THREE-DIMENSIONAL NUMERICAL SIMULATION OF BLOOD FLOW THROUGH A MODELED ANEURYSM


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Abstract: The objective of this work was to simulate blood flow through a three-dimensional aneurysm and to study particularly the influence of expansion ratio and eccentricity of the aneurysm on wall shear stress and pressure distributions. The non-Newtonian behaviour of blood viscosity was considered (Casson model). Numerical simulations were performed with a finite element package FIDAP7.6. The results showed existence of local maxima of shear stress and pressure near the end of aneurysm. These maxima could be a mechanical factor involved in the rupture of aneurysm. It also should be pointed out that in the range of geometric parameters and flow rate considered, the non-Newtonian effects could be neglected.

Key words: aneurysm, blood, numerical computation, finite element method, non-Newtonian fluid, rupture

Introduction

It is well known that hemodynamic conditions and interactions blood-vessel-cells play a key role in understanding blood circulation. Beside classical hemodynamic parameters such as pressure and flow rate which describe globally blood flow, the effects of local mechanical forces on endothelial cells (EC) and blood cell adhesion are also very important [1-7], especially in some vascular pathological phenomena like atherosclerosis, thrombosis, inflammation, etc.

Blood flow is governed by blood constitutive equation, vessel geometry and mechanical properties, and flow state (laminar or turbulent, steady or unsteady). During the last three decades, a lot of studies have been carried out on blood flow through curved vessels, stenoses and bifurcations. It can be noted some pioneer works on morphology changes of ECs as function of shear rate [8], clinical observation of atherosclerotic plaque formation in disturbed flow area [9,10], analysis of the flow near a stenose at low Reynolds number by conform transformation method [11], experimental and numerical studies of flow through stenoses [12-23], and the consideration of non–Newtonian behavior of blood in flow computation [16, 17, 24-27].

If blood flow through stenoses has been studied in detail, there are only few reports on flow pattern in aneurysms. Some authors have visualized the stream lines in modeled abdominal aortical and cerebral aneurysms [28-31]. From these experimental studies, we know that the flow in an aneurysm depends largely on two parameters: the expansion ratio and the eccentricity of the aneurysm. It can be also noted that most of the numerical simulations are limited in cases of two-dimensional axisymmetric aneurysms. However, the real aneurysms are usually asymmetric and three dimensional (Fig.1). In the few published numerical analysis of three-dimensional aneurysms [33,34], it’s regrettable that the post aneurysm re-establishment of the flow has not been discussed, though this factor plays a key role in reliability of numerical simulations.
Fig. 1 Abdominal aortic aneurysm observed in clinic (courtesy of Dr. B. LEHALLE, CHRU-NANCY, France).

Fig. 2. Aneurysm model considered and mesh used. The post aneurysm section is long enough to have an established flow at exit. Definition of the coefficients of expansion $\beta$ and eccentricity $e$. 
The objective of this work was to simulate blood flow through a modeled three dimensional abdominal aneurysm in taking into account the non–Newtonian behavior of blood and to elucidate the influence of the expansion ratio and eccentricity of the aneurysm on flow. We also determined global or local mechanical parameters such as the distributions of pressure and shear stress on vessel wall, because these parameters could be involved in rupture of aneurysms [35,36].

**Aneurysm model and numerical method**

We considered a steady laminar flow in a cylindrical vessel which had a three-dimensional aneurysm, Fig.2. The transverse sections of the vessel were circles defined by the following equations:

\[
\begin{cases}
x^2 + y^2 = R^2, & 0 \leq z \leq 3D \\
x - \frac{eR}{2} \left( 1 - \cos \left( \frac{z - 3D}{2D} \pi \right) \right)^2 + y^2 = \left( R \left( 1 + \frac{\beta - 1}{2} \left( 1 - \cos \left( \frac{z - 3D}{2D} \pi \right) \right) \right) \right)^2, & 3D < z \leq 7D \\
x^2 + y^2 = R^2, & z > 7D
\end{cases}
\]

where \( D \) was the cylindrical vessel’s diameter, \( \beta \) the expansion ratio which was defined as the ratio of the maximum diameter of the aneurysm \( D_{an} \) over \( D \) (\( \beta = D_{an}/D \)), and \( e \) the coefficient of eccentricity defined as \( e = 2E/D \) (Fig.1). We remarked that there was a plane of symmetry which was \( y=0 \).

The blood was considered as an incompressible viscous non–Newtonian fluid of apparent viscosity \( \mu_a \). Thus the flow was governed by Navier-Stokes equation:

\[
\rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ 2\mu_a e_{ij} \right] \tag{2}
\]

and the equation of continuity:

\[
\frac{\partial u_i}{\partial x_i} = 0 \tag{3}
\]

where \( u_i (i=x,y,z) \) was the velocity vector, \( p \) the pressure, \( \rho \) the density of the fluid, \( e_{ij} \) the strain rate tensor defined as:

\[
e_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \tag{4}
\]

In the case of a viscous non–Newtonian fluid, \( \mu_a \) is a function of the second invariant \( J_2 \) of \( e_{ij} \):

\[
J_2 = \frac{1}{2} e_{ij} e_{ij}. \tag{5}
\]
There are in literature several models of blood constitutive equation. We chose to use the classical Casson relationship [37]:

\[ \mu_a = \left[ \mu_0^{1/2} J_2^{1/4} + \left( \tau_0 / 2 \right)^{1/2} \right]^{2} J_2^{-\frac{1}{2}} \]  

(6)

where \( \mu_0 = 4.08 \times 10^{-3} \text{ Pa.sec} \) and \( \tau_0 = 2.25 \times 10^{-3} \text{ Pa} \) [38].

It can be noted that following equation (6), the blood apparent viscosity approaches a constant \( \mu_0 \) when shear rate \( \dot{\gamma} = (J_2/2)^{1/2} \) increases infinitely, but to infinite when \( \dot{\gamma} \to 0 \) (\( J_2 \to 0 \)). In order to overcome a numerical problem of division by zero, we supposed that when \( \dot{\gamma} \) was lower than \( 10^{-4} \), \( \mu_a \) would be set to a constant of 25 Pa.sec. This technical operation caused only a small error which could be neglected compared with the criterion of convergence which was set to be \( 10^{-3} \) in our computation.

We also should remark that the classical Reynolds number was no longer useful in our case because the viscosity of the fluid was not a constant. We would use the average velocity \( U_d (U_d = 4Q/\pi D^2, \text{ where } Q \text{ was the flow rate}) \) as reference in order to compare results obtained for Newtonian and non–Newtonian models.

The above system of equations would be completed by the following boundary conditions:

\[ \begin{align*}
    u_x &= u_y = 0 \text{ and } u_z = \text{developed profile,} & \text{at the inlet (z=0)} \\
    u_x &= u_y = u_z = 0, & \text{on vessel wall} \\
    p &= p_0 = 0. & \text{at the outlet}
\end{align*} \]

(7)

If we have a Newtonian fluid (\( \mu_a = \text{Constant} \)), the developed velocity profile in a cylindrical vessel would be parabolic. But in the case of a Casson fluid, the theoretical profile [37] was imposed at the inlet of the vessel.

Based on the above conditions, the system of partial derivative equations was solved by finite element method with the help of a numerical package FIDAP7.66 (Fluid Dynamics International, Inc., Evanston, Illinois, USA) [39]. The fluid field was meshed by isoparametric elements of 9 nodes. The velocity was interpolated by bi-quadratic functions and the pressure was computed by penalty method: \( p = -\frac{1}{\epsilon} \nabla \cdot \vec{v} \) where the parameter \( \epsilon \) was set to be \( 10^{-6} \). All computations were performed on a work station SUN Ultra170.

**Results**

1. Post aneurysm flow re-establishment

We have supposed that the pressure in the outlet section was constant. This hypothesis could stand only when the flow was completely re-established in that section. So we studied at first the variation of the re-establishment length \( L_e \) in different flow conditions. However, this length could be defined at least in three ways: i. comparison of the maximum axial velocity in a section with that of a steady Poiseuille flow; ii. uniformity of pressure distribution; iii. uniformity of wall shear stress. That's why in this study we defined three criteria \( k_v, k_{\tau_w}, \) and \( k_p \) as following:
where $V_{\text{max}}$, $\tau_{w\text{max}}$ and $P_{\text{max}}$ were maximum axial velocity, maximum pressure and maximum wall shear stress in a section respectively, $P_{\text{min}}$ and $\tau_{w\text{min}}$ the minimum values, $\bar{V}_{\text{max}}$ the maximum axial velocity for a steady Poiseuille flow in a cylindrical vessel.

If we admit an error of 2% on flow re-establishment, we observed that with an average velocity $U_d$ and a given set of parameters ($\beta$, $e$), the criteria $k_v$ gave the smallest value of $L_e$, but $k_p$ gave the biggest value (Table 1). Because of the great differences between the criteria, we have chosen to use the most punishing criteria $k_p$ in our numerical computation.

### Table 1.
Comparison of the values of establishment length from the end of aneurysm according to different criteria ($U_d=0.2$ m/sec, $\beta=2$, $e=1$). It can be noted that $k_p$ gives the biggest value.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Newtonian</th>
<th>Casson</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_v$</td>
<td>$6.0D$</td>
<td>$5.5D$</td>
</tr>
<tr>
<td>$k_{\tau w}$</td>
<td>$9.5D$</td>
<td>$8.5D$</td>
</tr>
<tr>
<td>$k_p$</td>
<td>$10.5D$</td>
<td>$9.5D$</td>
</tr>
</tbody>
</table>

![Fig.3. Variation of the establishment length post aneurysm according to the criteria $k_p$ for Newtonian and Casson models.](image)
Fig. 3 shows the variation of $L_e$ as a function of the imposed average velocity $U_d$ with ($\beta=2$, $e=1$) for a Newtonian model ($\mu=4.08 \times 10^{-3}$ Pa.sec) and the Casson model. The relative difference between the two models was lower than 10%. This difference reached a maximum and then decreased when $U_d$ increased. This meant that under fast flow conditions, the Casson model would approach the Newtonian model. Fig. 4 represents the influence of eccentricity $e$ on the establishment length ($U_d=0.2$ m/sec). A maximum can be noted at $e=0.5$.

Fig. 5. Axial velocity profiles at different sections of the vessel for Casson model ($\beta=2$, $e=1$) with ($e=1$) for a Newtonian model ($\mu=4.08 \times 10^{-3}$ Pa.sec) and the Casson model. The relative difference between the two models was lower than 10%. This difference reached a maximum and then decreased when $U_d$ increased. This meant that under fast flow conditions, the Casson model would approach the Newtonian model. Fig. 4 represents the influence of eccentricity $e$ on the establishment length ($U_d=0.2$ m/sec). A maximum can be noted at $e=0.5$.

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coefficient e on \( L_e \) with \((U_d=0.2 \text{ m/s, } \beta=2)\) for the Newtonian model. It can be noted that \( e \) played an important role in flow re-establishment and that \( L_e \) had a maximum with \( e=0.5 \). Further, the variation of \( L_e \) could reach 50% with \( e \) ranging from 0 to 1.

2. **Evolution of velocity field**

Fig.5 shows the evolution of the axial velocity at different section of the vessel with \((U_d=0.2 \text{ m/s, } \beta=2, e=1)\) for Casson model. We found that flow was only disturbed in a short distance before the aneurysm \((z<3D)\). But at the center of the aneurysm \((z=5D)\), a recirculation zone appeared in the expanded area. The velocity profile approached progressively the theoretical profile for a Casson fluid in post aneurysm area. The re-establishment point was found at \( z=16.5D \). It is also important to remark that the maximum axial velocity (the main flow) showed a very limited variation (<4%) along with the vessel.

![Diagram](image)

Fig.6. Pressure distributions on two contours of the vessel C1 and C2 \((U_d=0.2 \text{m/sec, } \beta=2, e=1)\).

3. **Pressure distribution**

Fig.6 represents the evolution of the relative pressure \((p-p_0)\) along with two contours C1 and C2 of the vessel in the plane of symmetry with \((U_d=0.2 \text{ m/s, } \beta=2, e=1)\). It was to note that a local maximum pressure was found near the end of the aneurysm. This maximum corresponded to the point of flow reattachment (the end of the recirculation zone). Similar results were obtained for different values of the coefficients of expansion and eccentricity.

4. **Wall shear stress**

Fig.7 shows the distribution of wall shear stress \( \tau_w \) along with C1 and C2 at \((U_d=0.2m/s, \beta=2, e=1)\). We observed \( \tau_w \) was almost constant before the aneurysm \((0<z<3D)\).
But once the vessel expanded, $\tau_w$ decreased rapidly, passed zero, and became negative which meant a flow separation and the occurrence of a recirculation zone. In down stream area of the reattachment point ($z=6.65D$, the second point where $\tau_w=0$), the wall shear stress increased rapidly and reached a maximum $\tau_{\text{wmax}}$ ($z=7.0D$) at the end of the aneurysm. This maximum could be 1.5 times of that in cylindrical part of the vessel. The bigger the eccentricity coefficient was, the more important $\tau_{\text{wmax}}$ would be (Fig.8). We also remarked that similar results were obtained for both the Newtonian and Casson models.

![Graph](image)

**Discussion and Conclusions**

In this work, we have simulated numerically the blood flow through a modeled three dimensional aneurysm taking into account the non–Newtonian behavior of blood. In particular, the effects of the expansion ratio and eccentricity of the aneurysm on the distributions of pressure and wall shear stress have been studied. The numerical results
showed the existence of local maximum pressure and maximum shear stress in the aneurysm. These maxima are sensitive to aneurysm geometry. It was also observed that in the range of geometric and dynamic conditions considered, the effects of the non–Newtonian behavior of blood could be neglected.

This work was limited in the case of steady laminar flow with the expansion ratio ranging from 0 to 1 and the coefficient of eccentricity variable from 1 to 2. It will be interesting to consider in the future other geometry coupled eventually with a pulsatile flow. The used aneurysm was an idealized model (rigid smooth sinusoidal envelop), but aneurysms observed clinically presented usually a much more complicated geometry [30,35,36]. That’s why we suggest to develop a numerical simulation with real geometry (reconstructed 3D vessel by magnetic resonance imaging MRI) which could be helpful in surgical diagnostics. Further, an anatomic study will allow us to verify if the numerically predicted maximum pressure and shear stress area are linked to aneurysm rupture.

References

Аневризма есть расширение кровеносного сосуда на отдельных участках вследствие заболеваний стенок сосуда (склероз, воспаление и др.) или их повреждений, так как ослабленное место не в состоянии долго противостоять давлению крови и подвергается растяжению. Известно, что величина аневризмы может быть от размеров зерна до размеров головы взрослого человека. Чаще всего встречается аневризма аорты, развивающаяся на почве атеросклероза. Большое значение имеет построение математических моделей, позволяющих сделать прогноз разрушения аорты. Цель...
данной работы: построить модель течения крови через трехмерную аневризму аорты и изучить влияние формы и размеров аневризмы на сдвиговые и нормальные напряжения на стенке сосуда. Для моделирования применялся конечноэлементный пакет FIDAP7.6. Свойства жидкости описывались определяющими соотношениями ньютоновской жидкости (модель Кассона). Результаты расчета показывают появление локальных максимумов напряжений, которые могут быть механической причиной разрушения аорты. Анализируется зависимость результатов от геометрических параметров аневризмы и вязкости крови. Библ. 39.

Ключевые слова: аневризма, кровь, численное моделирование, метод конечных элементов, ньютоновская жидкость, разрушение

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