigma_0$, and the Poisson's ratio ν = 0.3.

After the step change of load $\sigma = \sigma_1 = const$, therefore in equations (8) and (9) $\dot{\sigma} = 0$. We have linear differential equations with respect to $\varepsilon(t)$ in both cases.. For their numerical solution the Euler's scheme is used, with a constant time step $\Delta\, t$ equal to arbitrary unit of time. On the k–th time step the following non-dimensional equations are under solution:

$$\varepsilon_k = \varepsilon_{k-1} - E_{k-1}^{-1}(\varepsilon_{k-1} - 1)\varepsilon_{k-1}\Delta\tau, \quad k = 1,2\ldots, \tag{10}$$

$$\varepsilon_k = \varepsilon_{k-1} - E_{k-1}^{-1}\frac{C_4 u_h}{C_3 \varepsilon_h}(E_{k-1}\varepsilon_{k-1}{}^2 - 1)\varepsilon_{k-1}\Delta\tau, \quad k = 1,2\ldots, \tag{11}$$

where $E_0\left(C_3\varepsilon_h\right)^{-1}$ is a time scale factor; ε_h is a strain scale factor, and E_0 is Young's modulus scale factor. Initial conditions were the following: $E(0) = E_0$, $\sigma(0) = \sigma_0$, $\varepsilon(0) = \sigma_1 / E_0$. Homeostatic values of remodeling stimuli are determined as $\varepsilon_h = \sigma_0 / E_0$ and $u_h = \sigma_0^2 / 2E_0$.

Results and discussion

The response of the adaptable cylinder on instantaneous step change of compressive stress is shown on Fig. 1, two curves on it are corresponding with two adaptation models described by equations (8) and (9). These curves were calculated according to equations (10) and (11) with C_3 = C_4 = $6{\cdot}10^6$, $\Delta\tau$ =0.01135. They show that the remodeling processes in both cases run monotonously with equal rates. The relative strain, calculated by the equation (8), and the strain energy density reach their steady-state values, both equal to one. However, the relative strain, calculated by equation (9), reaches its steady-state value equal to $E^{-1/2}$, where E is the elastic modulus after the adaptation. Both models qualitatively right reflect the remodeling process but the preference is given to the relation (8) as it is more confirmed by experiments [5].

Thus, the results obtained in present paper show that proposed phenomenological approach allows to compare the influence of known mechanical stimuli on the remodeling process in an adaptable bone tissue with different kinetic equations of internal adaptation. As the deformational stimulus is experimentally confirmed, we may regard that equation (8) at simulation of bone tissue remodeling is more preferable than equation (9).

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Феноменологическая модель приспосабливающейся спонгиозной костной ткани взрослого человека

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В настоящее время в основном применяются две модели внутренней адаптации костных тканей, использующие уравнения баланса плотности костного матрикса и отличающиеся механической природой стимула (тензор деформации и плотность энергии деформации). В данной работе предлагается анализ обеих моделей с помощью их феноменологических аналогов. Идея применения феноменологического подхода к приспосабливающейся упругой среде состоит в очевидном предположении о том, что в процессе адаптации к новым нагрузкам модуль упругости, напряжения и деформации непрерывно монотонно изменяются и имеют непрерывные первые производные. Феноменологические аналоги обеих моделей являются нелинейными дифференциальными операторами первого порядка одинаковой структуры, однако отличаются порядком нелинейности по деформации. Реакция обеих моделей на ступенчатое изменение осевой нагрузки исследовалась на тестовой задаче об осевом сжатии упругого полого толстостенного цилиндра из приспосабливающегося материала. На основе результатов решения и известных экспериментальных данных, подтверждающих чувствительность адаптационной способности костной ткани к изменению именно деформаций, в работе отдается предпочтение модели с деформационным стимулом. Библ. 7.

Ключевые слова: внутренняя перестройка, костная ткань, перестроечные стимулы, феноменологические модели.

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